Università degli Studi di Padova

Radio-frequency network analysis

Lecture #15
Electronic measurements
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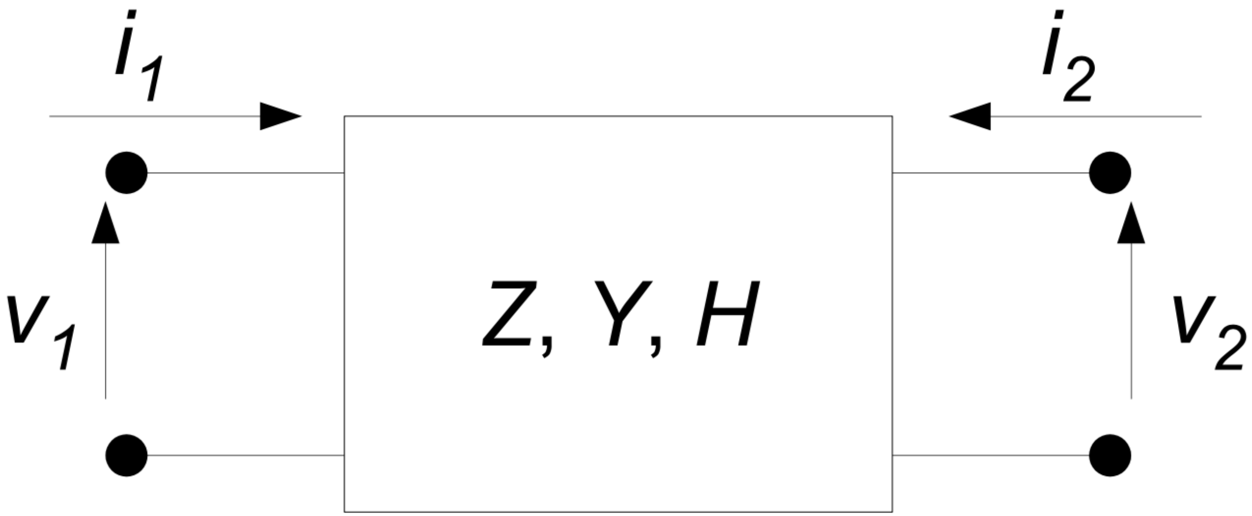
Scattering parameters

- **Two-port linear devices** → Linear equations expressing the link among voltages and currents

$$v_1 = z_{11} i_1 + z_{12} i_2 \qquad i_1 = y_{11} v_1 + y_{12} v_2$$

$$v_2 = z_{21} i_1 + z_{22} i_2 \qquad i_1 = y_{21} v_1 + y_{22} v_2$$

Impedances **Admittances**



Scattering parameters

- **Transistors** → Linearized equations → hybrid parameters

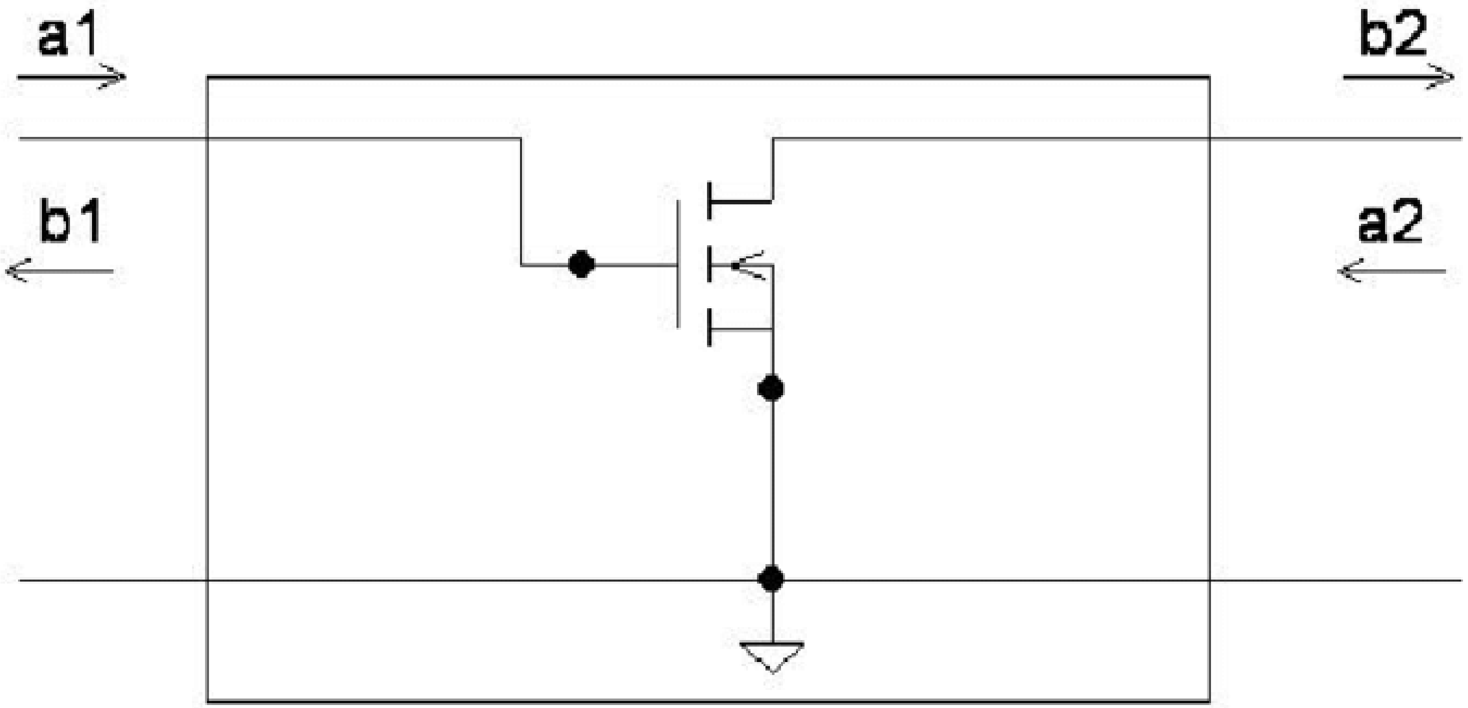
$$v_1 = h_{11} i_1 + h_{12} v_2$$
$$i_2 = h_{21} i_1 + h_{22} v_2$$

- h_{11} impedance
- h_{22} admittance
- h_{12} and h_{21} dimensionless parameters

- h_{11} determined measuring voltage v_1 and current i_1 at port 1 when port 2 is short-circuited ($v_2 = 0$)
- h_{22} determined measuring voltage v_2 and current i_2 at port 2 when port 1 is an open circuit ($i_1 = 0$)

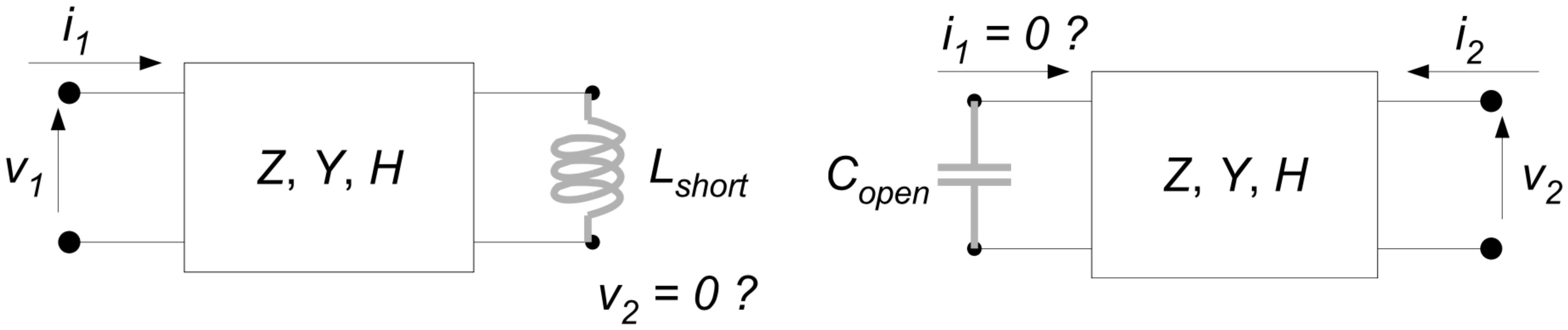
Scattering parameters

- **Two port model of a transistor**



Scattering parameters

- **Real conditions:**
  - a small inductance L_{short} is introduced between terminals when port 2 is short-circuited
  - a small capacitance C_{open} is found between terminals when port 1 is left open



Scattering parameters

- **High frequencies:**
  - **Adverse effects on accuracy** due to these parameters
  - **Signal reflections** at the terminations

- **High-frequency electronic circuits**: impedance-matched systems → ideally, signals propagate unaltered from one functional block to the next without reflection
- A two-port device might be characterized by just one parameter
  - **Attenuation (passive device)**
  - **Amplification (active device)**

- **Reflections** → Reflection coefficient Γ

Scattering parameters

- Measurements on **high-frequency devices under test** operating in **impedance matching conditions**:
  - Test conditions are less difficult to be set
  - Measurements are referred to test conditions that are closer to the normal operation of the device

- Measurements need to be referred to a different set of parameters:
  - Incident wave $\mathbf{V}_i$
  - Reflected wave $\mathbf{V}_r$

