UNIVERSITÀ DEGLI STUDI DI PADOVA

Spectrum analyzer II

Lecture #12
Electronic measurements
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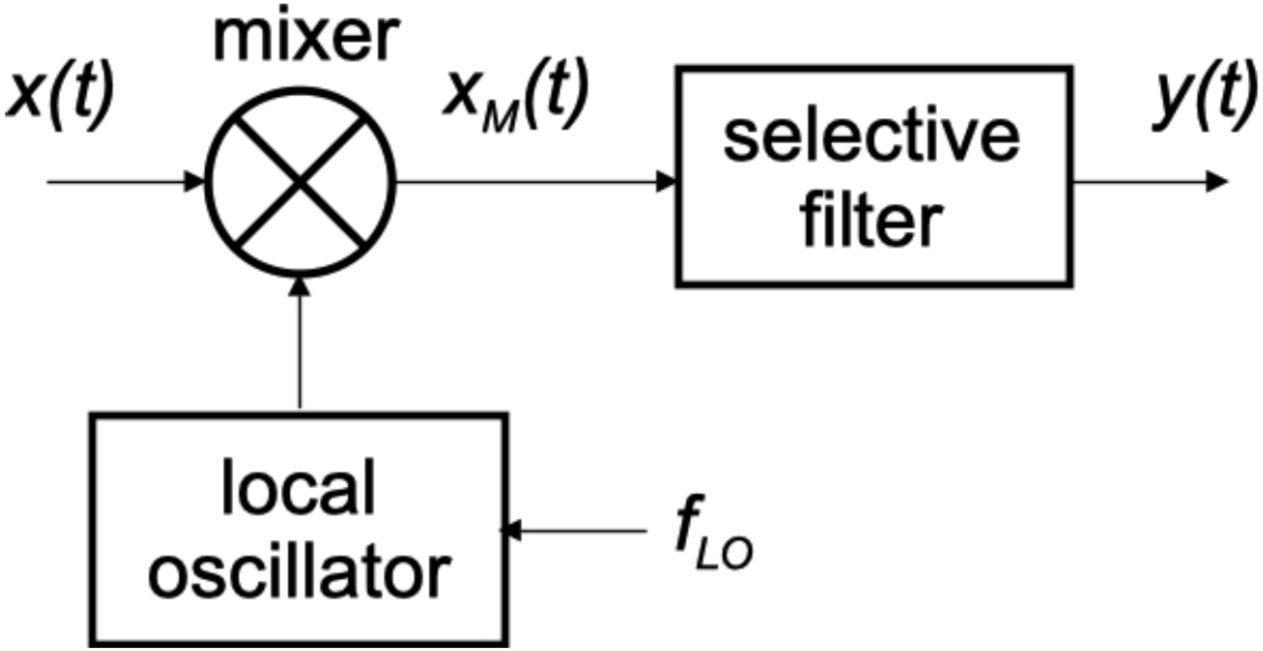
Swept frequency spectrum analyzer

Basic elements:

- Mixer
- Variable-frequency oscillator
- Selective filter



Swept frequency spectrum analyzer

Mixer

- **Two inputs**
  - Analyzed signal $x(t)$
  - Voltage controlled local oscillator $x_{LO}(t)$
- **One output** $x_M(t)$

- The **mixer output amplitude** is proportional to the amplitude of input signal $x(t)$ only $\Rightarrow x_M(t) = K_M \cdot x(t)$ (K_M conversion factor)
- The **mixer output frequency** $f_{x(M)}$ is related to f_x and f_{LO} by these relationships:

$$f_{x(M)} = f_{LO} \pm f_x \text{ if } f_{LO} > f_x$$

$$f_{x(M)} = f_x \pm f_{LO} \text{ if } f_{LO} < f_x$$

Swept frequency spectrum analyzer

Mixer

- **Mathematical product** between the two inputs ⇒ **Amplitude modulation**
  - $x_{LO}(t)$ Carrier
  - $x(t)$ Modulating signal

$$x_M(t) = K_M \cdot x(t) \cdot \cos(2\pi f_{LO} t)$$

$$X_M(f) = K_M \cdot \frac{1}{2} \cdot [X(f - f_{LO}) + X(f + f_{LO})]$$

Swept frequency spectrum analyzer

Selective filter

- Pass band filter whose centre frequency is fixed and precisely known ⇒ **Intermediate Frequency (IF) filter**
- Let only a narrow band of the input spectrum to pass through at the output

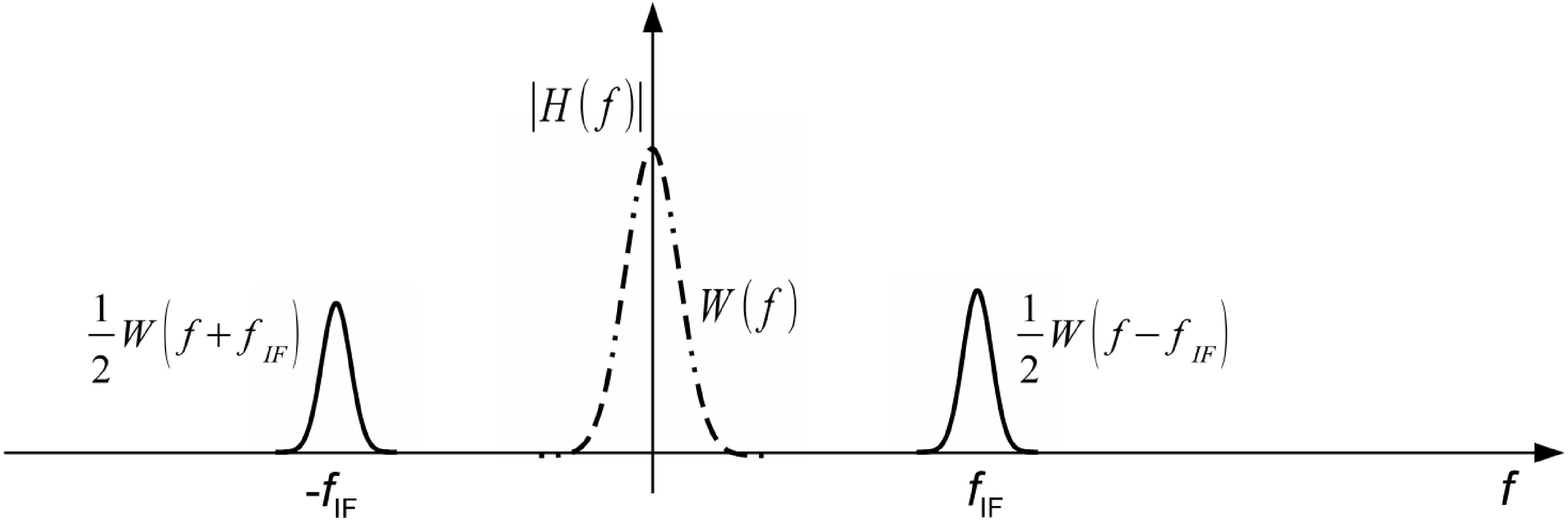
- **Impulse response** of a symmetric pass band filter

$$h(t) = w(t)\cos(2\pi f_{IF} t)$$
$$H(f) = \frac{1}{2}[W(f - f_{IF}) + W(f + f_{IF})]$$

