Università di Padova, Facoltà di Scienze Statistiche
Laurea Magistrale in Scienze Statistiche
Corso: **Teorie e modelli demografici**

The life table and single decrement process

Maria Letizia Tanturri
Dipartimento di Scienze Statistiche
tanturri@stat.unipd.it

Lezione 8

Riferimenti

- Preston et al. (2001), ch. 3

Life Table

- A statistical model for measuring the mortality (or any other type of "exit") experiences of a population, controlling for age distributions

Why is it so important?

- The life table is the most important device used in demography.
- It is only one way to summarize a cohort mortality experience (but simple!).
- It allows to calculate life expectancy at birth (or a certain age x), one of the most used indicators

Un’idea intuitiva della speranza alla vita alla nascita

- Ined

http://www.ined.fr/fr/tout-savoir-population/videos/animation-mesurer-esperance-vie/

Types of Life Tables

- Current/Period vs. Generation/Cohort
- Complete vs. Abridged
- Single vs. Multiple Decrement
- (Increment/Decrement Tables)

Università di Padova, Facoltà di Scienze Statistiche
Laurea Magistrale in Scienze Statistiche
Corso: **Teorie e modelli demografici**

Demographic probabilities

Lezione 8

Period & Cohort Rates & Probabilities

$$ {}_{n}M_{x}^{\text{Cohort } C} = \frac{\text{Deaths to Cohort C between ages x and x+n}}{\text{Person-years lived by Cohort C between ages x and x+n}} $$

$$ {}_{n}M_{x}^{\text{Year } t} = \frac{\text{Deaths between ages x and x+n in Year t}}{\text{Person-years lived between ages x and x+n in Year t}} $$

$$ {}_{n}q_{x}^{\text{Cohort } C} = \frac{\text{Deaths to Cohort C between ages x and x+n}}{\text{Number of persons in Cohort C who reached their x}^{\text{th}}\text{ birthday}} $$

Università di Padova, Facoltà di Scienze Statistiche
Laurea Magistrale in Scienze Statistiche
Corso: **Teorie e modelli demografici**

The life tables

Lezione 8

The Life Table

- One of the most important demographic techniques
- Describes the dying out of a cohort
- Age or more generally “duration” is the most important dimension along which a life table is organized

The Life Table

- Contains a number of columns
  - Age (age groups),
  - Numbers of deaths in each age group
  - Probability of dying in each age group
  - Number of survivors to the beginning of each age group
  - Number of person years lived in each age group
  - Average additional years to live for those who survive to beginning of each age group, etc.

Life Table Columns: l_x

l_x

- Number left alive at age x

Life Table Columns: $_nd_x$

$$_nd_x = l_x - l_{x+n}$$

- Number dying between ages x and x+n

Life Table Columns: ${}_nq_x$

$$ {}_nq_x = \frac{{}_nd_x}{l_x} $$

- Probability of dying between ages x and x+n

Life Table Columns: ${}_np_x$

$$ {}_np_x = \frac{l_{x+n}}{l_x} = \frac{l_x - {}_nd_x}{l_x} = 1 - {}_nq_x $$

- Probability of surviving from ages x to x+n

Life Table Columns: ${}_nL_x$

$$ {}_nL_x = n \cdot l_{x+n} + {}_na_x \cdot {}_nd_x $$

- Person-years lived between ages x and x+n

Life Table Columns: T_x

$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

- Person-years lived at ages older than x
- (=Serie retrocumulata degli anni vissuti)

Life Table Columns: e_x

$$e_x^0 = \frac{T_x}{l_x}$$

- Expectation of life at age x; average additional years of life that someone who survives to age x can expect to live

Life Table Columns: ${}_nm_x$

$$ {}_nm_x = \frac{{}_nd_x}{{}_nL_x} $$

- Death rate in the cohort between ages x and x+n

Life Table Columns: ${}_na_x$

$${}_na_x$$

- Average number of years lived in the age interval by those dying in the age interval

Note

- Some functions refers to a single exact age:
  - lx
  - Tx
  - e0x
- Other functions refers to age intervals, that begin with exact age x and extend for n years:
  - ndx
  - npx
  - nqx
  - nmx
  - nax

Esercitazione

- Come costruire una tavola di mortalità per una coorte
- (es.Lifetable1_preston_esercizio_Preston_p39)

Example: age at death and life line of an hypothetical cohort of births (10). Date of birth: January 1800

