FOGLIO 2

ESERCIZIO 1 : generatori

prendo un vettore qualsiasi di $\mathbb{R}^2$: (α, β) $\alpha, \beta \in \mathbb{R}$

$\Rightarrow$ dimostro che $\exists \lambda_1, \lambda_2 \in \mathbb{R}$ t.c. $(\alpha, \beta) = \lambda_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad \left(\begin{array}{cc|c} 1 & -1 & \alpha \\ 2 & 1 & \beta \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & -1 & \alpha \\ 0 & 3 & \beta - 2\alpha \end{array}\right)$$

$$\begin{cases} \lambda_1 = \alpha + \lambda_2 = (\beta + \alpha)/3 \\ \lambda_2 = (\beta - 2\alpha)/3 \end{cases}$$

il sistema ammette soluzioni

$rk A = rk A|b = rk MAX$

$\Rightarrow \exists \lambda_1, \lambda_2$ sol. unica

$\Downarrow$

sono l.i

$\Rightarrow \mathbb{R}^2 = \left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\rangle \rightarrow$ insieme di generatori

ESERCIZIO 2 criterio sottospazio

sottospazi vettoriali

1) $\exists 0_V$ el neutro
2) $\forall \lambda \in \mathbb{R}, \forall u \in U \Rightarrow \lambda u \in U$
3) $\forall u_1, u_2 \in U \Rightarrow u_1 + u_2 \in U$

a) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x^2 + y^2 = z \right\}$

1) $\exists 0_V = (0,0,0)$: $0^2 + 0^2 = 0$

2) $\lambda = 2 \in \mathbb{R}$ $u = (2, 2, 8) \in U$

$\lambda \cdot u \in U$? $\rightarrow (4, 4, 16)$ $4^2 + 4^2 = 16$ NO!

$\Rightarrow U$ no sottospazio di $\mathbb{R}^3$ (no chiuso rispetto a uno scalare!)

NB. un sottoinsieme difficilmente sarà sottospazio se le equazioni non sono lineari o se appare un termine noto

e) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + y + z + 1 = 0 \right\}$

1) $\exists 0_V = (0,0,0)$? NO! $0 + 0 + 0 + 1 = 0$ NO!

U non é sottospazio di $\mathbb{R}^3$

b) $U = \{ (x, y, z) \in \mathbb{R}^3 \mid |x| = |y| \}$

1) $\exists\, 0_V = (0, 0, 0)$: $|0| = |0|$

2) $\lambda(x, y, z) \in U$? $|\lambda x| = |\lambda y| \to |\lambda||x| = |\lambda||y|$

3) $u_1 = (x_1, y_1, z_1) \in U$ es $(1, 1, 0) = (x, x, z_1) \Rightarrow |x| = |x|$

$u_2 = (x_2, y_2, z_2) \in U$ es $(2, -2, 0) = (x, -x, z_2) \Rightarrow |x| = |-x|$

$u_1 + u_2 = (3, -1, 0) \notin U$! No sottospazio

$\overset{!}{=} (x + x, x - x, z_1 + z_2) \to |2x| = |0|$ NO!

c) $U = \{ (x, y, z) \in \mathbb{R}^3 \mid z = x + y \}$

1) $\exists\, 0_V$

2) $\lambda(x, y, z) \in U$ $\lambda z = \lambda x + \lambda y$

3) $u_1 + u_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2) \in U$

$z_1 + z_2 = x_1 + x_2 + y_1 + y_2$

U è sottospazio di $\mathbb{R}^3$

d) $U = \{ (x, y, z) \in \mathbb{R}^3 \mid xy + yz = 0 \}$

1) $\exists\, 0_V$

2) $\lambda(x, y, z) = (\lambda x, \lambda y, \lambda z) \in U$? $\lambda^2 xy + \lambda^2 yz = \lambda^2(xy + yz) = 0$ ✓

3) $u_1 + u_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$(x_1 + x_2)(y_1 + y_2) + (y_1 + y_2)(z_1 + z_2) \overset{?}{=} 0$

$\underbrace{x_1 y_1 + y_1 z_1}_{=0} + \underbrace{x_2 y_2 + y_2 z_2}_{=0} + x_1 y_2 + x_2 y_1 + y_1 z_2 + y_2 z_1 \neq 0$

es $u_1 = (7, 0, 11) \in U$ $7 \cdot 0 + 0 \cdot 11 = 0$ ✓

$u_2 = (1, 2, -1) \in U$ $1 \cdot 2 + 2(-1) = 0$ ✓

$u_1 + u_2 = (8, 2, 10)$ $8 \cdot 2 + 2 \cdot 10 \neq 0$

U no sottospazio di $\mathbb{R}^3$

ESERCIZIO 3 : somma, intersezione

a) $V + W = \left\langle \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \begin{pmatrix}0\\1\\0\\1\end{pmatrix} \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \right\rangle$ $\qquad V+W = \{v+w \mid v \in V, w \in W\}$

