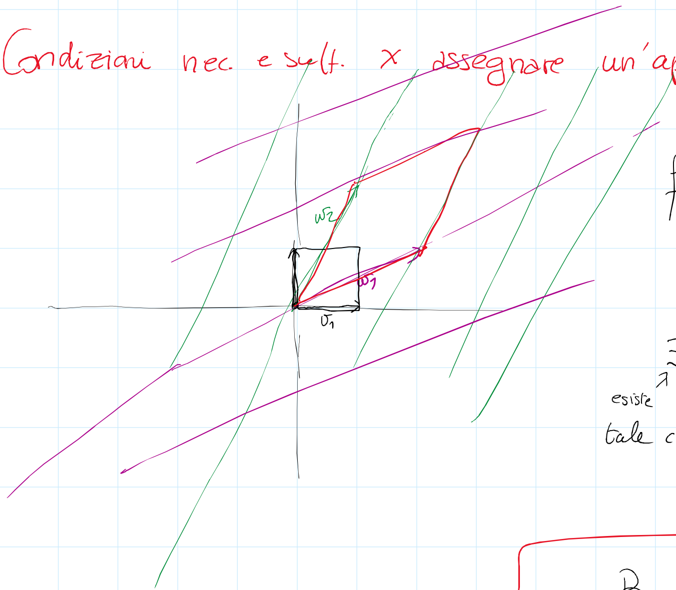
Condizioni nec. e suff. x assegnare un'applicazione lineare



$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$B_V = \left\{ v_1 = \begin{pmatrix}1\\0\end{pmatrix}, v_2 = \begin{pmatrix}0\\1\end{pmatrix} \right\}$

$w_1, w_2 \in \mathbb{R}^2$

$\exists!$ $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ lineare

esiste unica

tale che $\begin{cases} f(v_1) = w_1 \\ f(v_2) = w_2 \end{cases}$

$B_V = \{v_1, \dots, v_n\}$ base di V

$w_1, \dots, w_n$ in W

$f(v_i) = w_i \quad \forall i = 1, \dots, n$

$$f\left(\sum_{i=1}^n a_i v_i\right) = \sum_{i=1}^n a_i w_i$$

$f: V \longrightarrow W$

Applicazioni:

1) $f: \underset{V}{\mathbb{R}^2} \longrightarrow \underset{W}{\mathbb{R}^2}$ $\qquad B_V = \left\{ \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} \right\}$

$$\boxed{f\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} \qquad f\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}}$$

L'unica applicazione lineare è $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1 & -1\\2 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x-y\\2x+y\end{pmatrix}$

SOLO PER LA BASE CANONICA $f(e_1)$ $f(e_2)$

2) Cerchiamo $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ lineare tale che:

$f\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\3\end{pmatrix} \qquad f\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}1\\5\end{pmatrix} \qquad B_{\mathbb{R}^2} = \left\{ \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix} \right\}$

$f(v_1) = w_1 \qquad f(v_2) = w_2$ $\qquad$ è base di $\mathbb{R}^2$ perché lin. indip.

Si mettono in righe i vettori $v_1, \dots, v_n$

$$\left(\begin{array}{c|c} v_1 & f(v_1) \\ v_2 & f(v_2) \end{array}\right)$$

Riduciamo con Gauss $\left(\begin{array}{cc|c} 1 & 0 & f(e_1) \\ 0 & 1 & f(e_2) \end{array}\right) = \left(\begin{array}{c|c} e_1 & f(e_1) \\ e_2 & f(e_2) \end{array}\right)$

$$\left(\begin{array}{cc|cc} 1 & 1 & 0 & 3 \\ 2 & 1 & 1 & 5 \end{array}\right) \xrightarrow[2^\circ R - 2 \cdot 1^\circ R]{} \left(\begin{array}{cc|cc} 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & -1 \end{array}\right) \longrightarrow$$

$$\left(\begin{array}{c|c} v_1 & f(v_1) \\ v_2 & f(v_2) \end{array}\right) \longrightarrow \left(\begin{array}{c|c} v_1 & f(v_1) \\ v_2 - 2v_1 & f(v_2) - 2f(v_1) \end{array}\right) = \left(\begin{array}{c|c} v_1 & f(v_1) \\ v_2 - 2v_1 & f(v_2 - 2v_1) \end{array}\right)$$

$$f(v_2) - 2f(v_1) = f(v_2 - 2v_1)$$

$$\xrightarrow[-1\cdot 2^\circ R]{1^\circ R + 2^\circ R} \left(\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right) = \left(\begin{array}{c|c} e_1 & \boxed{f(e_1)} \\ e_2 & \boxed{f(e_2)} \end{array}\right) \qquad \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = A$$

$$\left(\begin{array}{c|c} v_2 - v_1 = e_1 & f(v_2 - v_1) = f(e_1) \\ e_2 & f(e_2) \end{array}\right) \qquad f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$f\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ 2x + y \end{pmatrix}$$