- $w(t)$ filter envelope

Swept frequency spectrum analyzer

Selective filter



- **Selective filter** $\Rightarrow B_H \ll f_{IF}$ (B_H filter bandwidth)
- $w(t)$ impulse response of a **base band filter** with frequency response with the same shape as $H(f)$

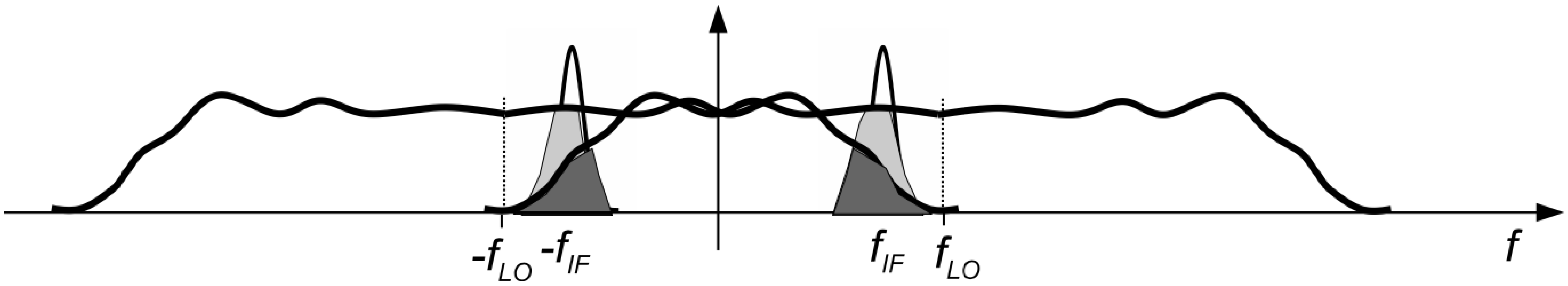
Swept frequency spectrum analyzer

Selective filter

- IF filter output $Y(f)$

$$Y(f) = X_M(f) \cdot H(f) = \frac{K_M}{4} \cdot [W(f - f_{IF}) + W(f + f_{IF})] \cdot [X(f - f_{LO}) + X(f + f_{LO})]$$

- Spectral interference

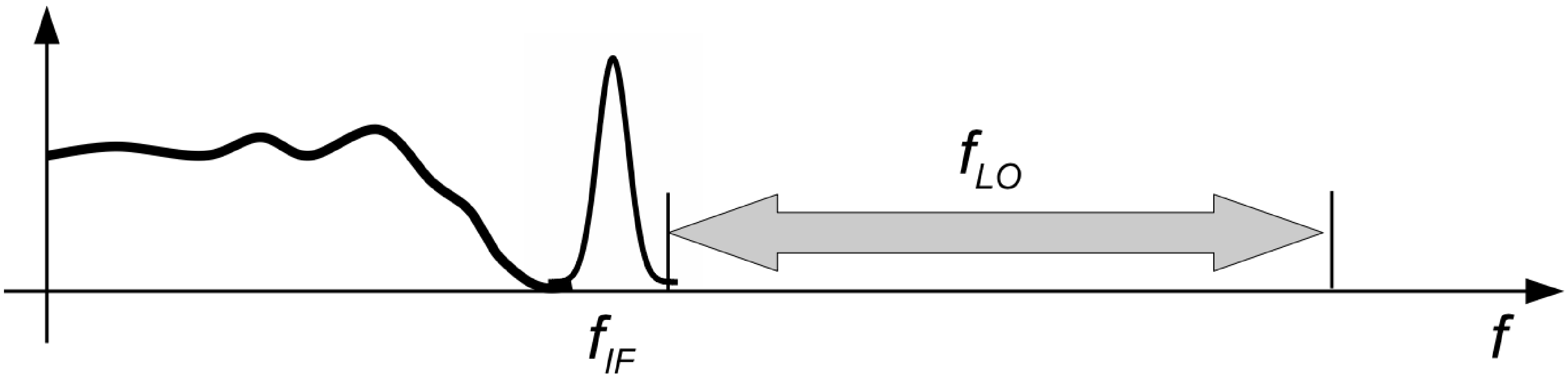


Swept frequency spectrum analyzer

Frequency scan

- f_{MAX} upper limit of the input frequency range
- Local oscillator frequency between $f_{LO_{min}}$ and $f_{LO_{MAX}}$