$V \cap W = \{ u \in \mathbb{R}^4 \mid u \in V \wedge u \in W \}$ $\qquad u = (x, y, z, t)$

$$\begin{cases} u = \alpha(1,0,0,1) + \beta(0,1,0,0) \\ u = \gamma(0,1,0,1) + \delta(1,0,0,0) \end{cases}$$

$$(\alpha, \beta, 0, \alpha) = (\delta, \gamma, 0, \gamma)$$

$$\begin{cases} \alpha = \delta \\ \beta = \gamma \\ 0 = 0 \\ \alpha = \gamma \end{cases} \Rightarrow \alpha = \delta = \gamma$$

$\Rightarrow V \cap W = \langle (1,1,0,1) \rangle$

ESERCIZIO 4

$W_1 = \{ (x,y,z) \in \mathbb{R}^3 \mid 2x + 3y - z = 0 \}$, $W_2 = \{ (x,y,z) \in \mathbb{R}^3 \mid x + 2y - z = 0 \}$

a) W_1 :

1) $\exists\, 0_V = (0,0,0)$: $2 \cdot 0 + 3 \cdot 0 - 0 = 0$

2) $\lambda(x,y,z) \in W_1$? $2\lambda x + 3\lambda y - \lambda z = 0$?

$$\lambda \underbrace{(2x + 3y - z)}_{=0} = 0 \ \checkmark$$

3) $w_1 = (x_1, y_1, z_1) \in W_1$, $w_2 = (x_2, y_2, z_2) \in W_1$

$\Rightarrow w_1 + w_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$2(x_1 + x_2) + 3(y_1 + y_2) - (z_1 + z_2) = 0$?

$$\underbrace{(2x_1 + 3y_1 - z_1)}_{=0} + \underbrace{(2x_2 + 3y_2 - z_2)}_{=0} = 0$$

W_2 : analogo ...

1) $\exists\, 0_V = (0,0,0)$: $0 + 2 \cdot 0 - 0 = 0$

2) $\lambda(x,y,z) \in W_2$? $\lambda x + 2\lambda y - \lambda z = \lambda(x + 2y - z) = 0$ ✓

3) $w_1 \in W_2$, $w_2 \in W_2$ $\Rightarrow w_1 + w_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2) \in ?\ W_2$

$(x_1 + x_2) + 2(y_1 + y_2) - (z_1 + z_2) = (x_1 + 2y_1 - z_1) + (x_2 + 2y_2 - z_2) = 0$ ✓

b) $W_1 \cap W_2 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{matrix} 2x + 3y - z = 0 \\ x + 2y - z = 0 \end{matrix} \right\}$

$$\begin{cases} 2a + 3b - c = 0 \\ a + 2b - c = 0 \end{cases} \qquad \left(\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & 2 & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right)$$

$$\begin{cases} 2a + 3b - c = 0 \\ b - c = 0 \end{cases} \qquad \begin{matrix} a = -\frac{3}{2}c + \frac{c}{2} = -\frac{3}{2}c + \frac{c}{2} = -c \\ b = c \end{matrix}$$

$\Rightarrow (a, b, c) = (-c, c, c) = c(-1, 1, 1)$

$\Rightarrow W_1 \cap W_2 = \langle (-1, 1, 1) \rangle$

c) W_1 è sottospazio di $\mathbb{R}^3$
W_2 è sottospazio di $\mathbb{R}^3$ $\Big\} \Rightarrow$ $W_1 \cap W_2$ è sottospazio di $\mathbb{R}^3$

$W_1 + W_2$ è il + piccolo sottosp contenente W_1 e W_2
$W_1 \cup W_2$ non sempre è ssv
è ssv $\Leftrightarrow$ $W_1 \subseteq W_2$ o $W_2 \subseteq W_1$

$\Rightarrow$ per il criterio di sottospazio $\exists \alpha \in \mathbb{R}$ t.c. $\forall v \in W_1 \cap W_2$
$\alpha v \in W_1 \cap W_1$

d) $W_1 + W_2 = \mathbb{R}^3$ per trovare dei generatori di W_1 e W_2 risolvo l'equazione/sistema che li definiscono

$W_1 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right\rangle \qquad W_2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle$

$W_1 + W_2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle$

Si dimostra che ogni $v \in \mathbb{R}^3$ può essere espresso come comb. lin. dei vettori appartenenti all'insieme di generatori (*) $W_1 + W_2$ $\Rightarrow$ da ciò $W_1 + W_2 = \mathbb{R}^3$

(*) per farlo si deve verificare che il sistema $\mathbb{R}^3 \ni \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda_4 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \lambda_5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ sia risolvibile!