Esercizio: determinare tutte le applicazioni lineari $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ tali che

$$f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

Svolg.

$$B_{\mathbb{R}^2} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \qquad f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad a, b, c \in \mathbb{R}$$

$$A = \begin{pmatrix} 2 & a \\ 1 & b \\ 5 & c \end{pmatrix} \qquad f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \qquad f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x + ay \\ x + by \\ 5x + cy \end{pmatrix}$$

$$f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

Es: determinare tutte le applicazioni lineari $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ tali che

$$\boxed{f\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}} \qquad f(v_1) = w_1 \qquad v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad B_{\mathbb{R}^2} = \left\{ \underset{v_1}{\begin{pmatrix} 1 \\ 3 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right\}$$

$$\boxed{f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}}$$

e_2 non è multiplo di $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\Rightarrow$ è base.

$$\left(\begin{array}{c|c} v_1 & w_1 \\ v_2 & w_2 \end{array}\right) \longrightarrow \left(\begin{array}{c|c} e_1 & f(e_1) \\ e_2 & f(e_2) \end{array}\right)$$

$$\left(\begin{array}{cc|ccc} 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & a & b & c \end{array}\right) \xrightarrow{1^\circ R - 3\cdot 2^\circ R} \left(\begin{array}{cc|ccc} 1 & 0 & -3a & -3b & -3c \\ 0 & 1 & a & b & c \end{array}\right)$$

$$A = \begin{pmatrix} -3a & a \\ -3b & b \\ -3c & c \end{pmatrix} \qquad f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \qquad A \in M_{3,2}(\mathbb{R})$$

$f(e_1)$ $f(e_2)$

$$\boxed{f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3ax + ay \\ -3bx + by \\ -3cx + cy \end{pmatrix}}$$

Calcolare $\dim \operatorname{Im} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}$ $\quad B_{\operatorname{Im} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}}$ $\quad \operatorname{Im} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}} = \left\langle \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rangle$

$\dim \operatorname{Ker} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}$ $\quad B_{\operatorname{Ker} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}}$ $\quad \begin{pmatrix} 1 \\ 3 \end{pmatrix} \in \operatorname{Ker} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}$ perché il testo imponeva $f\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Se $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad \dim \operatorname{Im} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}} = 1$ $\quad B_{\operatorname{Im} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}} = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\}$

$\dim \operatorname{Ker} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}} = 2 - 1 = 1$ $\quad B_{\operatorname{Ker} f_{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}} = \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$

Se $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad \dim \operatorname{Im} f_{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}} = 0$ $\quad B_{\operatorname{Im} f_{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}} = \emptyset$

$\dim \operatorname{Ker} f_{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}} = 2 - 0 = 2 \Rightarrow \operatorname{Ker} f_{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}} = \mathbb{R}^2 \Rightarrow B_{\mathbb{R}^2} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

Esercizio: esiste un'applicazione lineare $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ tale che

$$\boxed{f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}} \quad \boxed{f\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}} \quad ?$$

$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad w_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad w_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \qquad f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \\ y \end{pmatrix}$$

$$f\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$f\begin{pmatrix} 1 \\ 1 \end{pmatrix} = f\begin{pmatrix} 1 \\ 0 \end{pmatrix} + f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$\left(\begin{array}{c|c} v_1 & f(v_1) \\ v_2 & f(v_2) \\ v_3 & f(v_3) \end{array}\right)$$

$$\left(\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 3 \end{array}\right) \xrightarrow{3^\circ - 1^\circ} \left(\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 3 \end{array}\right) \rightarrow$$

$$\left(\begin{array}{c|c} e_1 & f(e_1) \\ e_2 & f(e_2) \\ 0\,0 & f(00) \end{array}\right)$$

$f(00) = (0\,0\,0)$ ESISTE

$$\xrightarrow{3^\circ - 2^\circ} \left(\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & \boxed{1 & -1 & 2} \end{array}\right)$$