- $\mathbf{V}$ complex number $\rightarrow$ **phasor** of a sinusoidal waveform

Scattering parameters

$$v(t) = \sqrt{2} \cdot |\mathbf{V}| \cos(2\pi f t + \angle\mathbf{V}) = \mathbb{R}\left[\sqrt{2} \cdot |\mathbf{V}| \cdot e^{j2\pi f t}\right]$$

- f generic **frequency value**
- $|\mathbf{V}|$ sinewave **root mean-square value**
- Z_0 **characteristic impedance**

Incident power $\frac{|\mathbf{V}_i|^2}{Z_0}$

Reflected power $\frac{|\mathbf{V}_r|^2}{Z_0}$

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Scattering parameters

- Normalized values referred to $\mathbf{V}_i$ and $\mathbf{V}_r$:

$$a = \frac{\mathbf{V}_i}{\sqrt{Z_0}}$$

$$\mathrm{b} = \frac{\mathbf{V}_r}{\sqrt{Z_0}}$$

- Two port equations:

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

- $s_{11}, s_{12}, s_{21}, s_{22}$ **scattering parameters** (s-parameters)
- $|a|^2$ and $|b|^2$ incident and reflected power

Scattering parameters

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \text{ from which } s_{11} = \left.\frac{\mathbf{V}_{r1}}{\mathbf{V}_{i1}}\right|_{\mathbf{V}_{i2}=0} = \Gamma_1$$

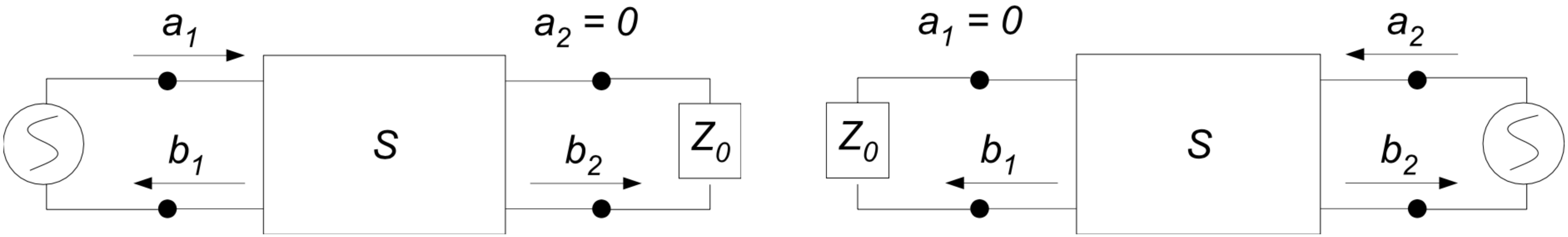
Complex **reflection** coefficient

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \text{ from which } s_{21} = \left.\frac{\mathbf{V}_{r2}}{\mathbf{V}_{i1}}\right|_{\mathbf{V}_{i2}=0}$$

Complex **transmission** coefficient

Scattering parameters

- Determination of s_{11} and s_{21}:
  - A **matched generator** providing the incident wave $\mathbf{V}_{i1}$ at port 1
  - A **load impedance** Z_0 on the opposite port 2, to ensure that wave b_2, "outgoing" from port 2, is not reflected back
  - **Two measuring units** for waves $\mathbf{V}_{r1}$ and $\mathbf{V}_{r2}$ on both ports



(a) measurement configuration, port 1

(b) measurement configuration, port 2

Scattering parameters

- Crucial requirements:
  - Values of *s*-parameters are **complex numbers** that are determined as ratios of two vectors. This requires the capability to measure both **a magnitude ratio and a phase difference** (between reflected and incident wave)
  - At any point along a transmission line measured voltage is the (vector) superposition of $\mathbf{V}_i$ and $\mathbf{V}_r$. The two components can only be separated on the basis of their **propagation direction**
  - Frequencies for *s*-parameter measurement may range from a **few MHz to several tens of GHz**

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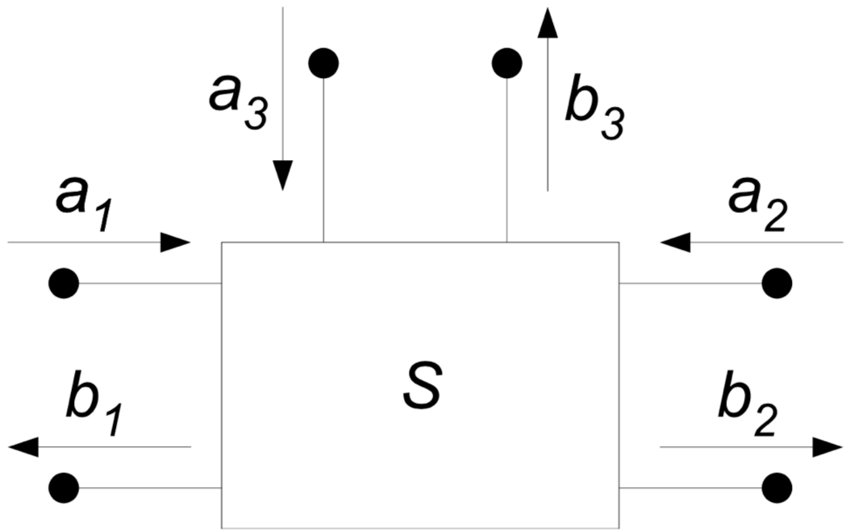
Network analyzers

Directional coupler

- Separates the incident and reflected waves
- **Three-port *s*-parameter model**

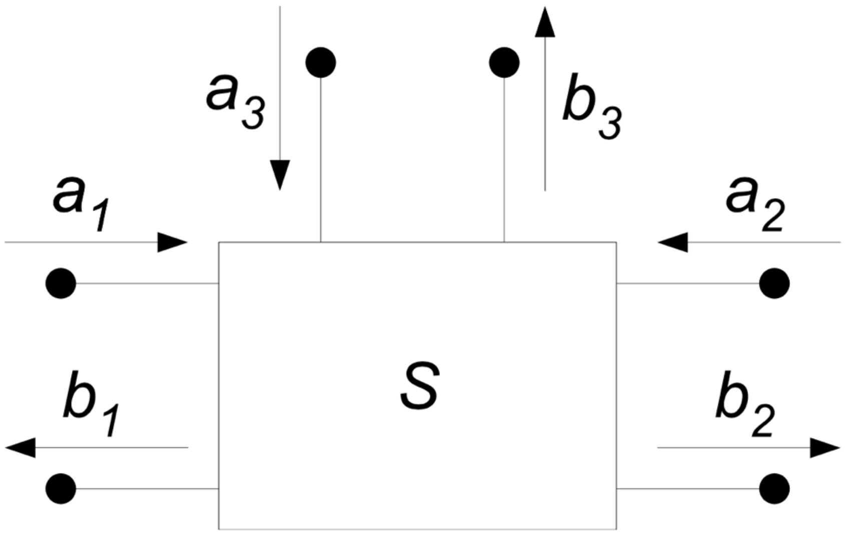
$$b_1 = s_{11}a_1 + s_{12}a_2 + s_{13}a_3$$
$$b_2 = s_{21}a_1 + s_{22}a_2 + s_{23}a_3$$
$$b_3 = s_{31}a_1 + s_{32}a_2 + s_{33}a_3$$



- The directional coupler is placed **between the matched generator** providing the test sinewave and **one port of the device under test**
- **Termination impedance** for port 3 is $Z_0 \rightarrow$ No reflection, $a_3 = 0$

Directional coupler

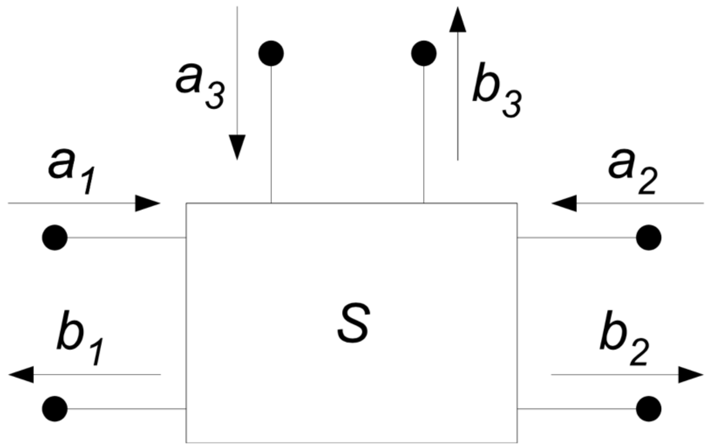
- Parameters of a directional coupler:
  - **Coupling factor**, defined by the power ratio: $\frac{|b_3|^2}{|a_2|^2}$ with $a_1 = 0, a_3 = 0 \rightarrow |s_{32}|^2$
  - **Reverse coupling factor**, or **isolation**, defined by the power ratio: $\frac{|b_3|^2}{|a_1|^2}$ with $a_2 = 0, a_3 = 0 \rightarrow |s_{31}|^2$
  - **Insertion loss**, defined by the power ratio: $\frac{|b_2|^2}{|a_1|^2}$ with $a_2 = 0, a_3 = 0 \rightarrow |s_{21}|^2$



Directional coupler

- **Passive device** → $|s_{32}| \leq 1$ for all s-parameters
- All parameters are expressed in dB
- "Minus" sign is usually omitted → implied in terms like "loss" or "attenuation"

- **Directivity:** capability to separate the incident wave from the reflected wave and present at port 3 an output proportional only to the latter



