Frequency Analysis of Waveforms - Digital Spectral Analysis Algorithms

The exercise involves the use of the mathematical function for calculating the amplitude spectrum of the signal, made available by a digital oscilloscope, to analyze the spectral composition of a certain number of signals provided by a Hewlett-Packard HP 54654 training board.

Introduction

The board is powered by a 9 V battery. A red LED indicates proper power. A button near the battery holder allows you to turn power on or off to the circuits.

The HP 54654 training board provides 14 different signal types, accessible through measurement points on the side of the board. The corresponding signal behavior is graphically illustrated next to each point. There are also two points for connecting to the ground reference. To avoid loading effects, it is necessary to use high-impedance attenuator probes (10:1) to collect the signal from the circuits.

NOTE: Using attenuating probes reduces the displayed amplitude by a factor of 10. Digital oscilloscopes can automatically account for this by setting an appropriate probe factor. The corresponding command is part of the CHANNEL command group. Note that the displayed trace does not change when the probe factor is changed, but the vertical scale factor does. Some instruments automatically recognize the presence of a probe and adjust the scale factor accordingly, without requiring operator intervention.

Spectrum Analysis

The mathematical function "spectrum analysis" is based on the numerical algorithm known as *Fast Fourier Transform* (FFT), combined with the use of mathematical functions for weighting the sequence of processed samples (*windows*) to improve the accuracy of the measurements. All measurements should be made using cursors, remembering that a linear scale is used for the frequency axis, while the amplitude scale for the vertical axis is usually logarithmic.

The effective value of a sinusoidal component is measured by direct reading. To do this, simply place a vertical cursor on the peak of the corresponding spectrum line and read its value, expressed in dBV. Remember that the following relationship holds:

$$V_x[\text{dBV}] = \frac{V_x[\text{V}]}{1[\text{V (RMS)}]}$$

The correspondence between the spectral peak and the amplitude of the sinusoid is more accurate if the instrument uses a window with a low *scalloping loss* in the spectral analysis. The attenuation introduced in the worst-case (*Worst-Case Scalloping Loss*, WCSL) is reported in the table for the most commonly used window types in a DSO. It should also be remembered that windows with higher sidelobes cause more spectral interference and can make estimating the noise component more difficult.

Table 1: WCSL for some commonly used windows in FFT-based analysis

Window	WCSL [dB]	minimum side lobe attenuation [dB]	main lobe width [bin]	2 · -6dB [bin]	ENBW [bin]
uniform	3.92	13	2	1.21	1
Hann (*Hanning*)	1.42	32	4	2	1.5
Blackman-Harris	1.13	71	6	2.27	1.71
flat-top	< 0.01	93	10	4.58	3.77

Characteristics of the signals to be measured

The signals to be measured must be acquired from some of the 14 *test points* of the Hewlett-Packard HP 54654 training card. Their characteristics are summarised below and some indicative values ??are provided, useful for setting up the measurements.

Test Point	Description
1	square wave, fundamental frequency ≅ 500 kHz, ≅ 3.7 V peak-to-peak amplitude.
3	impulse train with a repetition period of approximately 28.6 μs, ≅ 3.7 V peak-to-peak amplitude.
8	triangular wave, 1.2 V peak-to-peak amplitude, $\neq 0$ average value, 1.6 s period.
11	sinewave with noise; 1.1 kHz frequency, 1.4 V peak-to-peak amplitude.
14	amplitude modulated signal; carrier frequency: about 260 kHz, modulating frequency: about 1.1 kHz.

Spectrum of a periodic square wave

Connect the probe to ***Test Point n. 1*** (and the ground cable to one of the ground points on the board) and visualize the trend over time of the waveform to be measured.

Generally, when configuring the measurement parameters, it is useful to simultaneously display both the signal in the time domain and the corresponding spectrum.

