Compitino 15/11/25

Esercizio 1

(a) r per $P=(1,1,1)$ e $Q=(0,1,0)$.

Vettore direttore $\vec{v} = \overrightarrow{QP} = (1,0,1)$

Eq. parametrica di r: $(1,1,1)+\lambda(1,0,1)$

$$\begin{cases} x = 1+\lambda \\ y = 1 \\ z = 1+\lambda \end{cases} \iff \begin{cases} x - z = 0 \\ y = 1. \end{cases}$$

$$s : \begin{cases} x+y=1 \\ x-y-z=0 \end{cases} \Rightarrow (1,0,1), (0,1,-1) \in s$$

$\Rightarrow$ vettore direttore di s è $(1,-1,2)$

$s : (1,0,1)+\mu(1,-1,2)$.

$$s : \begin{cases} x = 1+\mu \\ y = -\mu \\ z = 1+2\mu \end{cases}$$

Eq. parametriche di π

$$(1,0,1) + \lambda(1,0,1) + \mu(1,-1,2)$$

$$\begin{cases} x = 1 + \lambda + \mu \\ y = -\mu \\ z = 1 + \lambda + 2\mu \end{cases} \qquad \begin{cases} x = 1 + \lambda - y \\ \mu = -y \\ z = 1 + \lambda - 2y \end{cases}$$

$$\begin{cases} \mu = -y \\ \lambda = z + 2y - 1 \\ x = 1 + z + 2y - 1 - y \end{cases}$$

$$\pi : \quad x - y - z = 0$$

(b) r ed s sono sghembe: non sono parallele (i loro vettori direttori non sono proporzionali). Inoltre:

$$r \cap s : \begin{cases} x - z = 0 & \Rightarrow \quad z = 0 \\ y = 1 & \\ x + y = 1 & \Rightarrow \quad x = 0 \\ x - y - z = 0 & \end{cases}$$

$$0 - 1 + 0 \neq 0.$$

Distanza: Impongo che $\vec{RS} \perp$ rette direttori:

$R = (1+\lambda, 1, 1+\lambda)$

$S = (1+\mu, -\mu, 1+2\mu)$

$\vec{RS} = (\mu-\lambda, -\mu-1, 2\mu-\lambda)$.

$$\vec{RS} \cdot (1,0,1) = 0 \qquad \Rightarrow \begin{cases} \mu-\lambda+2\mu-\lambda = 0 \\ \mu-\lambda+\mu+1+4\mu-2\lambda = 0 \end{cases}$$
$$\vec{RS} \cdot (1,-1,2) = 0$$

$$\begin{cases} 3\mu = 2\lambda \Rightarrow \lambda = \frac{3}{2}\mu \\ 6\mu - 3\lambda + 1 = 0 \Rightarrow 6\mu - \frac{9}{2}\mu + 1 = 0 \end{cases}$$

$$\begin{cases} \frac{12-9}{2}\mu = -1 \Rightarrow \frac{3}{2}\mu = -1 \Rightarrow \mu = -\frac{2}{3} \\ \lambda = \frac{3}{2}\mu \Rightarrow \lambda = -1 \end{cases}$$

$R = (0, 1, 0)$

$S = (1/3, 2/3, -1/3)$

$\vec{RS} = (1/3, -1/3, -1/3)$

$d = \sqrt{1/9 + 1/9 + 1/9} = \sqrt{1/3} = \sqrt{3}/3.$

Es. 2

(a) $\lim_{n} \left(\sqrt{n^2+5}-\sqrt{n^2-2n+8}\right) =$

$$= \lim_{n} \left(\sqrt{n^2+5}-\sqrt{n^2-2n+8}\right) \cdot \frac{\sqrt{n^2+5}+\sqrt{n^2-2n+8}}{\sqrt{n^2+5}+\sqrt{n^2-2n+8}} =$$

$$= \lim_{n} \frac{n^2+5-(n^2+2n+8)}{\sqrt{n^2+5}+\sqrt{n^2-2n+8}} = \lim_{n} \frac{-2n-3}{\sqrt{n^2+5}+\sqrt{n^2-2n+8}}$$

$$= \lim_{n} \frac{-\cancel{n}(2+3/n)}{\cancel{n}\left(\sqrt{1+5/n^2}+\sqrt{1-2/n+8/n^2}\right.} =$$

$$= \lim_{n} \frac{-(2+3/n)}{\sqrt{1+5/n^2}+\sqrt{1-2/n+8/n^2}} = -1.$$

(b)

$$\lim_{n} \frac{\sqrt[3]{n^2-n+5}-\ln(n^5+2)}{n+2} =$$

$$= \lim_{n} \frac{\sqrt[3]{n^2}\left(\sqrt[3]{1-1/n+5/n^2}-\frac{\ln(n^5+2)}{\sqrt[3]{n^2}}\right)}{n(1+2/n)} =$$

$$= \lim_n \frac{\sqrt[3]{1 - 1/n + 5/n^2} - \frac{\ln(n^5+2)}{\sqrt[3]{n^2}}}{\sqrt[3]{n}\,(1 + 2/n)} = 0.$$