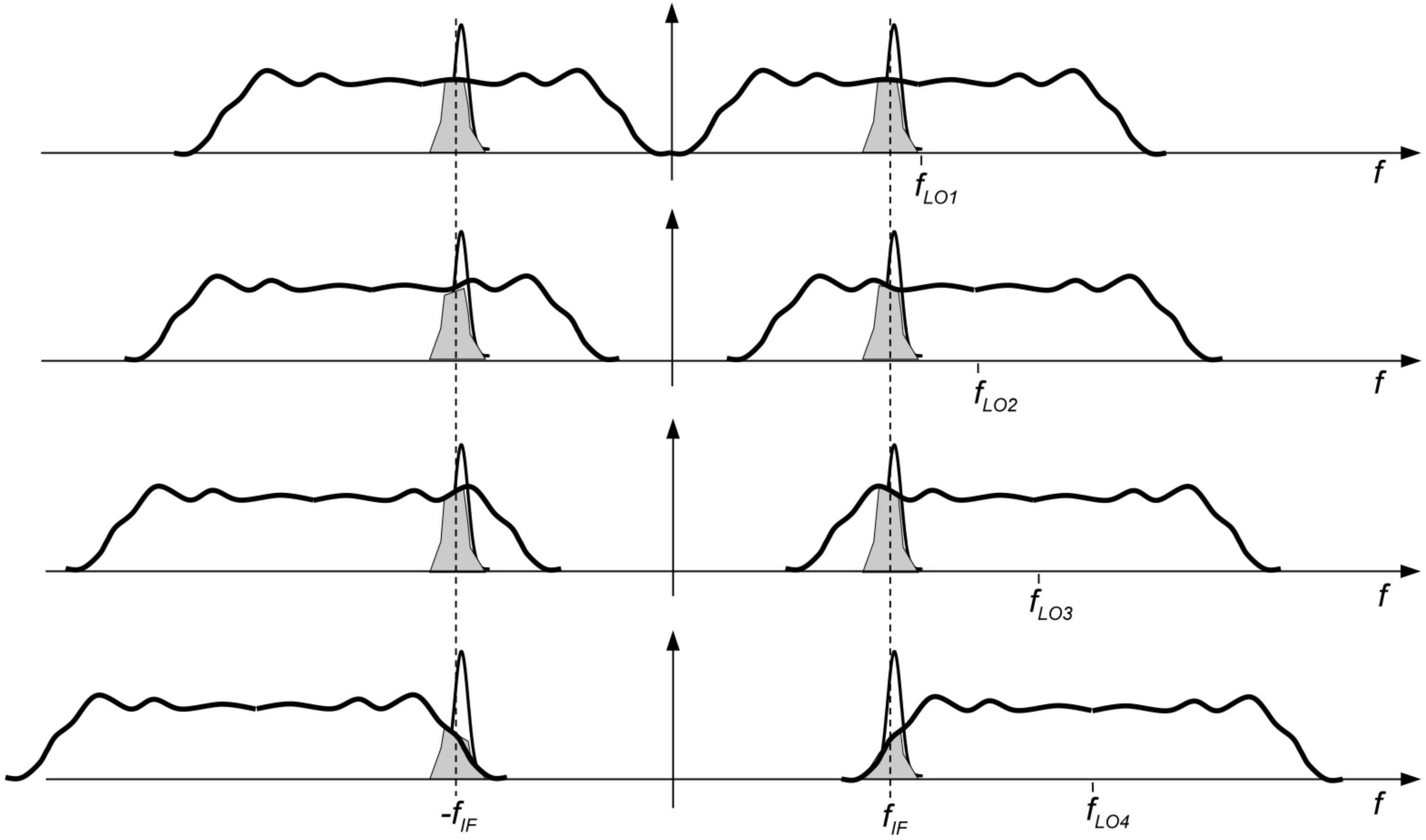
$$f_{MAX} < f_{IF} < f_{LO_{min}}$$



Swept frequency spectrum analyzer

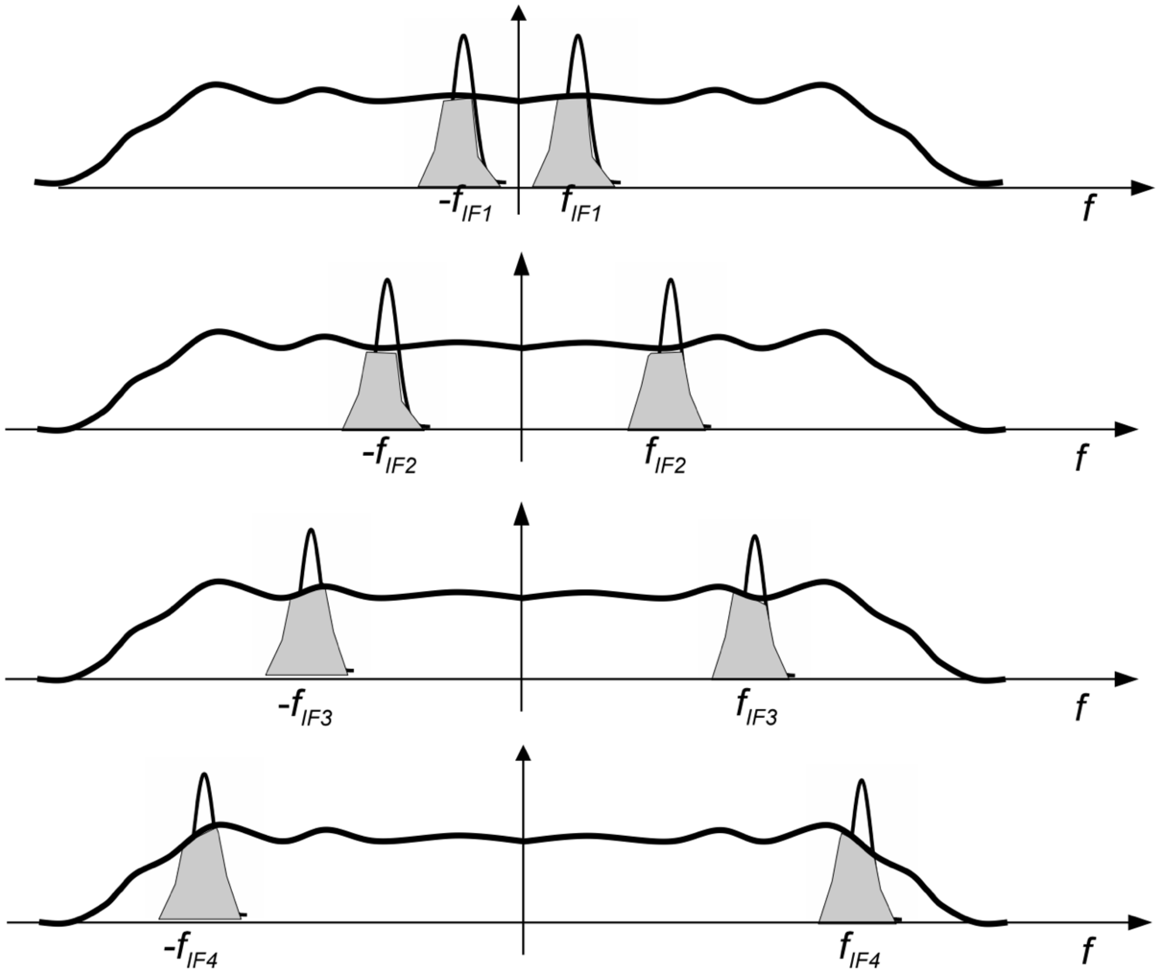
Frequency scan

$$Y(f) = X_M(f) \cdot H(f) = \frac{K_M}{4} \cdot [W(f - f_{IF}) \cdot X(f - f_{LO}) + W(f + f_{IF}) \cdot X(f + f_{LO})]$$



Swept frequency spectrum analyzer

Selective variable-frequency filter output



Swept frequency spectrum analyzer

Frequency transposition

- IF filter output constant
- Frequency information provided by the local oscillator control input

Selective variable-frequency filter output

- Variable output frequency

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Swept frequency spectrum analyzer

- Ideal IF filter ⇒ **infinitesimal bandwidth**
- **Filter output** ⇒ inverse Fourier transform of $Y(f)$ when $W(f) = W_0 \cdot \delta(f)$

$$y(t) = \frac{K_M \cdot W_0}{4} \cdot |X(f_{LO} - f_{IF})| \cdot \cos(2\pi f_{IF} t - \arg[X(f_{LO} - f_{IF})])$$

- Any spectral component of $x(t)$ between $f_{LO_{min}} - f_{IF}$ and $f_{LO_{MAX}} - f_{IF}$ can be selected

Swept frequency spectrum analyzer

- During a sweep the local oscillator frequency varies linearly with time from $f_1 \geq f_{LO_{min}}$ to $f_2 \leq f_{LO_{MAX}}$
- $f_{START} = f_1 - f_{IF}$
- $f_{STOP} = f_2 - f_{IF}$