Directivity

- Assume **that total reflection** occurs at port 2, so that $a_2 = b_2$
- Coupler output at port 3:

$$b_3 = s_{31}a_1 + s_{32}b_2 = s_{31}a_1 + s_{32}s_{21}a_1$$

- $s_{32}s_{21}a_1$ **coupling with port 2**
- $s_{31}a_1$ **"undesired" direct coupling of port 1 to port 3**

- **Power** measured at port 3:

$$P_3 = (|s_{32}| \cdot |s_{21}| \cdot |a_1|)^2 + (|s_{31}| \cdot |a_1|)^2 = |s_{32}|^2 \cdot |b_2| \left[1 + \frac{|s_{31}|^2}{|s_{32}|^2 \cdot |s_{21}|^2}\right]$$

Directivity

$$P_3 = (|s_{32}| \cdot |s_{21}| \cdot |a_1|)^2 + (|s_{31}| \cdot |a_1|)^2 = |s_{32}|^2 \cdot |b_2|^2 \left[1 + \frac{|s_{31}|^2}{|s_{32}|^2 \cdot |s_{21}|^2}\right]$$

- **Proportional to the power $|\boldsymbol{b_2}|^2$** reflected into port 2, times the coupling factor s_{32}
- The term in square brackets in should be as close to 1 as possible

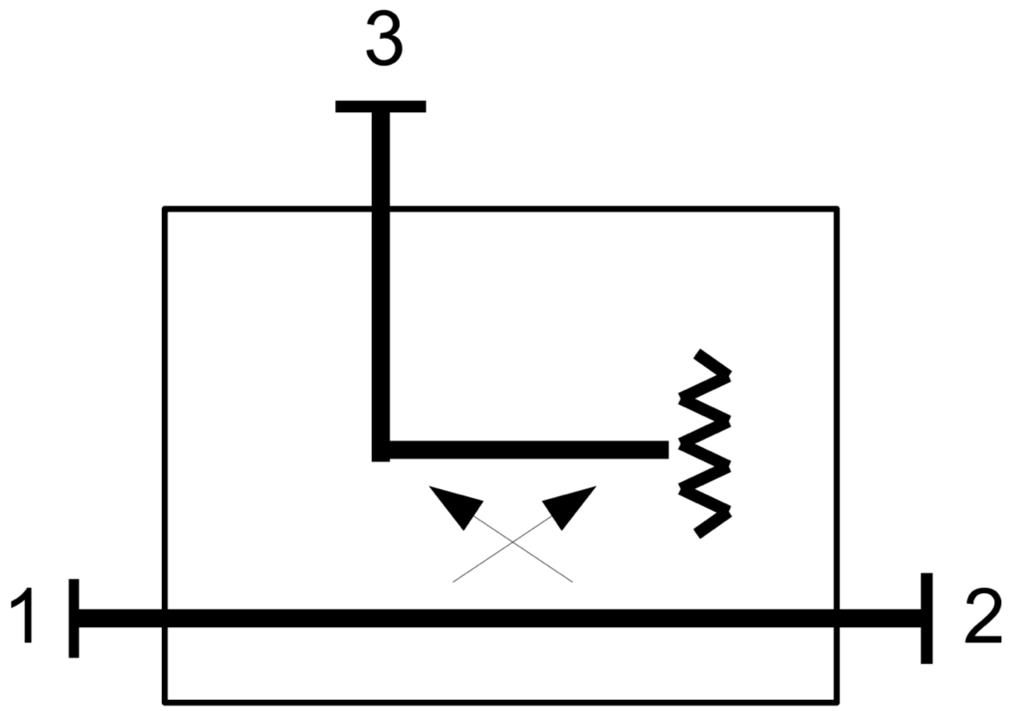
- **Directivity**: indication of the influence of power from the generator on the measurement of reflected waves
- Ratio of the isolation factor to the product between coupling factor and insertion loss

$$\text{Directivity}_{[dB]} = -20 \log_{10} \frac{|s_{31}|^2}{|s_{32}|^2 \cdot |s_{21}|^2} =$$

$$= \text{Isolation}_{[dB]} - \text{Coupling}_{[dB]} - \text{Insertion loss}_{[dB]}$$

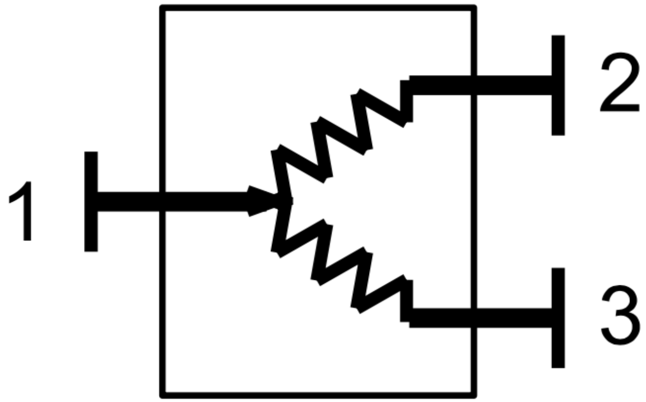
Directivity

- Directivity is defined as a **power ratio** → no phase characteristic
- Complete characterization of the measuring system input-output behaviour (magnitude and phase) → **preliminary calibration phase**

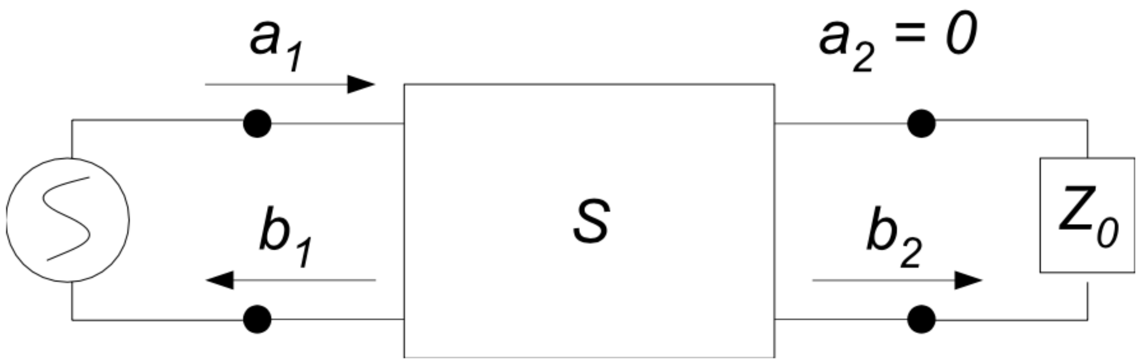


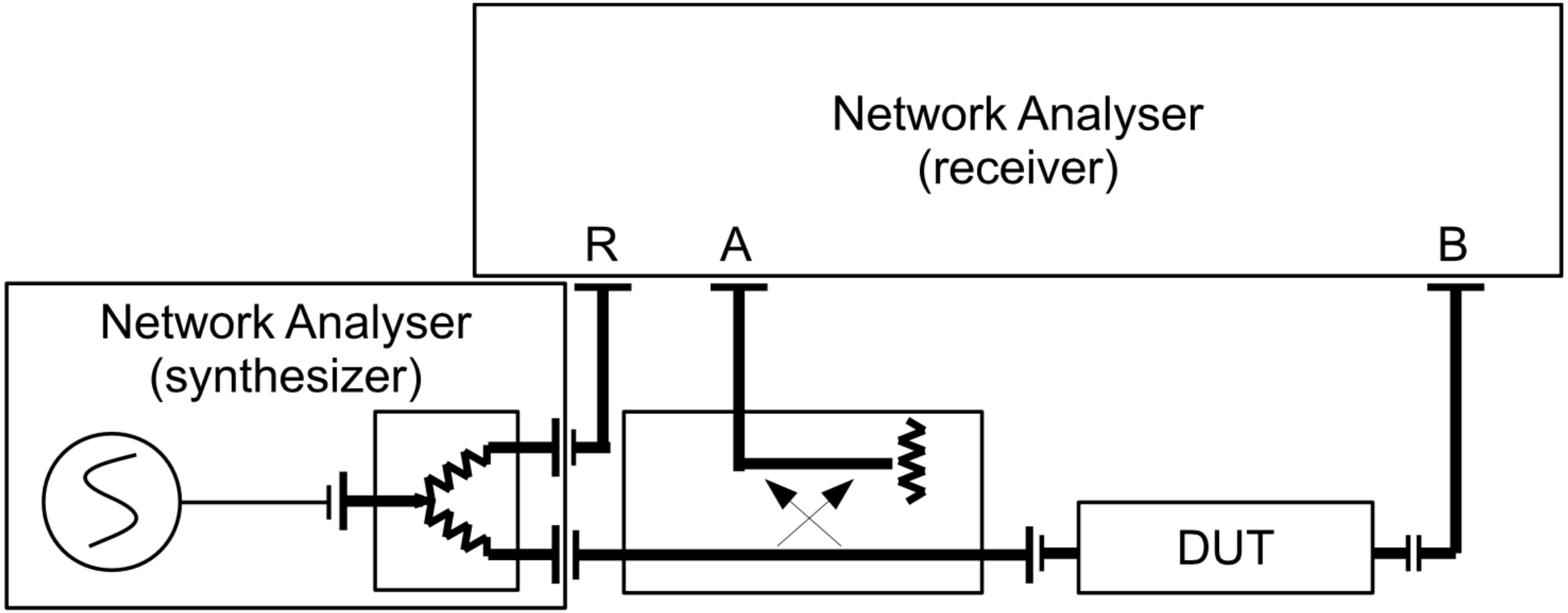
Power splitter

- Passive circuit which **divides the power input** into equal parts at the two outputs, while ensuring impedance matching
- Fully characterized by a set of three equations with **9 *s*-parameters**
- Ideal operation: **half of the input power** is found at each of the two output ports, corresponding to a 3 dB attenuation