NON è la riga nulla $\Rightarrow$ NON esiste

Esercizio: determinare i valori del parametro $h \in \mathbb{R}$ tale che esista un'applicazione lineare $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ tale che

$$f\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} \qquad f(v_1) = w_1$$

$$f\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad f(v_2) = w_2$$

$$f\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}2\\0\\h\end{pmatrix} \qquad f(v_3) = w_3$$

$$\left(\begin{array}{c|c} v_1 & w_1 \\ v_2 & w_2 \\ v_3 & w_3 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & f(e_1) \\ 0 & 1 & f(e_2) \\ 0 & 0 & f(00) \end{array}\right)$$

$0\,0\,0$

$$\begin{array}{c} 1^\circ \\ 2^\circ \\ 3^\circ \end{array} \left(\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 2 & 0 & h \end{array}\right) \quad \begin{array}{c} \frac{1}{2}\cdot 3^\circ \\ 2^\circ \\ 1^\circ \end{array} \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & h/2 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{3^\circ - 1^\circ} \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & h/2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1-\frac{h}{2} \end{array}\right) \rightarrow$$

$$\xrightarrow{3^\circ - 2^\circ} \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & h/2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\frac{h}{2} \end{array}\right)$$

esiste se e solo se $(0\ 0\ -\frac{h}{2}) = (0\ 0\ 0)$

solo per $h = 0$ $\qquad -\frac{h}{2} = 0$

$$f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\0\\y\end{pmatrix}$$

Esercizio 2. Si consideri l'endomorfismo f di $\mathbb{R}^3$ tale che:

$$f\begin{pmatrix}2\\1\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}, \quad f\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}-2\\-2\\0\end{pmatrix}, \quad f\begin{pmatrix}3\\1\\4\end{pmatrix} = \begin{pmatrix}-2\\0\\-2\end{pmatrix}.$$

1. Si determini una base di Ker(f) e una base di Im(f). L'applicazione lineare f è iniettiva; NO è suriettiva; NO è biiettiva; NO Si determini inoltre la matrice A associata ad f rispetto alla base canonica di $\mathbb{R}^3$ (sia nel dominio che nel codominio). (3 pti)
2. Si calcoli $f^{-1}\left\{\begin{pmatrix}2\\2\\0\end{pmatrix}\right\}$ e $f^{-1}\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$. (2 pti)

$$f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \quad f\overset{v_1}{\begin{pmatrix}2\\1\\3\end{pmatrix}} = \overset{w_1}{\begin{pmatrix}0\\0\\0\end{pmatrix}} \quad f\overset{v_2}{\begin{pmatrix}1\\1\\0\end{pmatrix}} = \overset{w_2}{\begin{pmatrix}-2\\-2\\0\end{pmatrix}} \quad f\overset{v_3}{\begin{pmatrix}3\\1\\4\end{pmatrix}} = \overset{w_3}{\begin{pmatrix}-2\\0\\-2\end{pmatrix}}$$

$\{v_1, v_2, v_3\} = B_v$ è base di $\mathbb{R}^3$

$$\left(\begin{array}{c|c} v_1 & w_1 \\ v_2 & w_2 \\ v_3 & w_3 \end{array}\right) \longrightarrow \left(\begin{array}{c|c} e_1 & f(e_1)^t \\ e_2 & f(e_2)^t \\ e_3 & f(e_3)^t \end{array}\right) \qquad \left(f(e_1)\ f(e_2)\ f(e_3)\right) = A$$

$$\mathrm{Im} f = \langle f(v_1), f(v_2), f(v_3) \rangle = \left\langle \begin{pmatrix} 0\\0\\0 \end{pmatrix}, \begin{pmatrix} -2\\-2\\0 \end{pmatrix}, \begin{pmatrix} -2\\0\\-2 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} 1\\1\\0 \end{pmatrix}, \begin{pmatrix} 1\\0\\1 \end{pmatrix} \right\rangle \begin{pmatrix} 1\\0\\0 \end{pmatrix}$$

$$B_{\mathrm{Im} f} = \left\{ \begin{pmatrix} 1\\1\\0 \end{pmatrix}, \begin{pmatrix} 1\\0\\1 \end{pmatrix} \right\} \quad \dim \mathrm{Im} f = 2 \qquad \dim \mathrm{Ker} f = \dim V - \dim \mathrm{Im} f = 3 - 2 = 1$$

$$\mathrm{Ker} f = \left\langle \begin{pmatrix} 2\\1\\3 \end{pmatrix} \right\rangle \qquad B_{\mathrm{Ker} f} = \left\{ \begin{pmatrix} 2\\1\\3 \end{pmatrix} \right\}$$