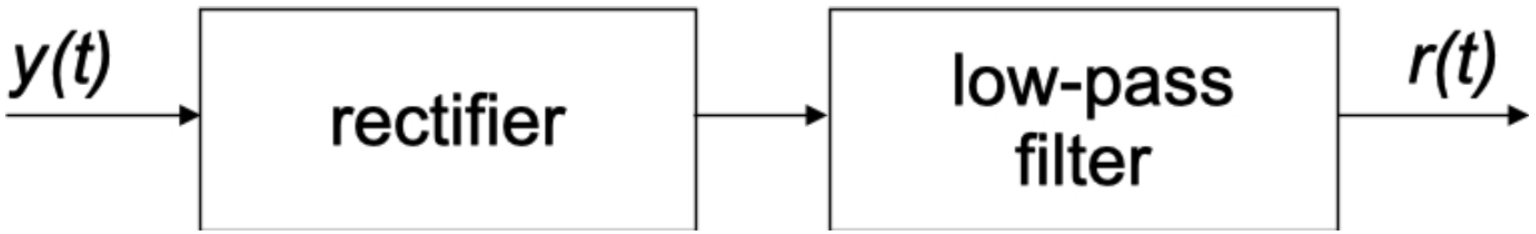
- **Frequency span** $f_{SPAN} = f_{STOP} - f_{START}$
- **Centre frequency** $f_{CENTRE} = \frac{f_{STOP} + f_{START}}{2}$

- **Sweep time** t_{SWEEP}
- **Sweep speed** f_{SPAN}/t_{SWEEP}

Envelope detector

- Converts the IF filter output waveform into amplitude (and power) information
- Converts a sinusoidal input into a voltage proportional to the amplitude

- **Features**:
  - Correct operation in a wide range of amplitude values (instrument amplitude dynamics, typically 70-80 dB)
  - Low distortion



Envelope detector

- **Rectifier**: produces both base-band components associated to the waveform envelope, and frequency components located at multiples of the IF filter centre frequency
- **Low pass filter**: eliminates the latter, leaving only the base-band component
- Envelope detector output $r(t)$ is proportional to $|X(f_{LO} - f_{IF})|$
- The output of the spectrum analyzer is a plot of $r(t)$ with the horizontal axis calibrated in frequency

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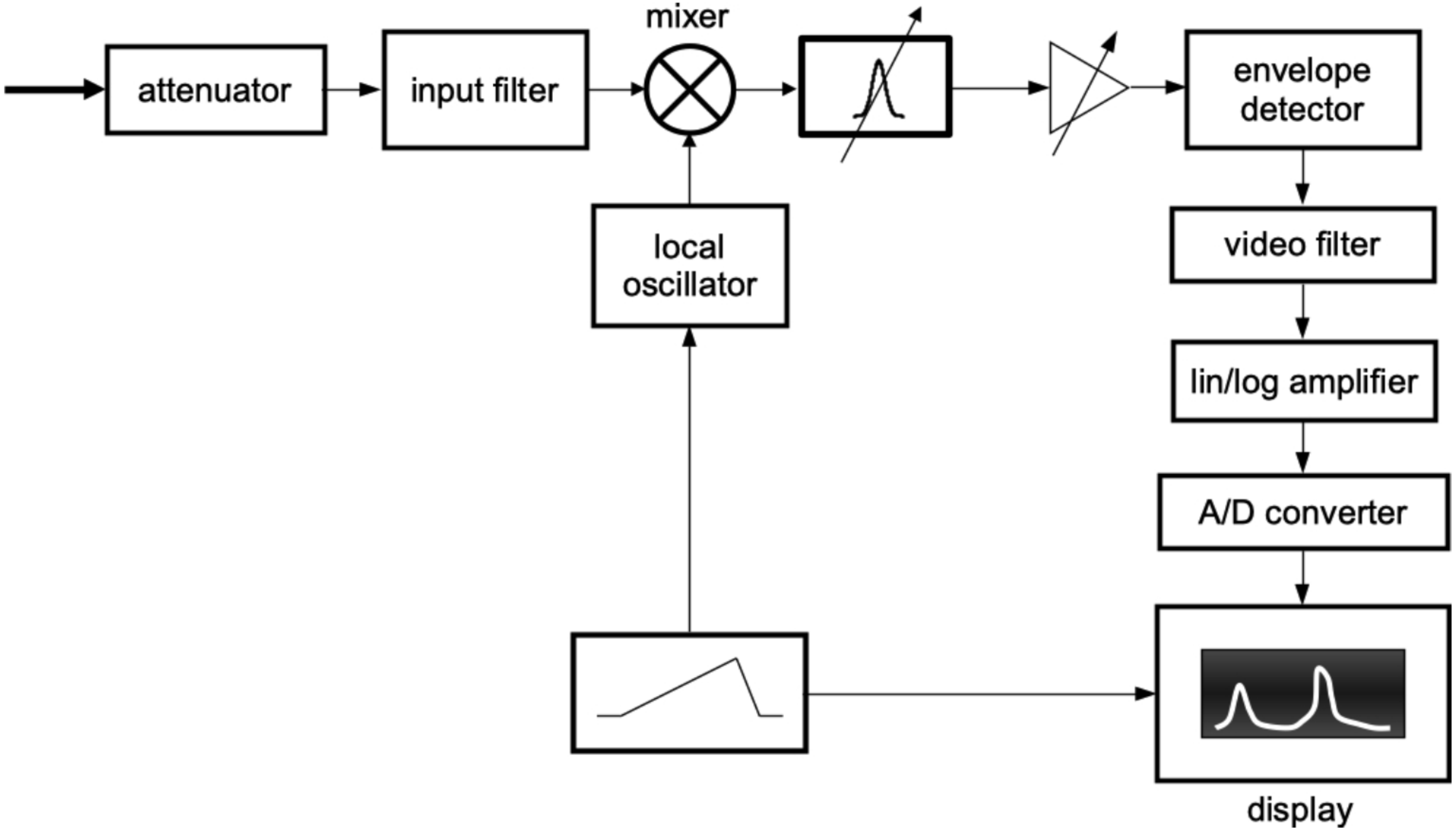
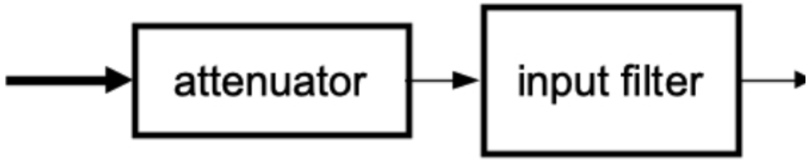
Functional diagram



Functional diagram



- **Input attenuator**: varies the amplitude of the signal that is sent to the mixer to prevent overloading → Possible permanent damage to the mixer
- **Maximum input power** for a spectrum analyzer is typically 1 W (+30 dBm) and the attenuator can reduce the signal by as much as 70 dB, so that RMS voltage into the mixer is usually a few mV.