Es. 3

(a) $$\sum_{n=1}^{\infty} \frac{2\sqrt{n^3+5}}{\ln(n) + \sqrt{n^3-n}}$$

$$\lim_n \frac{2\sqrt{n^3+5}}{\ln(n) + \sqrt{n^3-n}} = \lim_n \frac{2\sqrt{n^3}\left(\sqrt{1+5/n^3}\right)}{\sqrt{n^3}\left(\frac{\ln(n)}{\sqrt{n^3}} + \sqrt{1-1/n^2}\right)}$$

$$= 2 \lim_n \left(\frac{\sqrt{1+5/n^3}}{\frac{\ln(n)}{\sqrt{n^3}} + \sqrt{1-1/n^2}} \right) = 2 \neq 0,$$ quindi non converge

(b) $$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{n^2+2n+3}.$$

Leibnitz: decrescente

$$\frac{n+1}{(n+1)^2+2(n+1)+3} \leq \frac{n}{n^2+2n+3}$$

$$(n+1)(n^2+2n+3) \leq n((n+1)^2+2(n+1)+3)$$

$$n^3+2n^2+3n+n^2+2n+3 \leq n(n^2+2n+1+2n+2+3)$$

$$\cancel{n^3}+3n^2+5n+3 \leq \cancel{n^3}+4n^2+6n$$

$$-n^2-n+3 \leq 0$$

$$n^2+n-3 \geq 0, \quad \forall n \geq 2.$$

Infinitesima:

$$\lim_n \frac{n}{n^2+2n+3} = \lim_n \frac{1}{n+2+3/n} = 0.$$

(c) $$\sum_{n=0}^{\infty} \frac{2^{n+2}}{(n+1)!}$$

Rapporto:

$$\lim_n \left(\frac{2^{n+1+2}}{(n+2)!} \cdot \frac{(n+1)!}{2^{n+2}}\right) = \lim_n \left(\frac{2^{n+3}}{2^{n+2}} \cdot \frac{(n+1)!}{(n+2)(n+1)!}\right) =$$

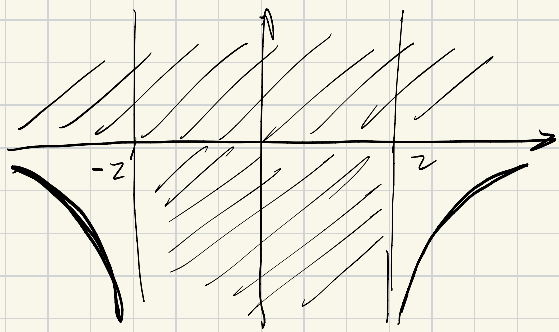
$$= \lim_n \left(\frac{2}{n+2}\right) = 0. \text{ Quindi converge}$$

Es. 4

$$\sum_{n=2}^{\infty}\left(\frac{1}{5}\right)^n = \frac{1}{1-\frac{1}{5}} - \left(1+\frac{1}{5}\right) = \frac{1}{\frac{4}{5}} - \left(\frac{6}{5}\right) = \frac{5}{4} - \frac{6}{5} = \frac{25-24}{20} = \frac{1}{20}.$$

Es. 5

(a) $\ln\left(\frac{x^2-4}{x^2}\right)$



Dominio: $x \neq 0$, $x^2-4>0 \Rightarrow x<-2, x>2$.

Simmetrie: Funzione pari

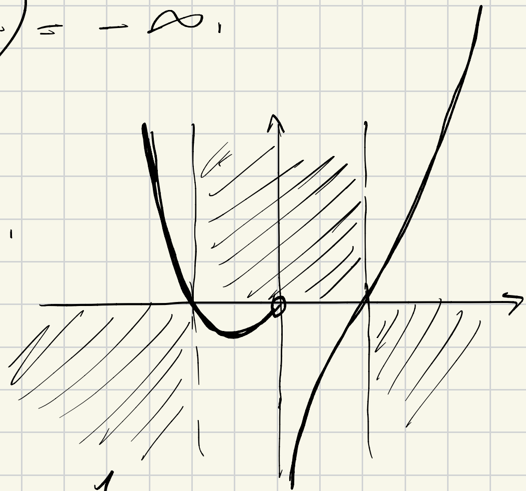
Segno: $f(x) \geq 0 \Leftrightarrow \frac{x^2-4}{x^2} \geq 1 \Leftrightarrow$

$$\Leftrightarrow x^2-4 \geq x^2$$

mai, quindi $f(x) \leq 0$, $\forall x$.

A.O. $\lim_{x\to\pm\infty} \ln\left(\frac{x^2-4}{x^2}\right) = \ln(1) = 0.$

A.V. $\lim_{x\to 2^+} \ln\left(\frac{x^2-4}{x^2}\right) = -\infty.$



(b) $f(x) = (x^2-1)e^{\frac{1}{x}}.$

Dominio: $x \neq 0.$

Simmetrie: nessuna.

Segno: $f(x) \geq 0 \iff (x^2-1)e^{\frac{1}{x}} \geq 0 \iff x \geq 1,\ x \leq -1.$

A.O. $\lim_{x\to+\infty} (x^2-1)e^{\frac{1}{x}} = +\infty$

$\lim_{x\to-\infty} (x^2-1)e^{\frac{1}{x}} = -\infty$ $\qquad \lim_{x\to-\infty} e^{\frac{1}{x}} = 1$

A.V. $\lim_{x\to0^+} (x^2-1)e^{\frac{1}{x}} = -\infty$ $\qquad \lim_{x\to0^+} e^{\frac{1}{x}} = +\infty$

$\lim_{x\to0^-} (x^2-1)e^{\frac{1}{x}} = 0$ $\qquad \lim_{x\to0^-} e^{\frac{1}{x}} = 0.$