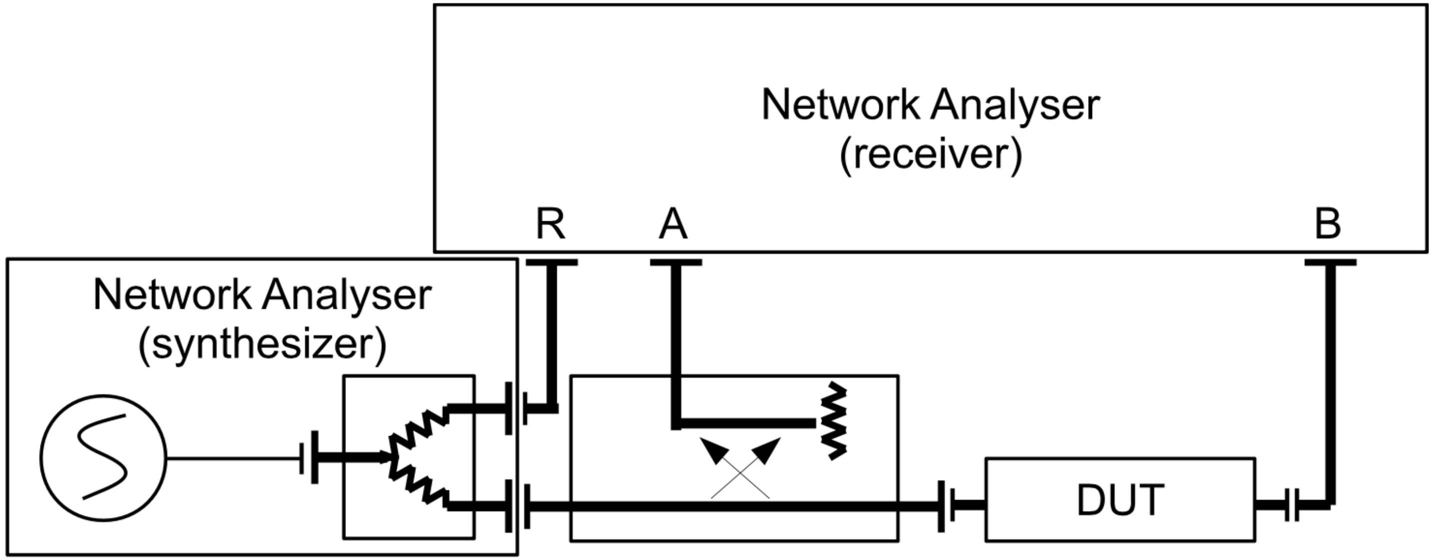


Vector network analyzer



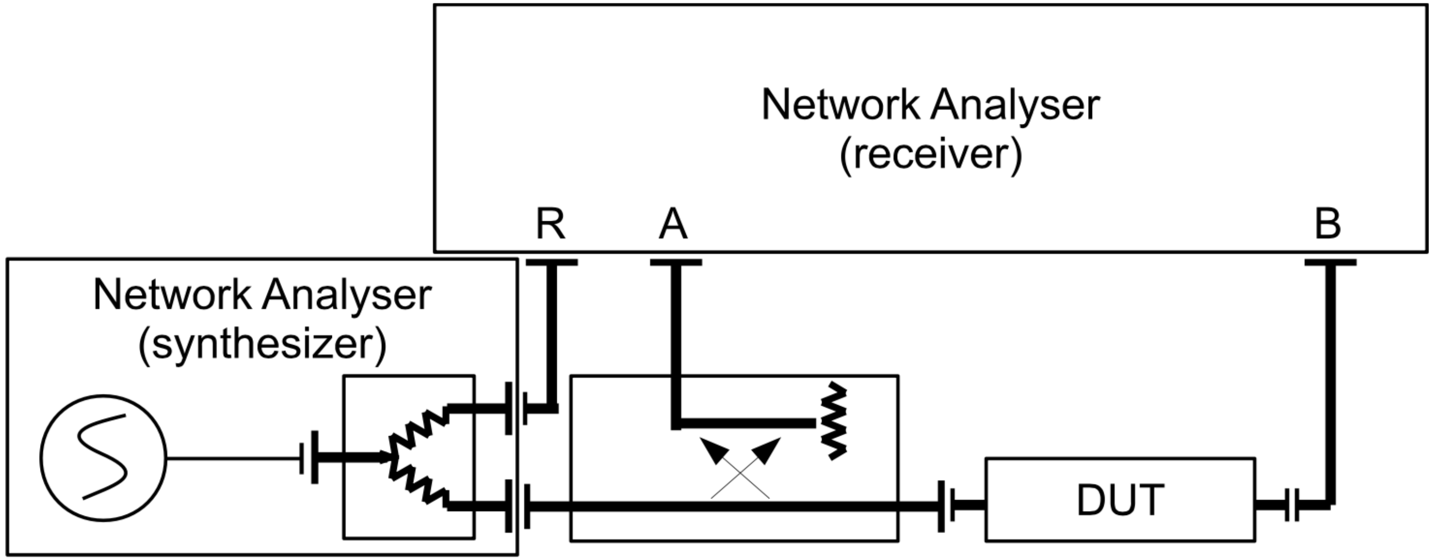


Vector network analyzer



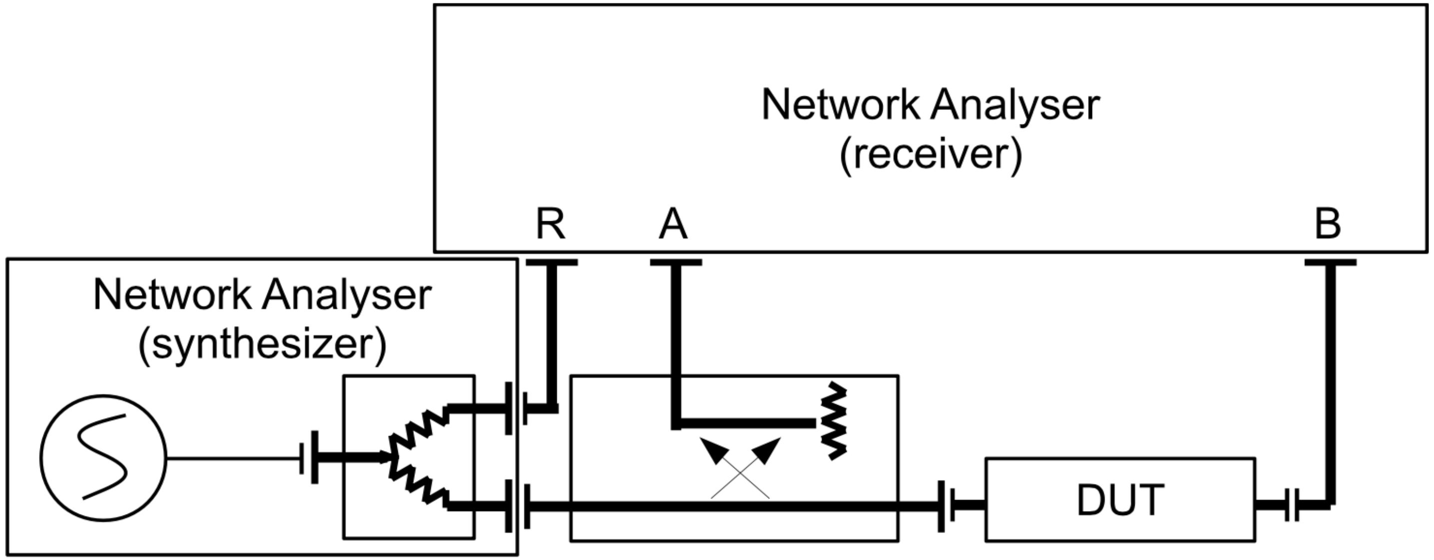
- A, B and R **measurement inputs**
- Through the power splitter, the signal from the radio-frequency generator is sent **both to R and to the DUT**
- Responses from the DUT are measured, respectively, at **input B** for transmission and at **input A** for reflection