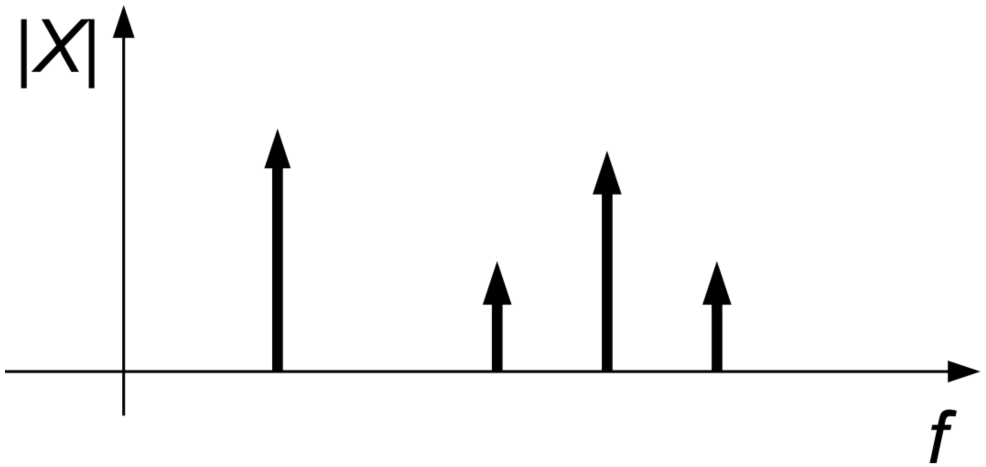
- **Broadband input** → Care must be taken to consider the total power into the mixer, not only that of the analyzed signal source → **Preselector filters**

Discrete-spectrum signals

- **Amplitude spectrum** (one sided-Fourier transform):

$$|X(f)| = \left| \sum_k A_k \delta(f - f_k) \right|$$

- $\delta(f)$ Dirac distribution
- A_k amplitude component
- f_k frequency component



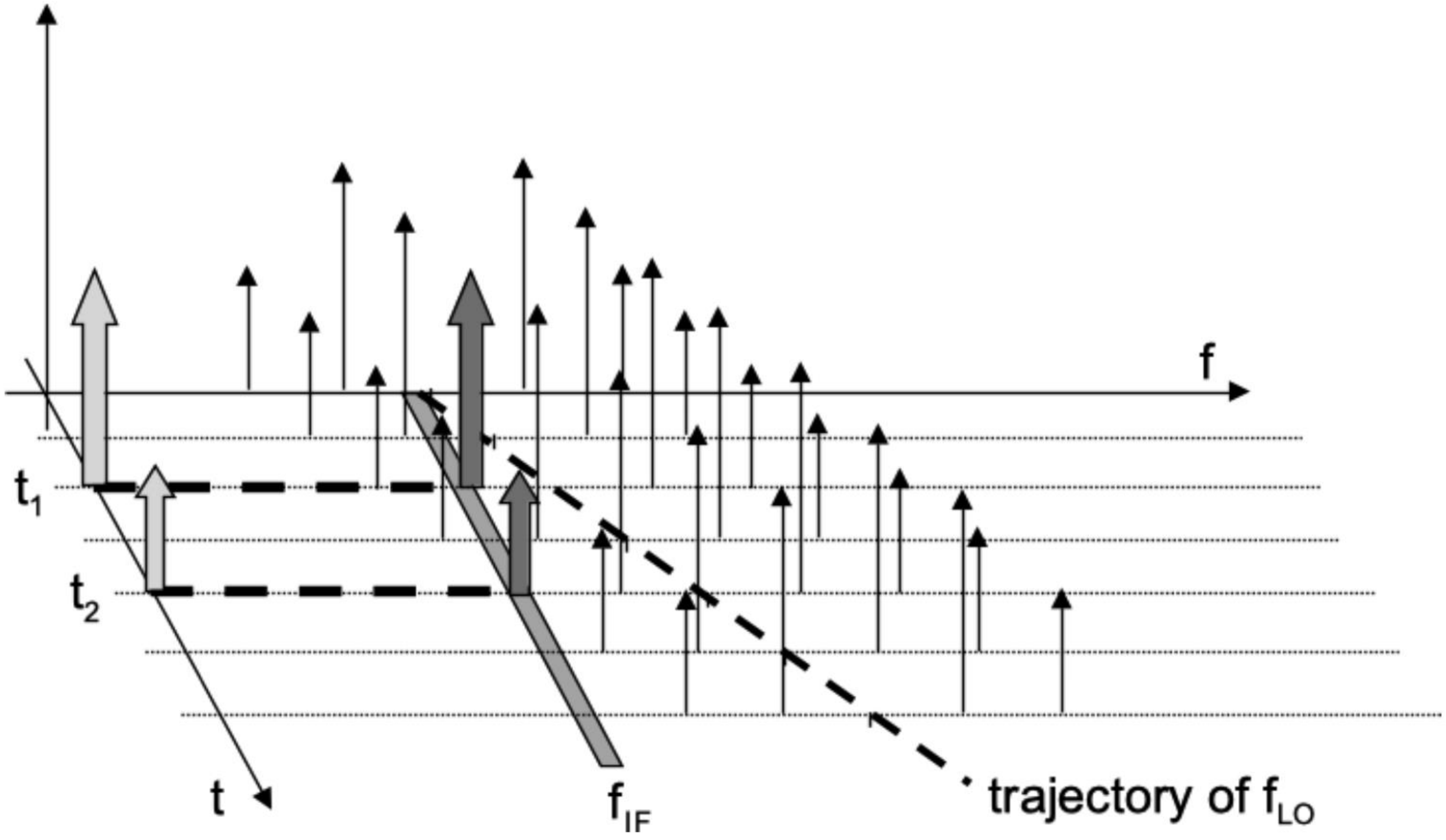
Discrete-spectrum signals

- **Example:** two sinusoids
- $f_1 = 30$ MHz
- $f_2 = 60$ MHz

- Local oscillator frequency f_{LO} varying between 200 and 300 MHz
- IF filter is centred at $f_{IF} = 200$ MHz
  - $f_{LO} - f_1$ varying between 170 and 270 MHz
  - $f_{LO} - f_2$ varying between 140 and 240 MHz

    f_{IF}
  - $f_{LO} + f_2$ varying between 230 and 330 MHz
  - $f_{LO} + f_1$ varying between 260 and 360 MHz