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 1 & x \\ 0 & 1 & 1 & 0 & y \\ 2 & 3 & 0 & 1 & z \end{array}\right) \xrightarrow{III - 2I} \left(\begin{array}{cccc|c} 1 & 0 & -2 & 1 & x \\ 0 & 1 & 1 & 0 & y \\ 0 & 3 & 4 & -1 & z - 2x \end{array}\right)$$

$$\xrightarrow{III - 3II} \left(\begin{array}{cccc|c} 1 & 0 & -2 & 1 & x \\ 0 & 1 & 1 & 0 & y \\ 0 & 0 & 1 & -1 & z - 2x - 3y \end{array}\right)$$

$\Rightarrow rk A = rk A|b = 3$ il sistema è risolvibile

$\Rightarrow W_1 + W_2 = \mathbb{R}^3 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

ESERCIZIO 6

$U = \{(x,y,z,t) \in \mathbb{R}^4 \mid x+y-z=0\}$ $\quad W = \left\{(x,y,z,t) \in \mathbb{R}^4 \;\middle|\; \begin{matrix} x+2y=0 \\ x-2z-t=0 \end{matrix} \right\}$

a) $\begin{cases} x = -y+z \\ y = y \\ z = z \\ t = t \end{cases}$

le variabili libere sono 3: y, z, t

una base di U si ottiene fissando i valori delle variabili libere in 3 modi indipendenti ad esempio

$(y,z,t) = (1,0,0), (0,1,0), (0,0,1)$

$$\Rightarrow U = \left\langle \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

b) $\begin{cases} x + 2y = 0 \\ x - 2z - t = 0 \end{cases} \xrightarrow{\text{f. a scala}} \begin{cases} x + 2y = 0 \\ 2y + 2z + t = 0 \end{cases}$

esplicito il primo termine di ogni equazione:

$$\begin{cases} x = -2y = +2z + t \\ y = -z - \frac{t}{2} \end{cases} \qquad \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 2z+t \\ -z-\frac{t}{2} \\ z \\ t \end{pmatrix}$$

le variabili libere sono (z,t) $\to (1,0) \Rightarrow (2,-1,1,0)$

$\to (0,2) \Rightarrow (2,-1,0,2)$

$$\Rightarrow W = \left\langle \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 0 \\ 2 \end{pmatrix} \right\rangle$$

c) $U \cap W = \left\{ \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{matrix} x+y-z=0 \\ x+2y=0 \\ x-2z-t=0 \end{matrix} \right\}$

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & -2 & -1 & 0 \end{array}\right) \xrightarrow[\text{III}-\text{I}-\text{II}]{\text{II}-\text{I}} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right) \Rightarrow \begin{cases} x = +z-y = 2z \\ y = -z \\ -t = 0 \end{cases}$$

$$U \cap W = \left\langle \begin{pmatrix} 2z \\ -z \\ z \\ 0 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

d) $U + W = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \\ -2 \end{pmatrix} \right\rangle$

ESERCIZIO 8

c) prendo un vettore di $\mathbb{R}^3$ (α, β, γ) $\alpha, \beta, \gamma \in \mathbb{R}$

$\Rightarrow$ dimostro che $\exists\ \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ t.c.

$$\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \lambda_1 \begin{pmatrix}1\\0\\1\end{pmatrix} + \lambda_2 \begin{pmatrix}1\\0\\0\end{pmatrix} + \lambda_3 \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \begin{pmatrix}1 & 1 & 1\\ 0 & 0 & 1\\ 1 & 0 & 1\end{pmatrix}\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix}$$

riduco in forma a scala

$$\left(\begin{array}{ccc|c}1 & 1 & 1 & \alpha\\ 0 & 1 & 0 & -\gamma+\alpha\\ 0 & 0 & 1 & \beta\end{array}\right) \qquad \begin{cases}\lambda_1 = \alpha - \beta + \gamma - \alpha = -\beta + \gamma\\ \lambda_2 = -\gamma + \alpha\\ \lambda_3 = \beta\end{cases}$$

$rkA = rkA|b \rightarrow \exists!$ soluzione

e) $$\begin{pmatrix}3 & 1 & -1\\ 2 & 2 & 2\\ 1 & 2 & 3\end{pmatrix}\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} \rightarrow \left(\begin{array}{ccc|c}1 & 2 & 3 & \gamma\\ 2 & 2 & 2 & \beta\\ 3 & 1 & -1 & \alpha\end{array}\right) \xrightarrow{\substack{II - 2I\\ III - 3I}}$$

$$\left(\begin{array}{ccc|c}1 & 2 & 3 & \gamma\\ 0 & -2 & -4 & \beta - 2\gamma\\ 0 & -5 & -10 & \alpha - 3\gamma\end{array}\right) \xrightarrow{5II - 2III} \left(\begin{array}{ccc|c}1 & 2 & 3 & \gamma\\ 0 & -2 & -4 & \beta - 2\gamma\\ 0 & 0 & 0 & 5(\beta - 2\gamma) - 2(\alpha - 3\gamma)\end{array}\right)$$