Vector network analyzer



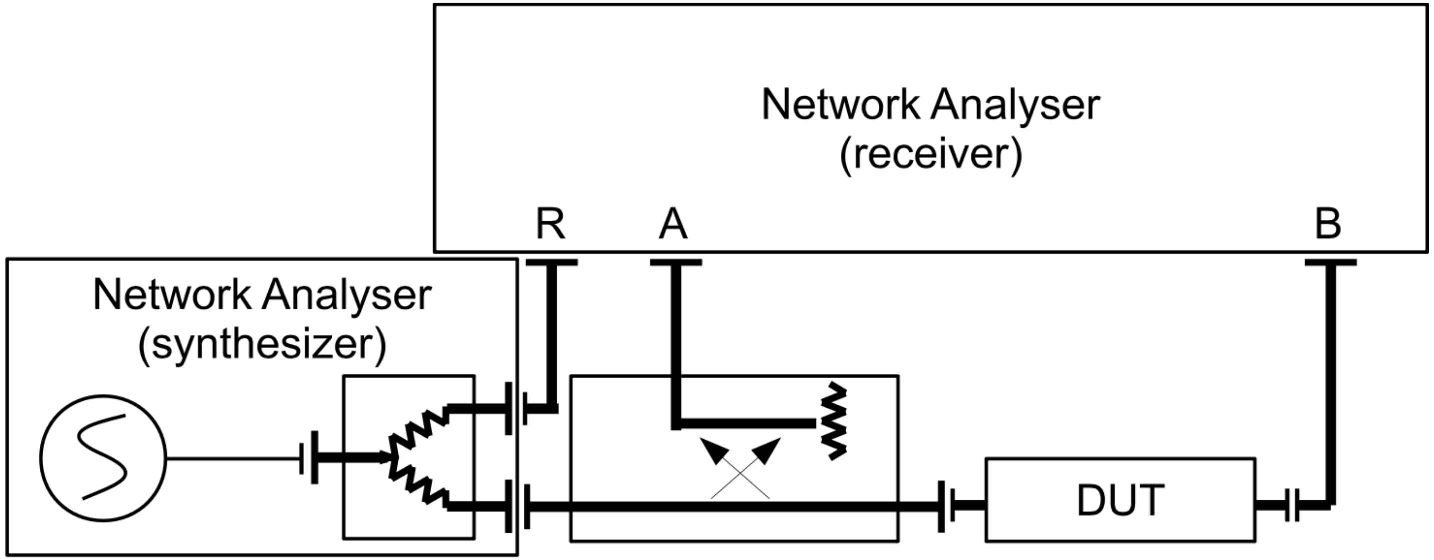
- Signal power at the DUT input is **lower than power measured at input R** because of the **insertion loss** of the directional coupler
- Power measured at input A **is proportional to power reflected by the DUT.** The coupling factor of the directional coupler also has to be considered

Vector network analyzer



- **Synthesizer**: generates a variable frequency sinewave with controlled, stable amplitude, that provides the input stimulus to the device under test. Frequency may vary between a minimum value of about 100 kHz and a maximum value that may reach up to around a hundred GHz

Vector network analyzer



- **Receiver**: measures sinewave magnitudes and phases at each of the three input channels. Indicating by $\mathbf{V}_R$, $\mathbf{V}_A$ and $\mathbf{V}_B$ the resulting three phasors, measurements of the complex transmission coefficient s_{21} and of the complex reflection coefficient s_{11} are given by the two ratios:

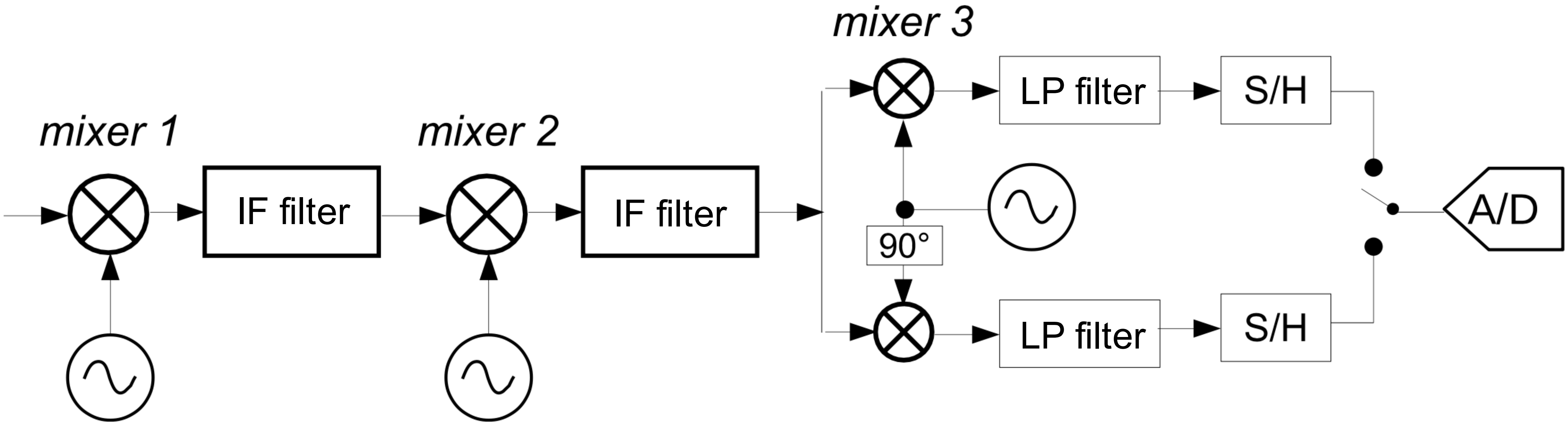
$$\hat{s}_{21} = \frac{\mathbf{V}_B}{\mathbf{V}_R} \qquad \hat{s}_{11} = \frac{\mathbf{V}_A}{\mathbf{V}_R}$$

Vector network analyzer

- **Accurate phase measurement** → No distortion
- DUT (e.g. radio-frequency amplifier) **may add distortion** even when designed to operate as a linear device
- **Non-linearities should be avoided** → the aim of network analyzer measurements is the determination of parameters in a linear model

- The response of analyzed devices may depend quite significantly on **frequency** → large amplitude dynamics (at least 100 dB)

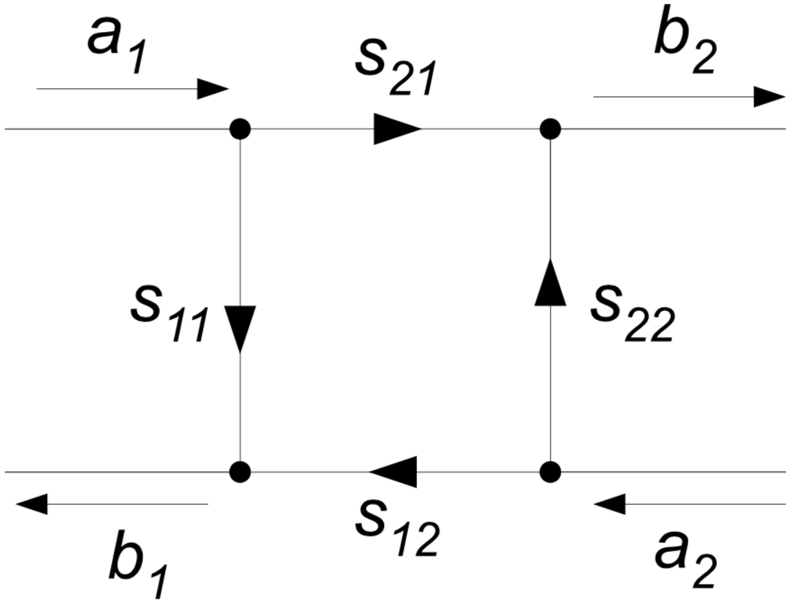
Network analyzer Receiver



- Swept-frequency spectrum analyzer → **Superheterodyne**
- Three **cascaded mixer stages** → high frequency selectivity and wide amplitude dynamics
- The outputs of the final mixer stage are then **sampled and digitized**
- Input frequency is known → synthesizer and receiver frequencies are tuned

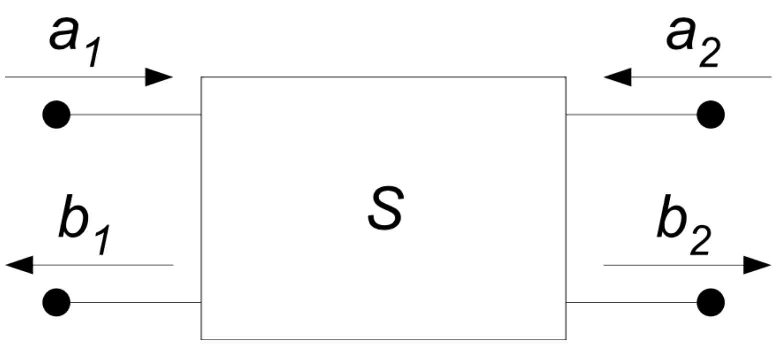
Measurement set-up calibration

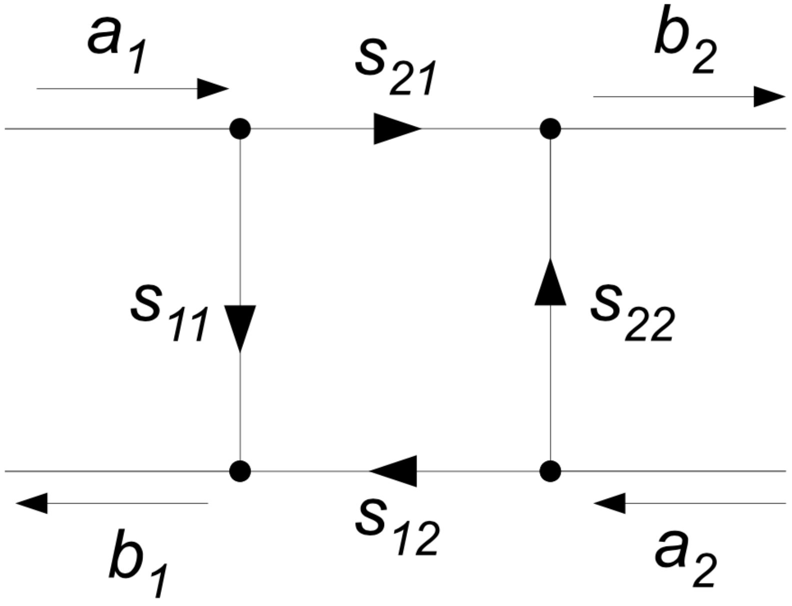
- **System calibration** → ensure the accuracy of measurement results
- **Radio-frequency measurements** → At high frequencies the influence of minor elements such as cables and connectors may become significant
- In network analyzers system calibration is essential to guarantee the validity of results