Discrete-spectrum signals



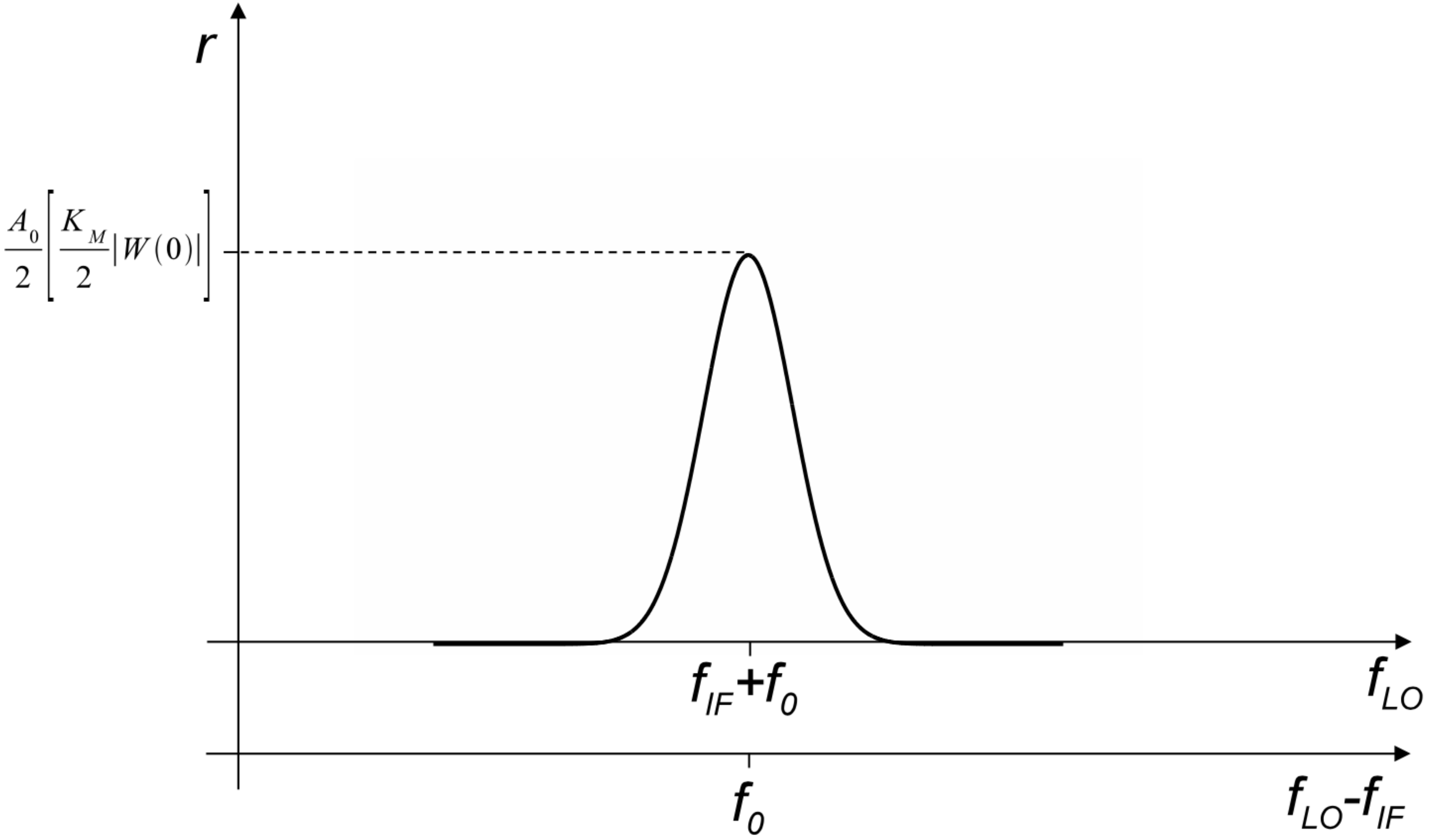
The IF filter output is, ideally, a sinusoid at frequency f_{IF}= 200 MHz, but the two responses can be distinguished since they are obtained at the two different times t_1 and t_2

Sinusoidal signals

- **Signal frequency** f_0
- $f_{LO} - f_{IF} = f_0$

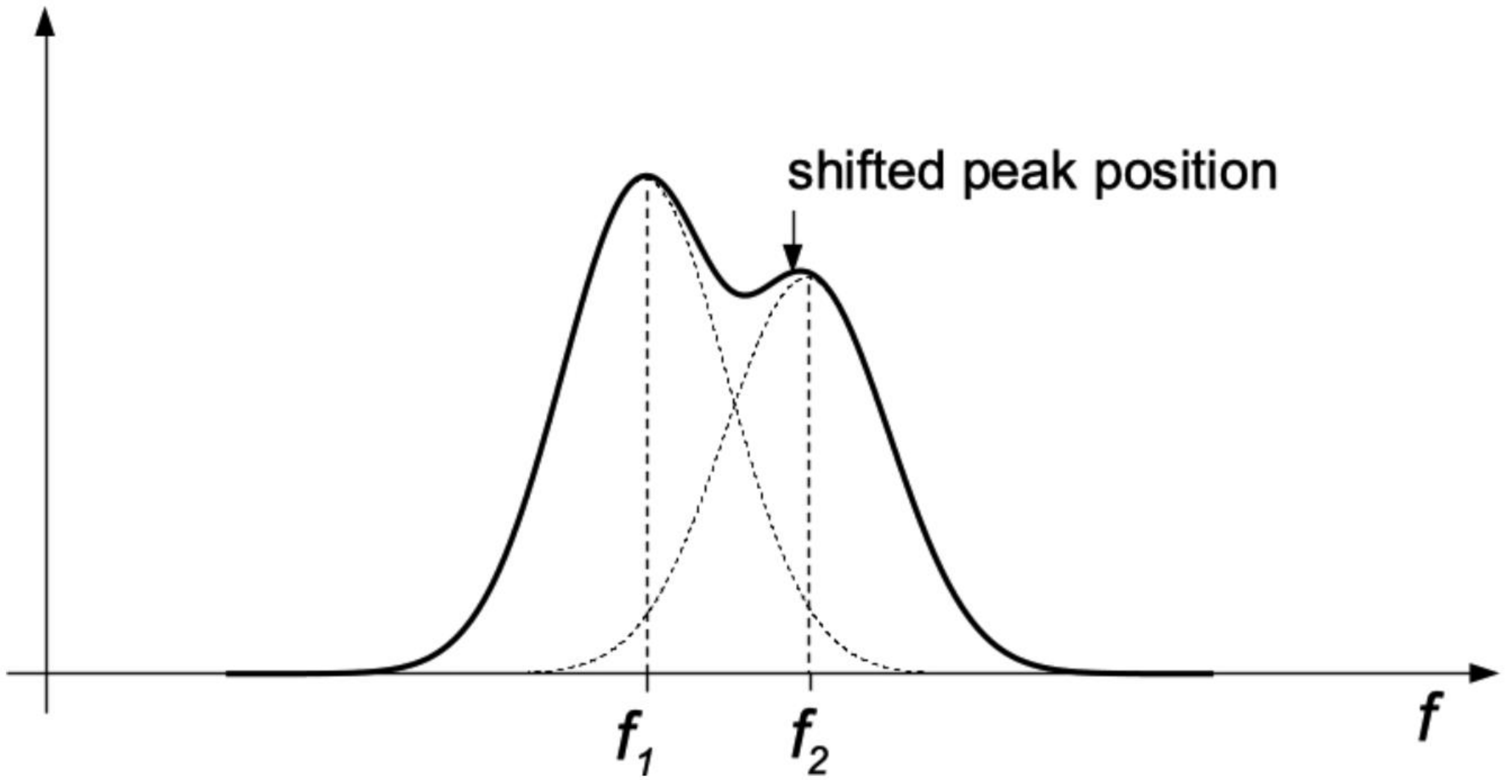
- Filter bandwidth is finite → non zero response also at close but slightly different frequencies
- IF filter function $|W(f)|$ centered at the sinewave frequency
- Amplitude of the peak → IF filter centre frequency