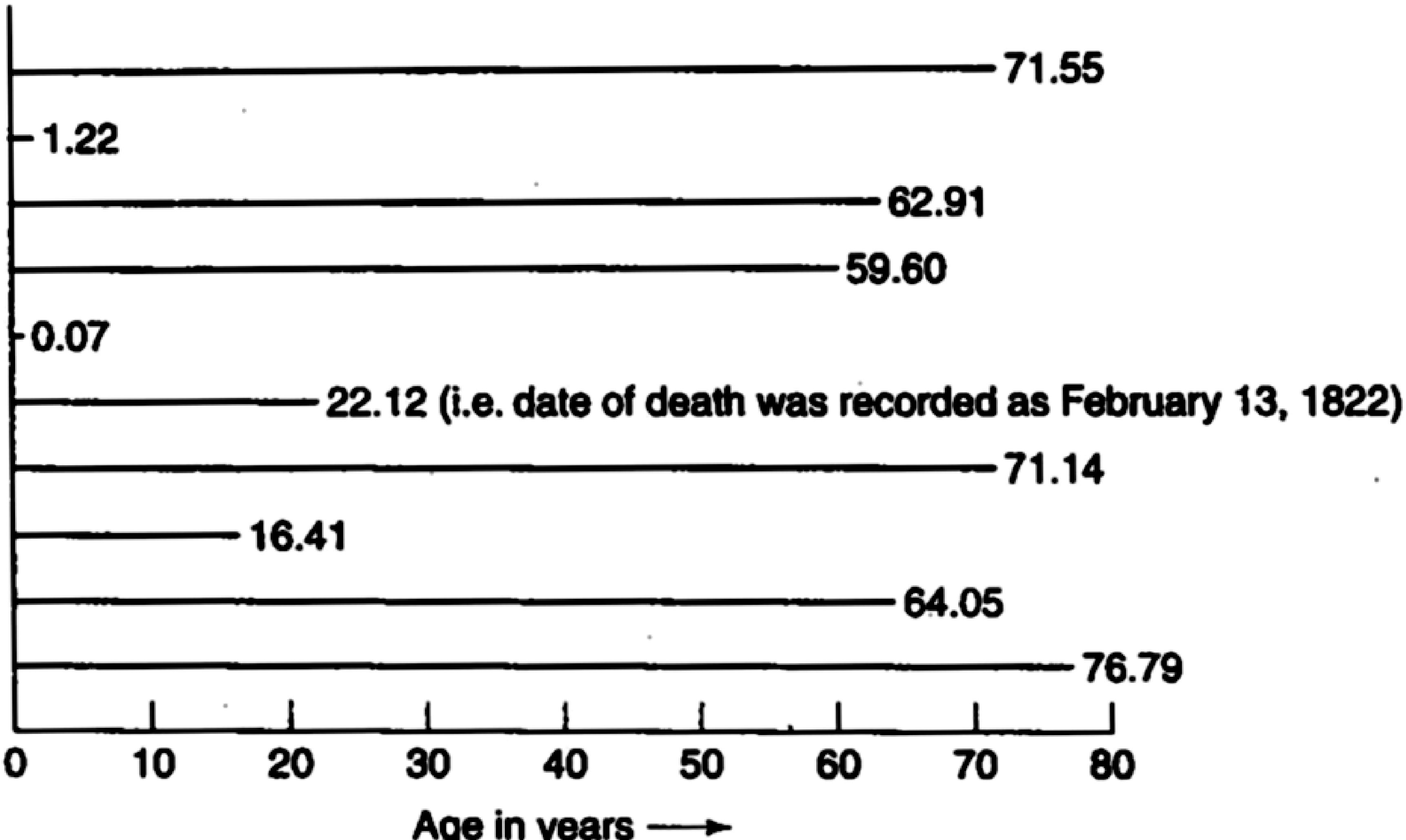

Table 3.1: *Life table for hypothetical cohort of 10 births shown in figure 3.1*

Exact age x	Number left alive at age x l_x	Number dying between ages x and $x+n$ ${}_nd_x$	Probability of dying between ages x and $x+n$ ${}_nq_x$	Probability of surviving from age x to age $x+n$ ${}_np_x$	Person-years lived between ages x and $x+n$ ${}_nL_x$	Person-years lived above age x $T_x = \sum_{a=x}^{\infty} {}_nL_a$	Expectation of life at age x $e_x^o = T_x/l_x$	Death rate in the cohort between ages x and $x+n$ ${}_nm_x$	Average person-years lived in the interval by those dying in the interval ${}_na_x$
0	10	1	1/10	9/10	$9 + .07 = 9.07$	$436.79 + 9.07 = 445.86$	$\frac{445.86}{10} = 44.586$	$\frac{1}{9.07}$	.07
1	9	1	1/9	8/9	$8 \cdot 4 + .22 = 32.22$	$404.57 + 32.22 = 436.79$	$\frac{436.79}{9} = 48.532$	$\frac{1}{32.22}$	.22
5	8	0	0	1	$8 \cdot 5 = 40$	$364.57 + 40 = 404.57$	$\frac{404.57}{8} = 50.571$	0	—
10	8	1	1/8	7/8	$7 \cdot 10 + 6.41 = 76.41$	$288.16 + 76.41 = 364.57$	$\frac{364.57}{8} = 45.571$	$\frac{1}{76.41}$	6.41
20	7	1	1/7	6/7	$6 \cdot 10 + 2.12 = 62.12$	$226.04 + 62.12 = 288.16$	$\frac{288.16}{7} = 41.166$	$\frac{1}{62.12}$	2.12
30	6	0	0	1	$6 \cdot 10 = 60$	$166.04 + 60 = 226.04$	$\frac{226.04}{6} = 37.673$	0	—
40	6	0	0	1	$6 \cdot 10 = 60$	$106.04 + 60 = 166.04$	$\frac{166.04}{6} = 27.673$	0	—
50	6	1	1/6	5/6	$5 \cdot 10 + 9.60 = 59.60$	$46.44 + 59.60 = 106.04$	$\frac{106.04}{6} = 17.673$	$\frac{1}{59.60}$	9.60
60	5	2	2/5	3/5	$3 \cdot 10 + 6.96 = 36.96$	$9.48 + 36.96 = 46.44$	$\frac{46.44}{5} = 9.288$	$\frac{2}{36.96}$	$(2.91 + 4.05)/2 = 6.96/2 = 3.48$
70	3	3	3/3	0	9.48	9.48	$\frac{9.48}{3} = 3.16$	$\frac{3}{9.48}$	$(1.55 + 1.14 + 6.79)/3 = 9.48/3 = 3.16$
80	0	0	—	—					