Measurement set-up calibration

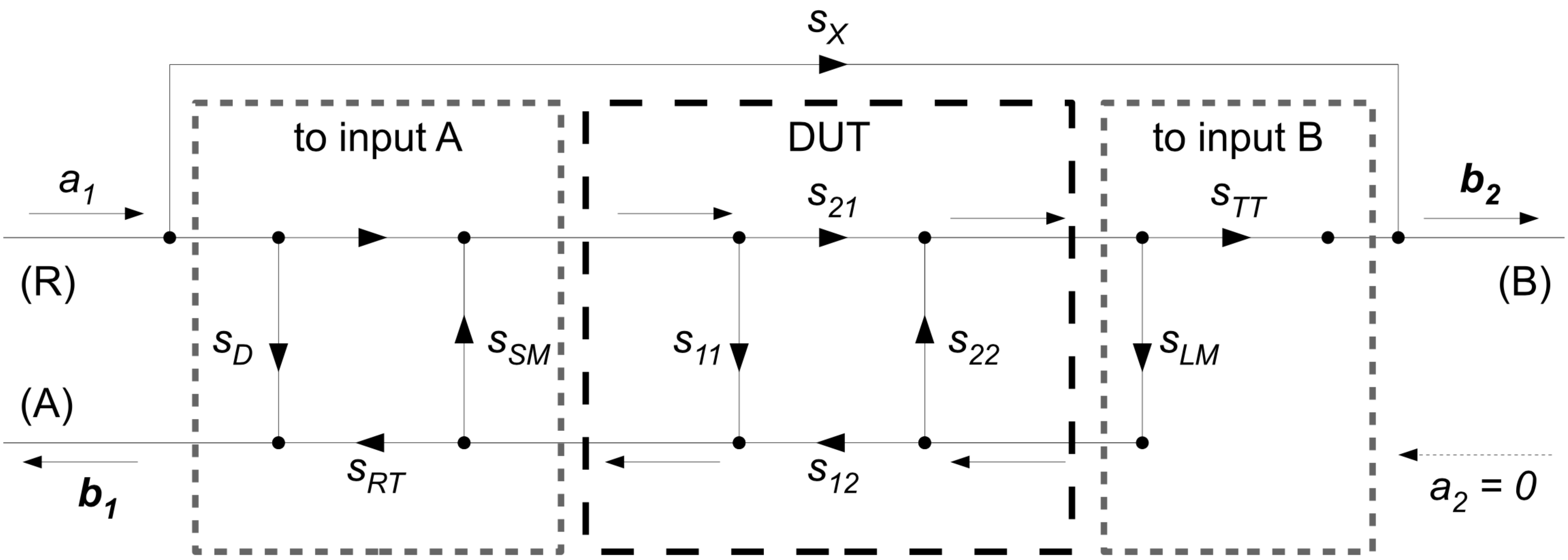
- Quantities a and b propagate along **arcs**, whose orientation is shown by arrows
- **Nodes** are summing points, with arcs converging towards a node or spreading out from it, according to orientation
- Parameter s_{ij} denotes that if a_j is the quantity entering an arc, its output will be $s_{ij}a_j$





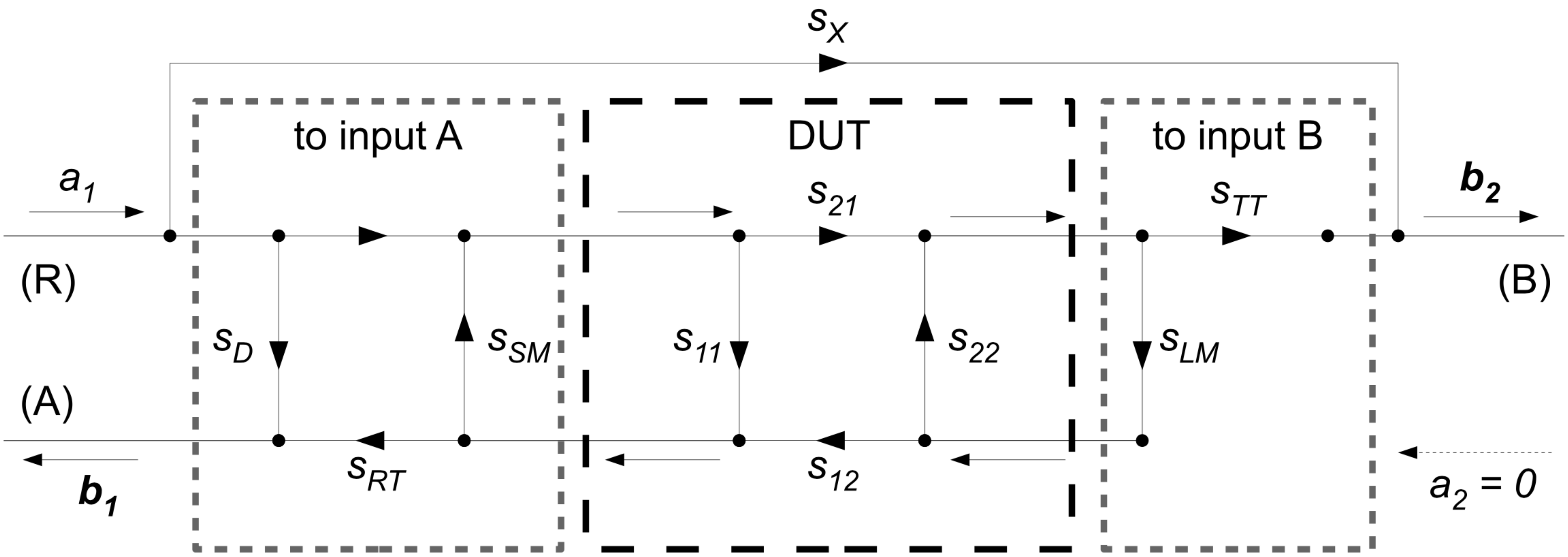
Measurement set-up calibration

- Factors affecting the measurement result
- Effects of test set-up non-ideality



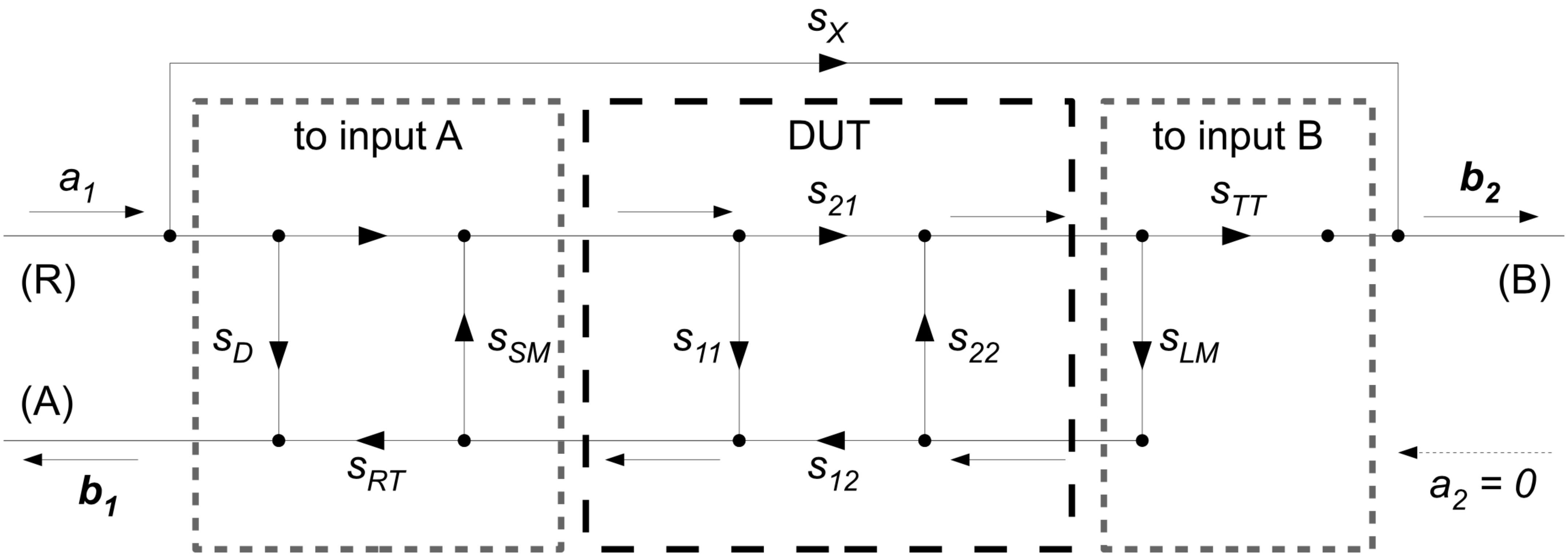
Measurement set-up calibration

- s_D: limitations in the **directivity of the directional coupler**, that affect input A of the network analyzer. It is a complex parameter, which allows to also consider the effects on the measurement of phase