Sinusoidal signals



Sinusoidal signals

- **Sinewaves with different amplitudes**
- Peak position shifting
- **Selectivity** in addition to frequency resolution



Spectrum analyzer dynamic behaviour

- **Very slow variation of** f_{LO} $\rightarrow$ **Quasi-static measurement**
- Continuous IF filter output variation
- Discrete spectrum $\rightarrow$ Response to a sequence of sinusoidal inputs

- IF filter response calibration $\rightarrow$ **centre frequency gain** $|W(0)|$
- Quasi-static measurement $\rightarrow$ **transient response**

⬇

Sweep speed and IF filter bandwidth

Spectrum analyzer dynamic behaviour

- **Interdepent parameters**
  - Frequency span F_{SPAN}
  - Sweep time t_{SWEEP}
  - Resolution bandwidth B_R

- **Time** a frequency-swept sinewave remains within the IF bandwidth → ratio of bandwidth to sweep time
- **Quasi-static requirement** → This **time** is long enough to allow the IF filter output to reach steady state
- IF filter transient response is inversely proportional to its bandwidth

$$\frac{1}{B_R} < \frac{B_R \cdot t_{SWEEP}}{F_{SPAN}} \quad \text{from which:} \quad t_{SWEEP} > \frac{F_{SPAN}}{B_R^2}$$

Spectrum analyzer dynamic behaviour

- **Interdepent parameters**
  - Frequency span F_{SPAN}
  - Sweep time t_{SWEEP}
  - Resolution bandwidth B_R

- Sweep time should be increased in inverse proportion to the square of the resolution bandwidth
- Measurement time is significantly increased when a very narrow bandwidth analysis is required
- When a wide frequency span is of interest, measurement time may become impractically long
- B_R needs to be adjustable, so that resolution is adequate but not overspecified

Continuous-spectrum signals

- Continuous-spectrum signal $\Rightarrow$ Resolution bandwidth B_R **narrow** if compared to the signal bandwidth B
- The PSD of $x(t)$ is approximately **constant** within the selective filter bandwidth
- Sweep speed is low enough to consider a **quasi-static succession of constant values**

- Selective filter **impulse response**

$$h(t) = w(t)\cos(2\pi f_{IF} t)$$

- Output **amplitude spectrum**

$$Y(f) = X_M(f) \cdot H(f) = \frac{K_M}{4} \cdot [W(f - f_{IF}) + W(f + f_{IF})] \cdot [X(f - f_{LO}) + X(f + f_{LO})]$$

Continuous-spectrum signals

- For a given value of f_{LO}

$$S_{xx}(f) \equiv S_{xx}(f_{LO} - f_{IF}) = S_{xx}(f_x)$$

$$S_{yy}(f) \propto S_{xx}(f_x)[|W(f - f_{IF})|^2 + |W(f + f_{IF})|^2]$$

- Since the **output of the detector** after the selective filter is proportional to **RMS value**, its indication is related to **power** rather than PSD

$$P_y(f_x) = \frac{\int_{-\infty}^{+\infty} S_{yy}(f)df}{R_i} \propto \frac{2S_{xx}(f_x)}{R_i} \int_{-\infty}^{+\infty} [W(f)]^2 df$$

Continuous-spectrum signals

- The instrument calibration accounts for the IF filter centre-frequency gain $|W(0)|$
- **Power indication** depends on the whole response ($\int_{-\infty}^{+\infty}[W(f)]^2 df$)
- This quantity depends on the resolution bandwidth

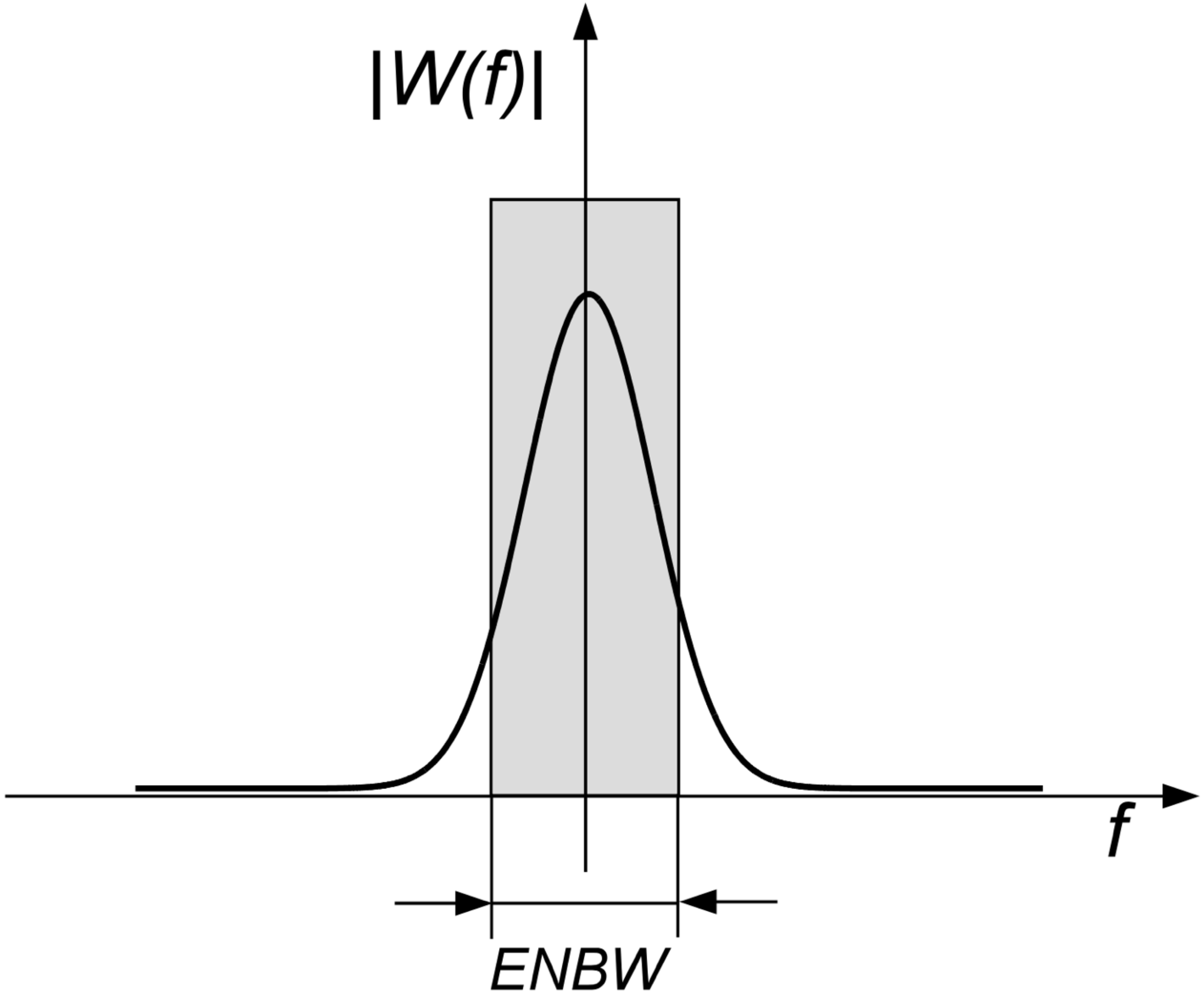
- **Equivalent Noise Bandwidth (ENBW)**:

