

Fondamenti di Algebra Lineare e Geometria

2025/2026

Ingegneria Meccanica, UniPD

Settimana 1

Esercizio 1

Scrivere in forma algebrica i seguenti numeri complessi:

1. $z = (1 + i)^{10}$

2. $z = (1 - i)(2 + i)$

3. $z = 3e^{\frac{5}{6}\pi i}$

4. $z = \frac{(\sqrt{2}i + \sqrt{3})^3}{\sqrt{2} - \sqrt{3}i}$

5. $z = \frac{(1 + i)^{10}}{(1 - i)^8}$

6. $z = i^{2014}$

Esercizio 2

Scrivere in forma trigonometrica i seguenti numeri complessi:

1. $z = \sqrt{3} - i$

2. $z = \sqrt[3]{i} - 1$

3. $z = \left(\frac{i - 1}{i + 1}\right)^3$

4. $z = \frac{4i}{\sqrt{3} + i}$

5. $z = \left(\frac{i + \sqrt{3}}{i(\sqrt{3} - i)}\right)^{22}$

6. $z = e^{-\frac{\pi}{2}i} + e^{-\frac{\pi}{6}i}$

Esercizio 3

Trovare tutte le soluzioni complesse delle seguenti equazioni:

1. $z^5 = \frac{\sqrt{3} - i}{\sqrt{3} + i}$

2. $z^3 = \frac{i - 1}{i + 1}$

3. $z^4 = 1$

4. $z^3 = \frac{(i - 1)^4}{(i + 1)^2}$

5. $z^2 = \frac{-2 + 2\sqrt{3}i}{\sqrt{3} - i}$

6. $z^3 = \frac{\sqrt{3} - i}{-2 + 2\sqrt{3}i}$

7. $z^4 = \frac{-2 - 2\sqrt{3}i}{\sqrt{3} + i}$

8. $z^5 = \frac{\sqrt{3} + i}{-2 - 2\sqrt{3}i}$

9. $z^2 = \frac{3i}{i + 1}$

10. $z^3 = \frac{-2i}{i - 1}$

11. $z^4 = \frac{3i}{\sqrt{3}i + 1}$

12. $z^4 = \frac{5i}{\sqrt{3} - i}$

Esercizi con soluzioni

Esercizio 1

Scrivere in forma algebrica i seguenti numeri complessi:

1. $z = (1 + i)^4$ [R: -4]

2. $z = (1 - i)(2 + i)$ [R: $3 - i$]

3. $z = 3e^{\frac{5}{6}\pi i}$ [R: $-\frac{3\sqrt{3}}{2} + \frac{3}{2}i$]

4. $z = \frac{\sqrt{2}i + \sqrt{3}}{\sqrt{2} - \sqrt{3}i}$ [R: i]

5. $z = \frac{(1+i)(3-2i)}{i}$ [R: $1-5i$]
 6. $z = i^{2022}$ [R: -1]

Esercizio 2

Scrivere in forma trigonometrica i seguenti numeri complessi:

1. $z = \sqrt{3} - i$ [R: $2 \left(\cos(-\frac{\pi}{6}) + i \sin(-\frac{\pi}{6}) \right)$]
 2. $z = (1-i)^5$ [R: $4\sqrt{2} \left(\cos(\frac{3}{4}\pi) + i \sin(\frac{3}{4}\pi) \right)$]
 3. $z = \left(\frac{i-1}{i+1} \right)^3$ [R: $\cos(\frac{3}{2}\pi) + i \sin(\frac{3}{2}\pi)$]
 4. $z = \frac{4i}{\sqrt{3}+i}$ [R: $2 \left(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}) \right)$]
 5. $z = (1+i)^2(3+\sqrt{3}i)$ [R: $4\sqrt{3} \left(\cos(\frac{2}{3}\pi) + i \sin(\frac{2}{3}\pi) \right)$]
 6. $z = \frac{1+i}{(1-i)(\sqrt{3}+i)}$ [R: $\frac{1}{2} \left(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}) \right)$]

Esercizio 3

Trovare tutte le soluzioni complesse delle seguenti equazioni:

1. $z^5 = \frac{\sqrt{3}-i}{\sqrt{3}+i}$ [R: $\cos(-\frac{\pi}{15} + \frac{2}{5}k\pi) + i \sin(-\frac{\pi}{15} + \frac{2}{5}k\pi)$]
 2. $z^3 = \frac{1-i}{i+1}$ [R: $\cos(-\frac{\pi}{6} + \frac{2}{3}k\pi) + i \sin(-\frac{\pi}{6} + \frac{2}{3}k\pi)$]
 3. $z^4 = 1$ [R: $\cos(\frac{k\pi}{2}) + i \sin(\frac{k\pi}{2})$]
 4. $z^3 = \frac{(i-1)^4}{(i+1)^2}$ [R: $\sqrt[3]{2} \left(\cos(\frac{\pi}{6} + \frac{2}{3}k\pi) + i \sin(\frac{\pi}{6} + \frac{2}{3}k\pi) \right)$]
 5. $z^2 = \frac{-2i}{i-1}$ [R: $\sqrt[4]{2} \left(\cos(\frac{3}{8}\pi + k\pi) + i \sin(\frac{3}{8}\pi + k\pi) \right)$]
 6. $\bar{z}^4 = \frac{1+i}{i}$ [R: $\sqrt[8]{2} \left(\cos(\frac{\pi}{16} + \frac{k}{2}\pi) + i \sin(\frac{\pi}{16} + \frac{k}{2}\pi) \right)$]
 7. $\bar{z}^3 = -8i$ [R: $2 \left(\cos(\frac{\pi}{6} + \frac{2}{3}k\pi) + i \sin(\frac{\pi}{6} + \frac{2}{3}k\pi) \right)$]
 8. $z^4 = 2\bar{z}^2$ [R: $0; \sqrt{2} \left(\cos(\frac{k\pi}{3}) + i \sin(\frac{k\pi}{3}) \right)$]
 9. $z^2 = (\sqrt{6} - \sqrt{2}i)|z|^3$ [R: $0; \frac{\sqrt{2}}{4} \left(\cos(-\frac{\pi}{12} + k\pi) + i \sin(-\frac{\pi}{12} + k\pi) \right)$]
 10. $z^3 = \frac{4+4i}{|z|^2}$ [R: $\sqrt[2]{2} \left(\cos(\frac{\pi}{12} + \frac{2}{3}k\pi) + i \sin(\frac{\pi}{12} + \frac{2}{3}k\pi) \right)$]

$$11. \quad z^4 = \frac{i}{(2\bar{z})^3}$$

$$[\text{R: } \frac{1}{\sqrt[7]{8}} \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right)]$$

$$12. \quad z^4 = \frac{i\bar{z}^3}{8}$$

$$[\text{R: } 0; \frac{1}{8} \left(\cos\left(\frac{\pi}{14} + \frac{2}{7}k\pi\right) + i \sin\left(\frac{\pi}{14} + \frac{2}{7}k\pi\right) \right)]$$