$f^{-1}\left\{ \underbrace{\begin{pmatrix} 2\\2\\0 \end{pmatrix}}_{w} \right\}$ Notiamo che $f\begin{pmatrix} 1\\1\\0 \end{pmatrix} = \begin{pmatrix} -2\\-2\\0 \end{pmatrix}$ quindi $w \in \mathrm{Im} f = \left\langle \begin{pmatrix} 1\\1\\0 \end{pmatrix}, \begin{pmatrix} 1\\0\\1 \end{pmatrix} \right\rangle$

$$f\underbrace{\begin{pmatrix} -1\\-1\\0 \end{pmatrix}}_{v_0} = \begin{pmatrix} 2\\2\\0 \end{pmatrix}$$

$$f^{-1}\{w\} = v_0 + \mathrm{Ker} f = \begin{pmatrix} -1\\-1\\0 \end{pmatrix} + \left\langle \begin{pmatrix} 2\\1\\3 \end{pmatrix} \right\rangle$$

$\begin{pmatrix} 1\\0\\0 \end{pmatrix} \notin \mathrm{Im} f = \left\langle \begin{pmatrix} 1\\1\\0 \end{pmatrix}, \begin{pmatrix} 1\\0\\1 \end{pmatrix} \right\rangle$ quindi $f^{-1}\left\{ \begin{pmatrix} 1\\0\\0 \end{pmatrix} \right\} = \emptyset.$

Come associare una matrice ad un'applicazione lineare.

Ingredienti

1. $f: V \longrightarrow W$ applicazione lineare
2. $B_V = \{v_1, \dots, v_n\}$ base di V
3. $B_W = \{w_1, \dots, w_m\}$ base di W.

$\Rightarrow$ $A \in M_{m,n}(\mathbb{R})$ matrice associata a f rispetto alle basi B_V e B_W.

$$A_{f, B_V, B_W} = \begin{pmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{pmatrix}$$

Metodo: Prendere il primo vettore di B_V v_1

calcoliamo
$$f(v_1) = a_{11} w_1 + a_{21} w_2 + \dots + a_{m1} w_m$$
$$f(v_2) = a_{12} w_1 + a_{22} w_2 + \dots + a_{m2} w_m$$
$$\vdots$$
$$f(v_n) = a_{1n} w_1 + a_{2n} w_2 + \dots + a_{mn} w_m$$

Esempi:

$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ ① $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x-y \\ x+y \\ x \end{pmatrix}$ $\begin{matrix} x=1 \\ y=0 \end{matrix}$ $\begin{matrix} x=0 \\ y=1 \end{matrix}$ $A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$

② $B_{\mathbb{R}^2} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

③ $B_{\mathbb{R}^3} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} = A$

$f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = -1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Esercizio: ① $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x-y \\ x+y \\ x \end{pmatrix}$ $\begin{matrix} x=0 \\ y=2 \end{matrix}$ $\begin{matrix} x=1 \\ y=0 \end{matrix}$

② $B_{\mathbb{R}^2} = \left\{ \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$ $\begin{pmatrix} -2 & 2 \\ 2 & 1 \\ 0 & 1 \end{pmatrix} = B$

③ $B_{\mathbb{R}^3} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ $\begin{pmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} = A$

$f(v_1) = f\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$f(v_2) = f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Esempio: ① $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x-y \\ x+y \\ x \end{pmatrix}$

② $B_{\mathbb{R}^2} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ $A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & -1 \end{pmatrix} = C$

③ $B_{\mathbb{R}^3} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$ $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$

$$f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \underbrace{a_{11}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + a_{21}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + a_{31}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{1\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} a_{11}\cdot 0 + a_{21} 0 + a_{31} 1 \\ a_{21} \\ a_{11} \end{pmatrix} = \begin{pmatrix} a_{31} \\ a_{21} \\ a_{11} \end{pmatrix}$$

$$f\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-1\\1\\0\end{pmatrix}=0\begin{pmatrix}0\\0\\1\end{pmatrix}+1\begin{pmatrix}0\\1\\\alpha\end{pmatrix}+(-1)\begin{pmatrix}1\\0\\\alpha\end{pmatrix}=$$

$$=a_{12}w_1+a_{22}w_2+a_{32}w_3$$

$f: \mathbb{R}^2 \to \mathbb{R}^3$ con $\dim \mathrm{Im} f = 2$ $\exists$ sempre $B_{\mathbb{R}^2}$ e $B_{\mathbb{R}^3}$

$$\begin{pmatrix}1&0\\0&1\\0&0\end{pmatrix}$$