$$ENBW = \frac{\int_{-\infty}^{+\infty}[W(f)]^2 df}{|W(0)|^2}$$

- Bandwidth of an **equivalent filter**, whose **output power** with white noise applied at the input is the same as W(f) and having a rectangular frequency response with the same centre-frequency gain $|W(0)|$

Continuous-spectrum signals

- Equivalent Noise Bandwidth (ENBW):



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Continuous-spectrum signals

- Spectrum analyzer reading in case of a continuous-spectrum signal:

$$\frac{2 \cdot S_{xx}(f)}{R_i} \cdot \frac{\pi}{4} \cdot ENBW$$

- $\frac{\pi}{4} \rightarrow$ Detector related constant
- $ENBW/B_R$ is a constant shape factor for a given class of filters $\rightarrow$ It does not change when B_R is changed

$$\frac{2 \cdot S_{xx}(f)}{R_i} \cdot B_R \cdot \left(\frac{\pi}{4} \cdot \frac{ENBW}{B_R}\right)$$

- Power Spectral Density:

$$PSD_x\left[\frac{\mathrm{dBm}}{\mathrm{Hz}}\right] = P_x[\mathrm{dBm}] - 10\log_{10} B_R - 10\log_{10}\left(\frac{\pi}{4} \cdot \frac{ENBW}{B_R}\right)$$

Continuous-spectrum signals

- Standard spectrum analyzer reading indicates the power **corresponding to the RMS value of a sinusoidal component** (dBm)
- **Noise marker**: directly provides the correct PSD indication (dBm/Hz)

- $S_{xx}(f)$ must be regarded as a mean value related to a random process
- Spectrum analyzer trace may evidence amplitude variability, that may be reduced by averaging if the spectrum can be assumed to be stationary:
  - **Reduction** of the video filter bandwidth B_V relative to the resolution bandwidth B_R
  - **Average** of the displayed trace

Zero span measurement

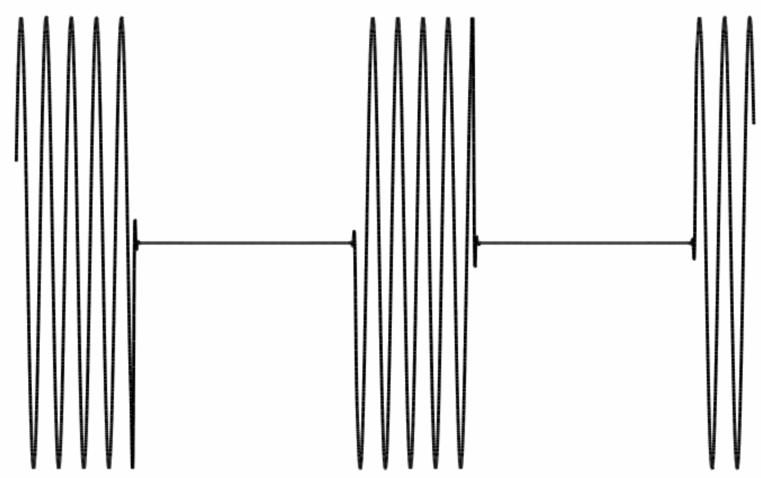
- Different operation mode → **No sweep** → Constant local oscillator frequency
- IF filter tuned to the indicated **centre frequency**
- Horizontal axis referred to **time**

- **Power vs time** display
- Observation interval set by setting t_{SWEEP}

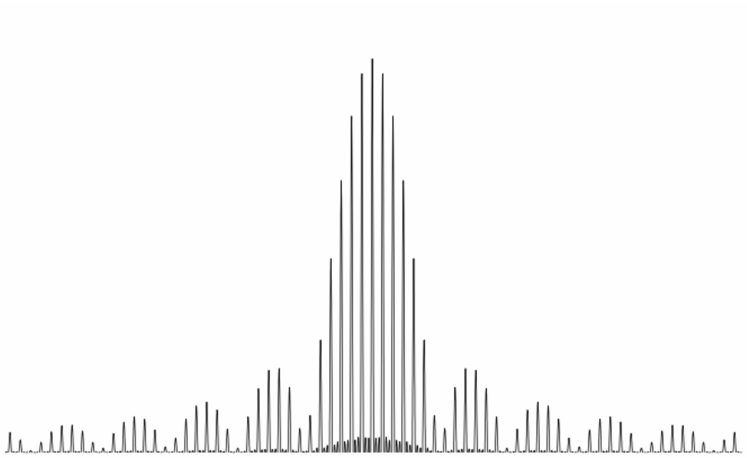
- Resolution bandwidth should correspond to **signal bandwidth**
- IF filter turned into a **preselector** for the frequency interval of interest

Zero span measurement

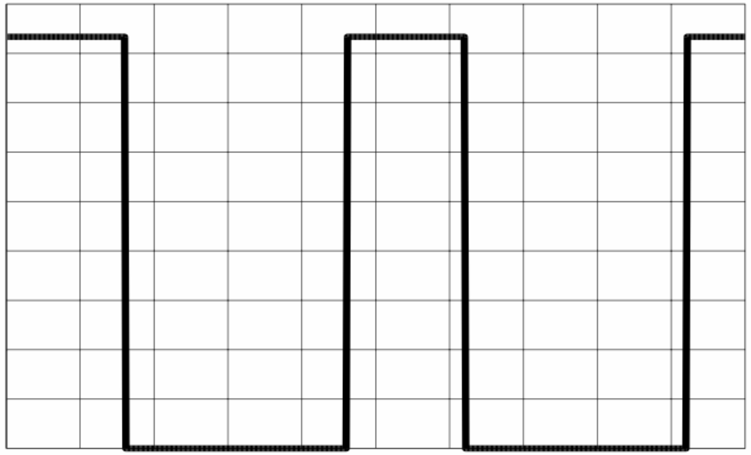
- **Frequency-selective voltmeter**
- Pulse modulated signals, amplitude-modulated signals, time-multiplexed communications, etc…



(a) pulse modulation;



(b) spectrum (linear scale);



(c) zero span measurement

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