$rkA \neq rkA|b$ non ha soluzioni

Foglio 2 es. 3

$V \cap W$? $\to$ cerchiamo $u \in V \cap W$

(1ª)

$\Rightarrow$ essendo che $u \in V \Rightarrow \begin{cases} u = \alpha \begin{pmatrix}1\\0\\0\\1\end{pmatrix} + \beta \begin{pmatrix}0\\1\\0\\0\end{pmatrix} & \leftarrow \text{ho espresso } u \text{ come combinazione lineare dei generatori di } V \\ u \in W \Rightarrow u = \gamma \begin{pmatrix}0\\1\\0\\1\end{pmatrix} + \delta \begin{pmatrix}1\\0\\0\\0\end{pmatrix} & \leftarrow u \text{ come comb lin dei generatori di } W \end{cases}$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{array}\right) \xrightarrow[\mathrm{III} = \mathrm{IV} - \mathrm{I}]{\mathrm{IV} = \mathrm{III}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} -\gamma + \delta = 0 \\ \beta - \gamma = 0 \\ \alpha - \delta = 0 \end{cases} \Rightarrow \begin{cases} \gamma = \delta \\ \beta = \gamma = \delta \\ \alpha = \delta \end{cases}$$

$$\Rightarrow u = \alpha \begin{pmatrix}1\\0\\0\\1\end{pmatrix} + \beta \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \delta \begin{pmatrix}1\\0\\0\\1\end{pmatrix} + \delta \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \delta \begin{pmatrix}1\\1\\0\\1\end{pmatrix}$$

$$\left(\text{analogo} \quad u = \gamma \begin{pmatrix}0\\1\\0\\1\end{pmatrix} + \delta \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \delta \begin{pmatrix}1\\1\\0\\1\end{pmatrix} \right)$$

$$\Rightarrow V \cap W = \left\langle \begin{pmatrix}1\\1\\0\\1\end{pmatrix} \right\rangle$$

(2ª) ricavo eq. cartesiana di V:

$$V \ni u = a \begin{pmatrix}1\\0\\0\\1\end{pmatrix} + b \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}a\\b\\0\\a\end{pmatrix} \Rightarrow \begin{cases} x = a \\ y = b \\ z = 0 \\ t = a \end{cases} \qquad \begin{cases} \cancel{a = x} \\ \cancel{b = y} \\ z = 0 \\ t = x \end{cases}$$

sono l.i.

4inc - 2par = 2 eq.

$$V: \begin{cases} z = 0 \\ t = y \end{cases}$$

scrivo W in forma parametrica:

$$W \ni u = a \begin{pmatrix}0\\1\\0\\1\end{pmatrix} + b \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}b\\a\\0\\a\end{pmatrix} = \begin{pmatrix}x\\y\\z\\t\end{pmatrix}$$

verifico per quali a, b $u \in W$ soddisfa l'equazione di V

$$\begin{cases} 0 = 0 \ \checkmark \\ a = b \end{cases} \qquad u = \begin{pmatrix}a\\a\\0\\a\end{pmatrix} \begin{matrix} \in V \\ \in W \end{matrix} \Rightarrow V \cap W = \left\langle \begin{pmatrix}1\\1\\0\\1\end{pmatrix} \right\rangle$$

Trovare le equazioni cartesiane del sottospazio:

$$V = \left\langle \begin{pmatrix} 3 \\ -1 \\ 1 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

1. ricavo i vettori l.i. dai generatori di V: li metto in riga e riduco in forma a scala (NB. le operazioni elementari sulle righe non alterano il sottospazio)

$$\begin{matrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 3 & -1 & 1 & 4 \end{matrix} \xrightarrow{III - 3I} \begin{matrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & -4 & -2 & -2 \end{matrix} \xrightarrow{III + 2II} \begin{matrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$\Rightarrow$ $V = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix} \right\rangle$ esprimo $v \in V$ in forma parametrica:

$$v = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = a \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} + b \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ a+2b \\ a+b \\ 2a+b \end{pmatrix}$$

NB. 4inc - 2par = 2eq

$$\begin{cases} x = a \\ y = a + 2b \\ z = a + b \\ t = 2a + b \end{cases} \rightarrow \begin{cases} \cancel{a = x} \\ y = x + 2b \\ z = x + b \\ t = 2x + b \end{cases} \rightarrow \begin{cases} y = x + 2(z - x) \\ \cancel{b = z - x} \\ t = 2x + (z - x) \end{cases}$$

$$\Rightarrow \begin{cases} x + y - 2z = 0 \\ x + z - t = 0 \end{cases}$$