Measurements to be made:

- Using the cursors, determine the amplitude (in dBV – logarithmic scale) and frequency of the signal component at the fundamental frequency. Verify that the measured frequency value corresponds to the reciprocal of the signal period, measured in the time domain. Considering the only source of uncertainty to be the resolution in cursor positioning ($\pm\Delta_t$ on the time axis for the period measurement, $\pm\Delta_f$ on the frequency axis to determine the peak position), compare the uncertainties of the two measurements, expressed in relative form (% of the measured value);
- Set the vertical scale to **linear** (unit of measurement: Vrms) and compare the measured amplitude values for the component at the **fundamental frequency**, using the following windows:
  - uniform;
  - Hanning;
  - flat-top, or Blackmann-Harris.
- using the window with the lowest *scalloping loss* value, measure the amplitude (in dBV – logarithmic scale) and frequency of the spectral components of the signal up to the 30^{th} harmonic;

- Let A_1 be the amplitude of the component at the fundamental frequency. For a periodic square wave, the ratio A_k/A_1 (which indicates the attenuation of the harmonic components with respect to the fundamental) is expressed by the formula:

$$\frac{A_k}{A_1} = \frac{1}{k} \cdot \frac{\sin \pi k D}{\sin \pi D}$$

where D is the square wave's duty cycle, i.e., the ratio of the duration of the high-level waveform segment to the period duration. Once the duty cycle has been measured in the time domain, verify the behavior of this theoretical expression, calculated based on the measured value of D, comparing it with the measurements obtained in the previous step.

Spectrum of a impulse train

Connect a probe to ***Test Point No. 3***. Adjust the horizontal scale factor, taking into account the approximate period of this signal, so that the observation interval again covers approximately 10-20 signal periods. This allows for clearly distinguishable and separate spectral components to be displayed.

The impulse train includes, within a period, multiple points at which the synchronization condition relating to the signal level and slope is satisfied. In the time domain, this results in instability of the trace on the screen, which can be avoided by correctly setting the *hold-off* time. This instability, however, is irrelevant for measuring the signal's amplitude spectrum. In this case, in fact, it translates into variability of the instantaneous phase values, which, however, only affects the spectral interference effect.

This can be verified by comparing the spectrum displayed when the time-domain trace is not synchronized with the one obtained after synchronizing it by setting an appropriate hold-off duration. The effects of spectral interference are, of course, made more evident by using a rectangular window.

Measurements to be made:

- Using the cursors, determine the amplitude (in dBV – logarithmic scale) and frequency of the signal component at the fundamental frequency. Verify the frequency separation between the spectral components, which should be equal to the fundamental frequency;

- Once the (stable) signal trace is displayed in the time domain, check the correspondence between the measured fundamental frequency and the reciprocal of the signal period.

Spectrum of a low frequency signal

Connect a probe to ***Test Point n. 8***. The signal has a very low fundamental frequency: as mentioned in the previous point, the stability of the trace in the time domain is not necessary for a spectrum measurement, therefore it is not necessary to intervene on the *trigger* settings. Remember that, given the indicative value of the signal period, an adequate observation interval lasts about

20 s. The acquisition in this case is **very** slow and even the updating of the trace takes a long time (tens of seconds – this does not mean that the instrument is not operating correctly).

Measurements to be made:

Using the cursors, determine the amplitude and frequency of the spectral components of the signal.

Sinusoidal signal spectrum with additive noise

Connect a probe to ***Test Point No. 11***. In this case, the observed signal comprises a sine wave and a noise component, characterized by a power spectral density approximately constant in the frequency range to be analyzed.

Measurements to be made:

Using the cursors, determine the amplitude (in dBV) and frequency (in Hz) of the sinusoid's spectrum. Compare the dBV measurement with the RMS value of the sinusoid measured in the time domain:

- directly via the instrument's RMS measurement function;
- dividing the value obtained through the instrument's peak-to-peak measurement function by $2\sqrt{2}$.

Spectrum of an amplitude modulated signal

Connect a probe to ***Test Point No. 14***. The observed signal comprises a periodic signal (called the modulation *carrier*), amplitude-modulated by a sine wave. According to the theory, the spectrum of the signal is made up of the set of lines that make up the spectrum of the carrier, around which, at a distance equal to the modulation frequency, the spectral lines associated with the modulating wave appear.

Measurements to be made:

- Using the cursors, determine the amplitude (in dBV – logarithmic scale) and frequency of the signal components. Verify that they match the expected behavior;
- evaluate the effects of using a window on the measured spectrum:
  - uniform;
  - Hanning;
  - flat-top, or Blackmann-Harris.