Calculus 2

(AY 2025/26 course presentation)

Paolo Guiotto

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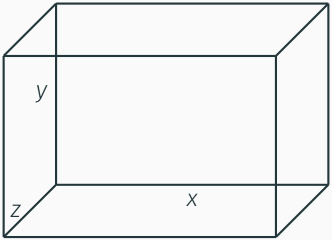

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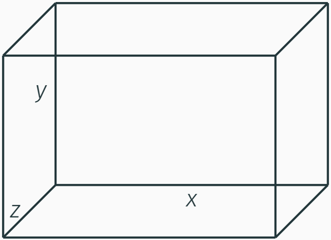

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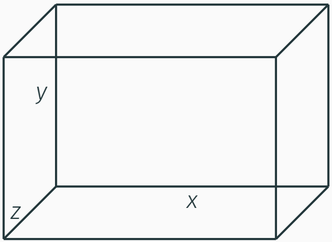

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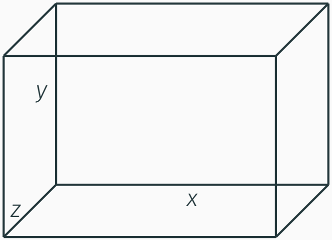

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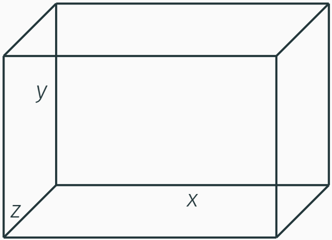

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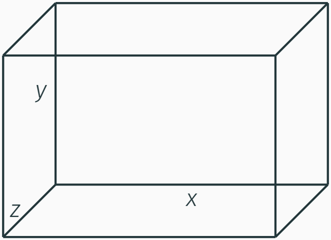

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$$\boxed{\max_{x,y,z \in]0,+\infty[,\ xy+yz+xz=\frac{S}{2}} xyz = \max_{(x,y,z)\in D} f(x,y,z)}$$

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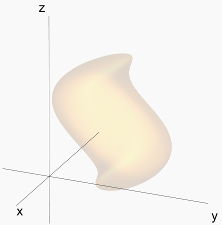
$\Longrightarrow$ we need differential calculus.

Problem: compute the volume of a solid figure



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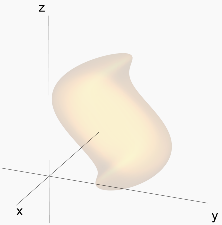

This problem reminds of the analogous one to compute area of plane figures $\rightsquigarrow \int_a^b f(x)\,dx$

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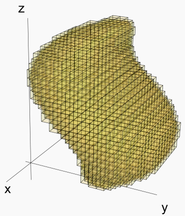

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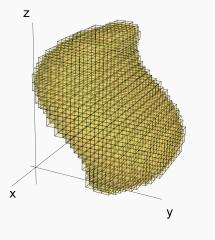

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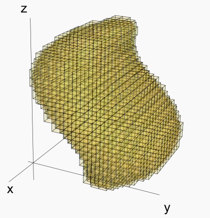

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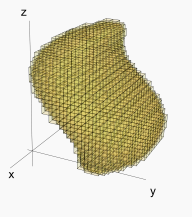

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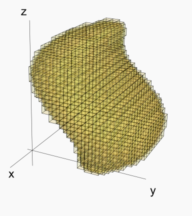

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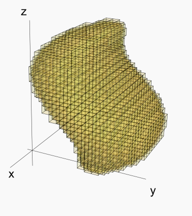

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$\implies$ **we need integral calculus**

Table of contents

PREREQUISITES

From **Calculus 1**:

- Calculus of limits
- Differential Calculus (one real variable)
- Integral Calculus (in one real variable)
- Basic Differential equations (linear first and second order equations, separable variables equations)
- Convergence of a Numerical Series.

From **Linear Algebra**:

- definition of vector space
- algebra of matrices (product line by column, invertible matrix, symmetric matrix, determinant, rank of a matrix)

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- discuss differentiability for functions of complex variable and apply powerful methods of holomorphic functions

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4. **Oral Exam:** in exceptional circumstances, where further assessment is deemed necessary, an oral supplementary exam may be required.