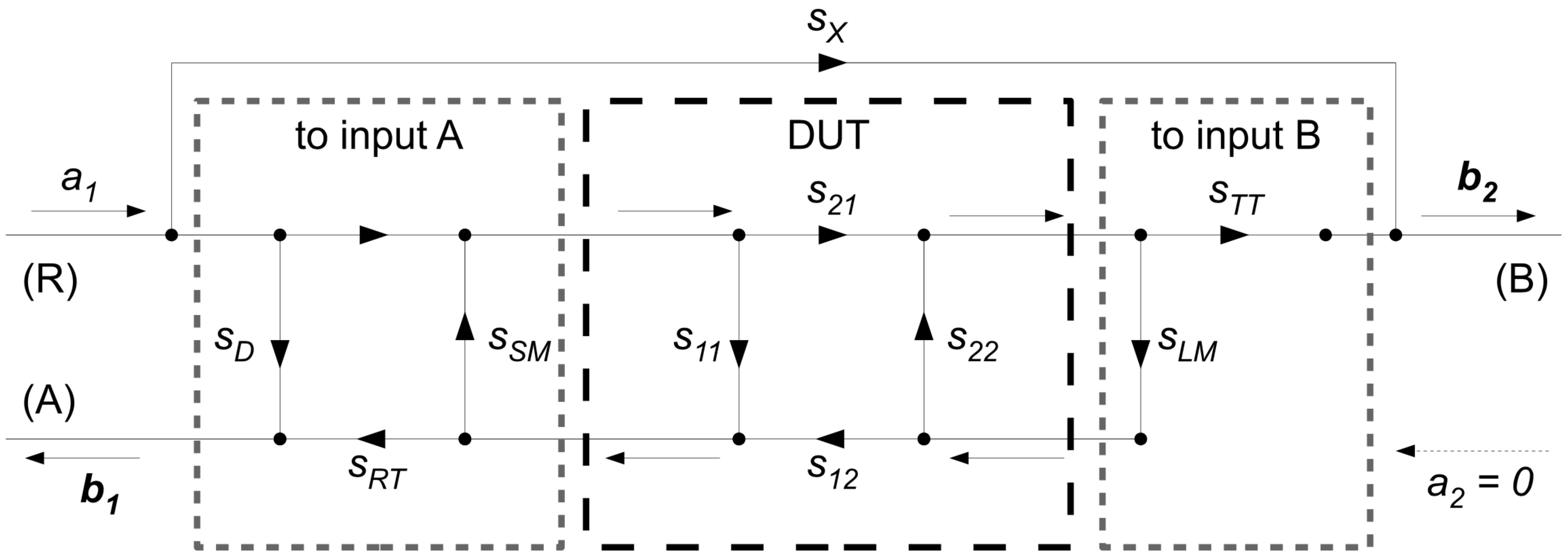
Measurement set-up calibration

- s_{SM} and s_{LM}: possible effects of impedance mismatch between the measuring system and DUT ports (impedances not corresponding to Z_0. On the synthesizer side this is called **source mismatch**. Similarly, **load mismatch** (LM) may occur on the side of the termination load



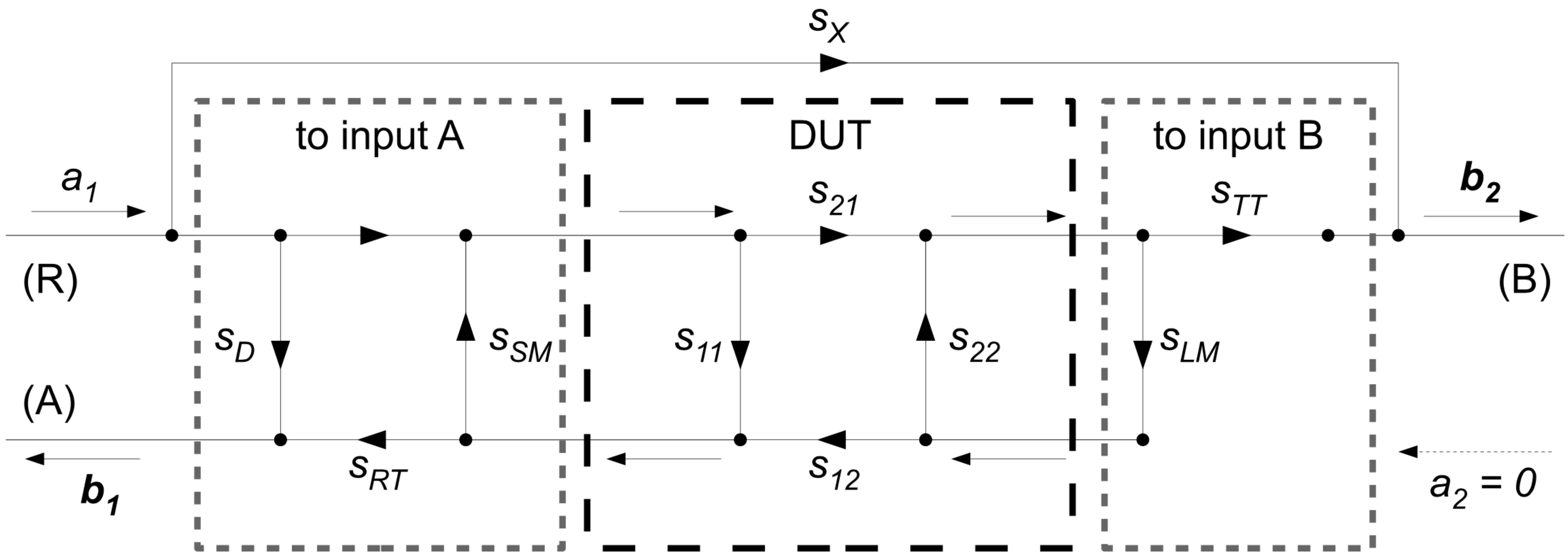
Measurement set-up calibration

- s_{RT} and s_{TT}: **tracking errors** caused by differences between the frequency responses of input channels A and R (in this case, **reflection tracking**), or of channels B and R (**transmission tracking**)



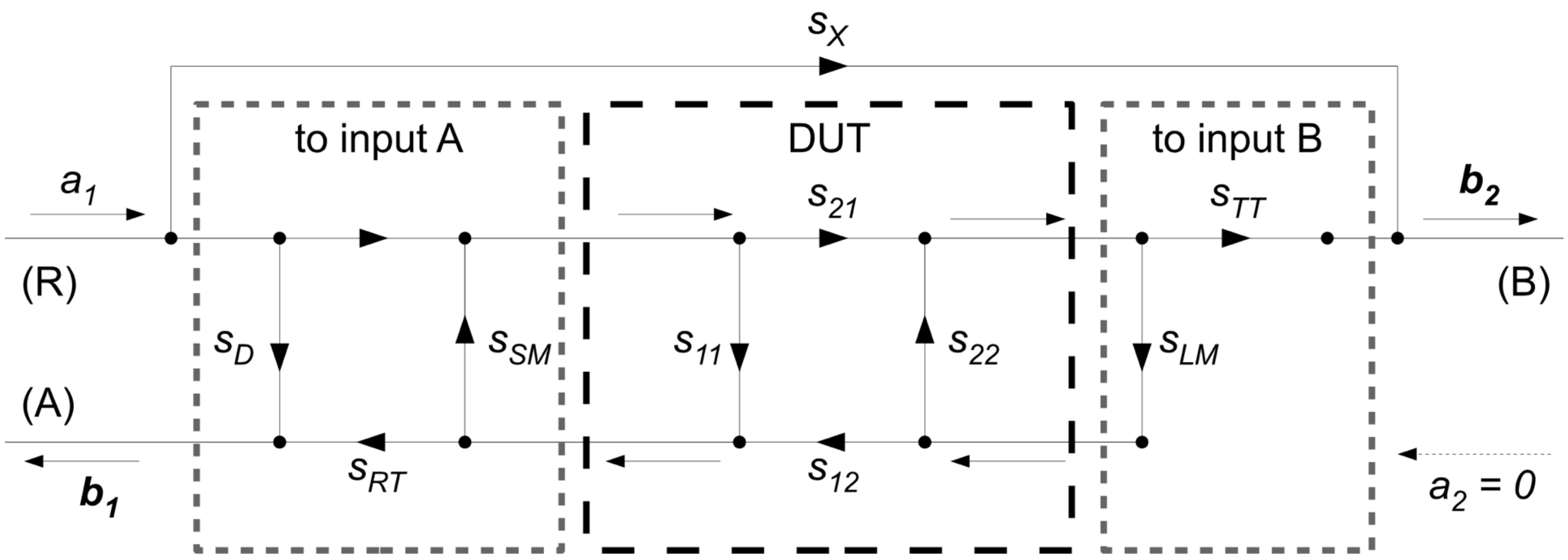
Measurement set-up calibration

- s_X: **crosstalk** effects caused by undesired coupling between the DUT ports



Measurement set-up calibration

- 6 parameters (dependent on frequency) → calibration the test set-up in the configuration where the input stimulus is sent to DUT port 1
- 6 parameters for port 2
- **12 complex parameters** at each frequency



Measurement set-up calibration

- Commonly used calibration method → set of reference conditions by means of four circuit elements:
    - Short circuit (S)
    - Open circuit (O)
    - Load impedance of known value, usually Z_0 (load, L)
    - Matched line (through, T)

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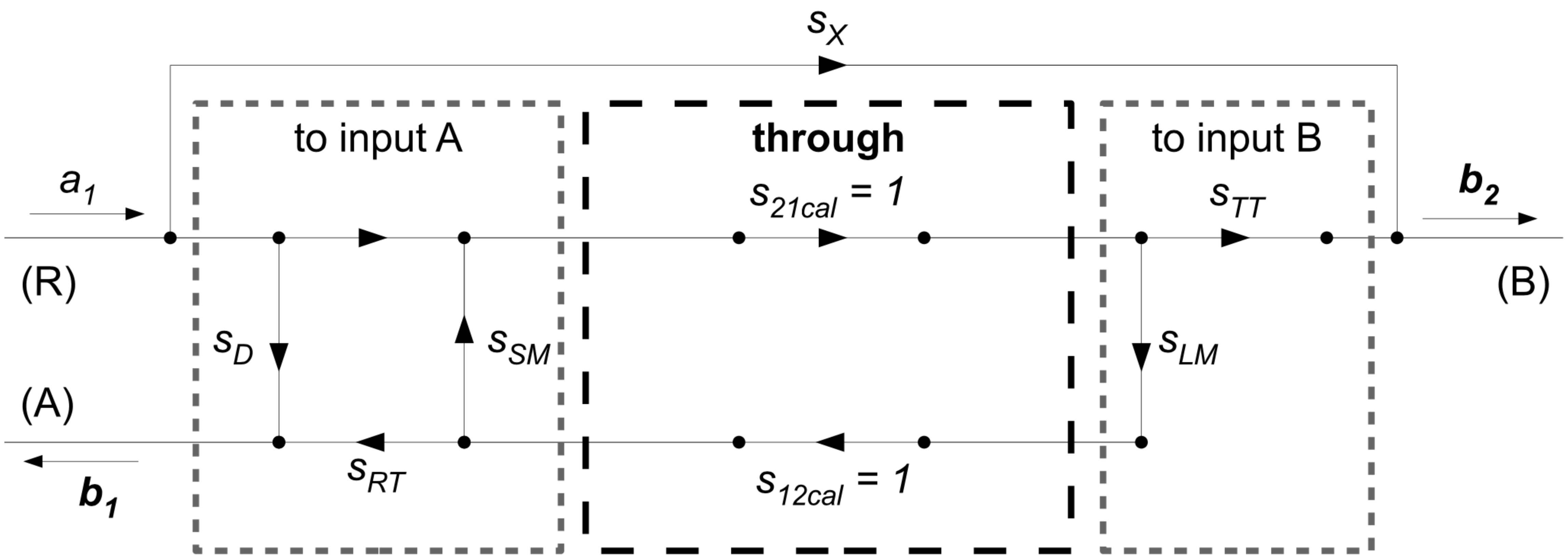
- **Calibration**: set of equations correlating measurements to calibration parameters only

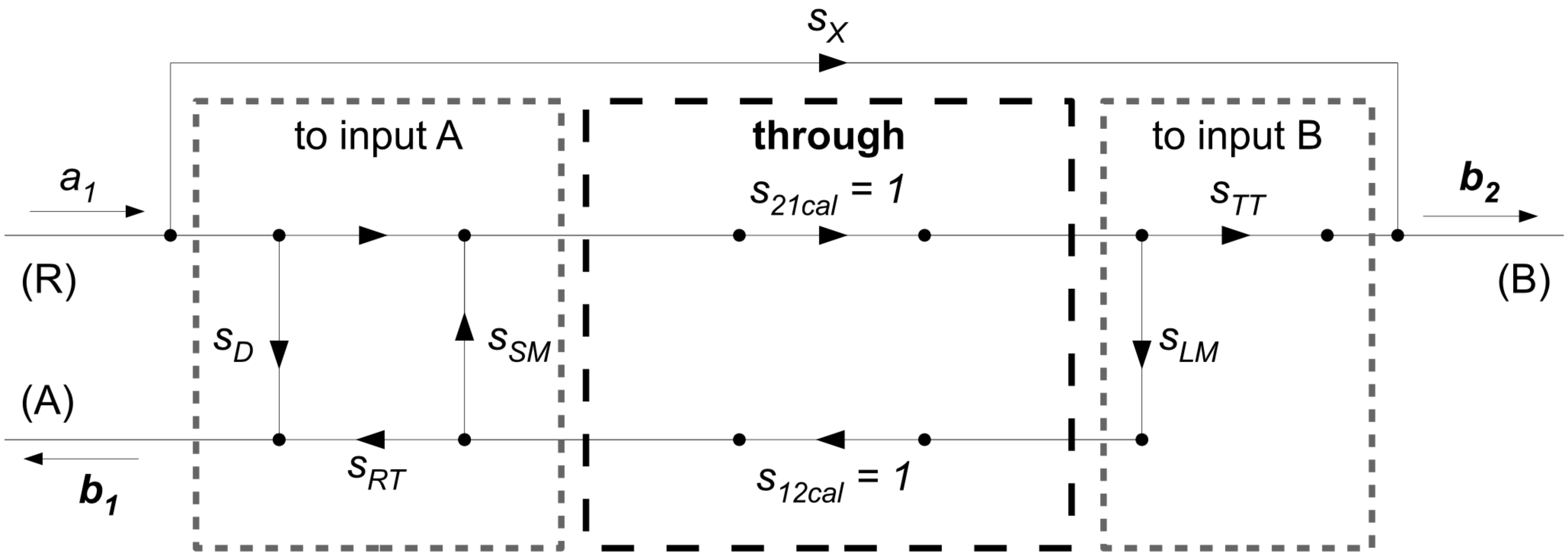
Measurement set-up calibration

- Example: use of a length of matched line (**through**) as the calibration reference



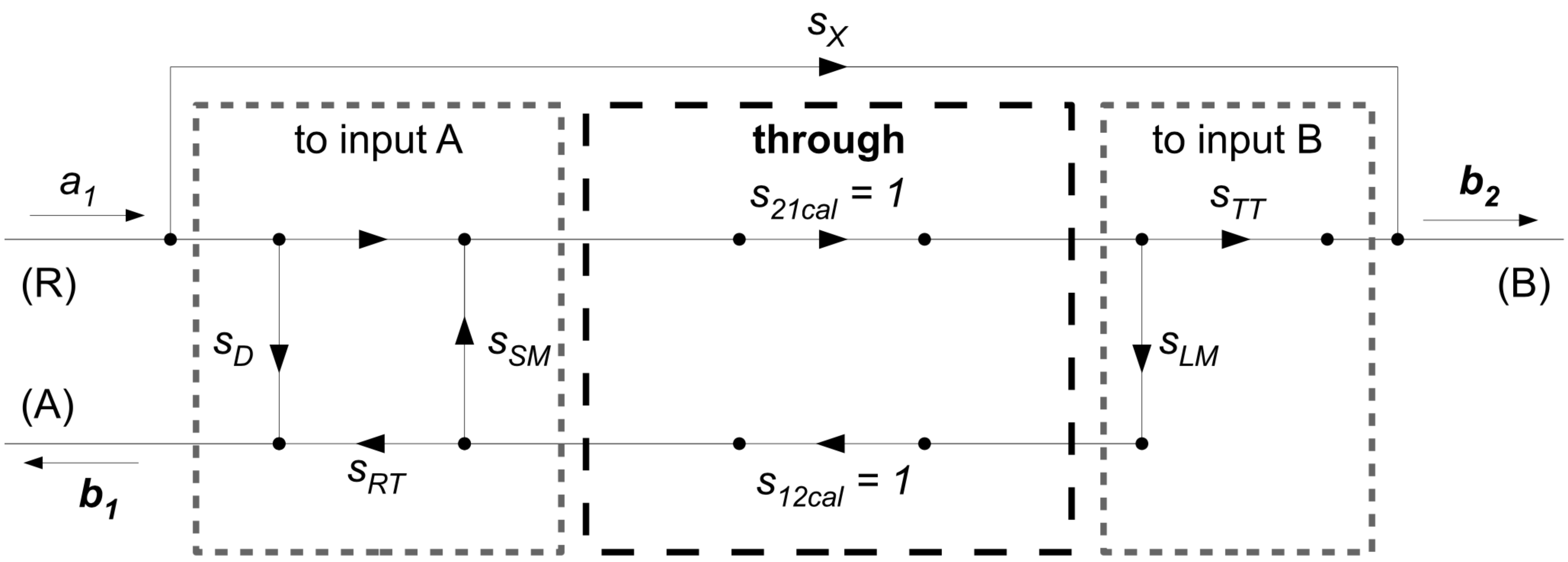
Measurement set-up calibration

- **s**-parameters: $s_{12cal} = s_{21cal} = 1,\ s_{11cal} = s_{22cal} = 0$
- By measuring ratios $\frac{\mathbf{V}_A}{\mathbf{V}_R}$ and $\frac{\mathbf{V}_B}{\mathbf{V}_R}$, that are primarily affected by tracking and mismatch errors (parameters s_{RT}, s_{TT}, s_{SM} and s_{LM}), the actual frequency response of the measuring system alone is determined



Measurement set-up calibration

- **Ideal through:** frequency response having unit magnitude and linear phase characteristic proportional to the line length
- Correction factors based on the measured response can be calculated and stored as calibration data



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