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Spectrum analyzer

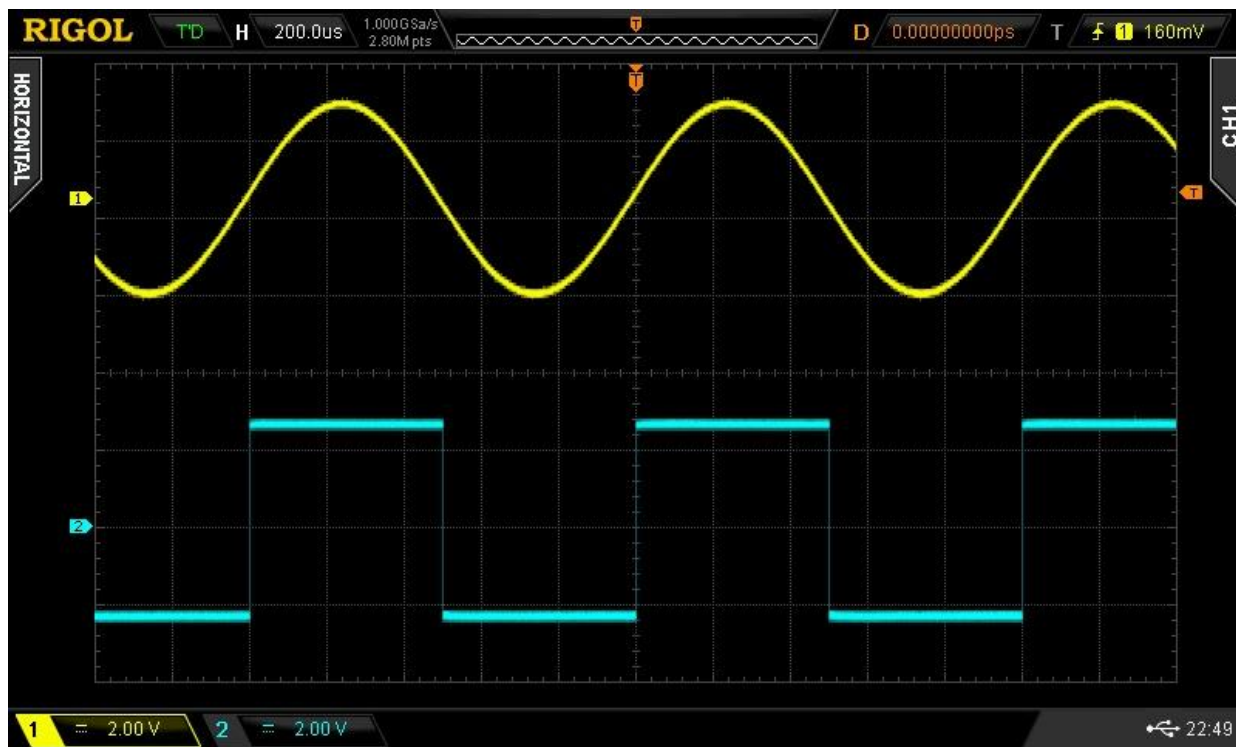
Lecture #11

Electronic measurements

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Time domain vs Frequency domain





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The spectrum analyzer





- **Signal source characterization**
- Determining signal composition in frequency
- Verifying the conformance to reference spectral behaviours
- Total Harmonic Distortion (THD)



- **Radiofrequency measurement**
- Characterization of signal modulation
- Test and verification of communication systems
- Spectrum monitoring and policing

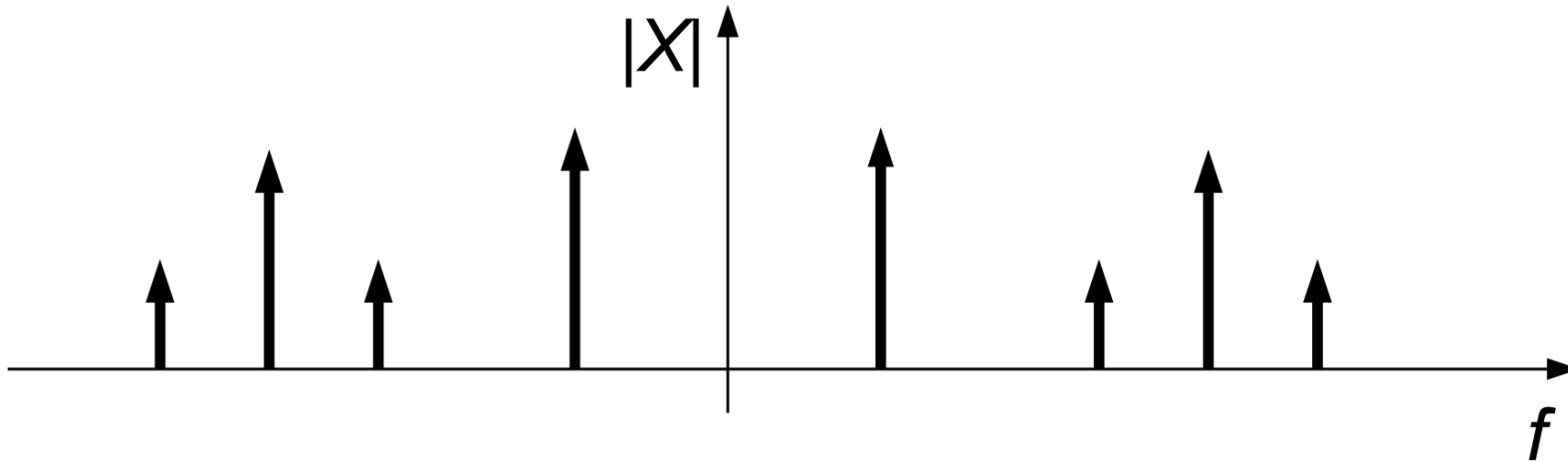


- **Electromagnetic compatibility testing**
- Disturbances **radiated** in the surrounding environment
- Disturbances **conducted** towards other devices through power supply lines
- **Electromagnetic compatibility (EMC) tests**
 - Check that emission intensities are below certain thresholds
 - EMC regulations



Discrete-spectrum signals

- Signal composed of sinusoidal terms



- If the frequencies of individual components are unrelated, the signal itself is not periodic
- $x(t)$ real function of time
- $X(f) \Rightarrow$ Hermitian symmetry: $X(-f) = X^*(f)$



Discrete-spectrum signals

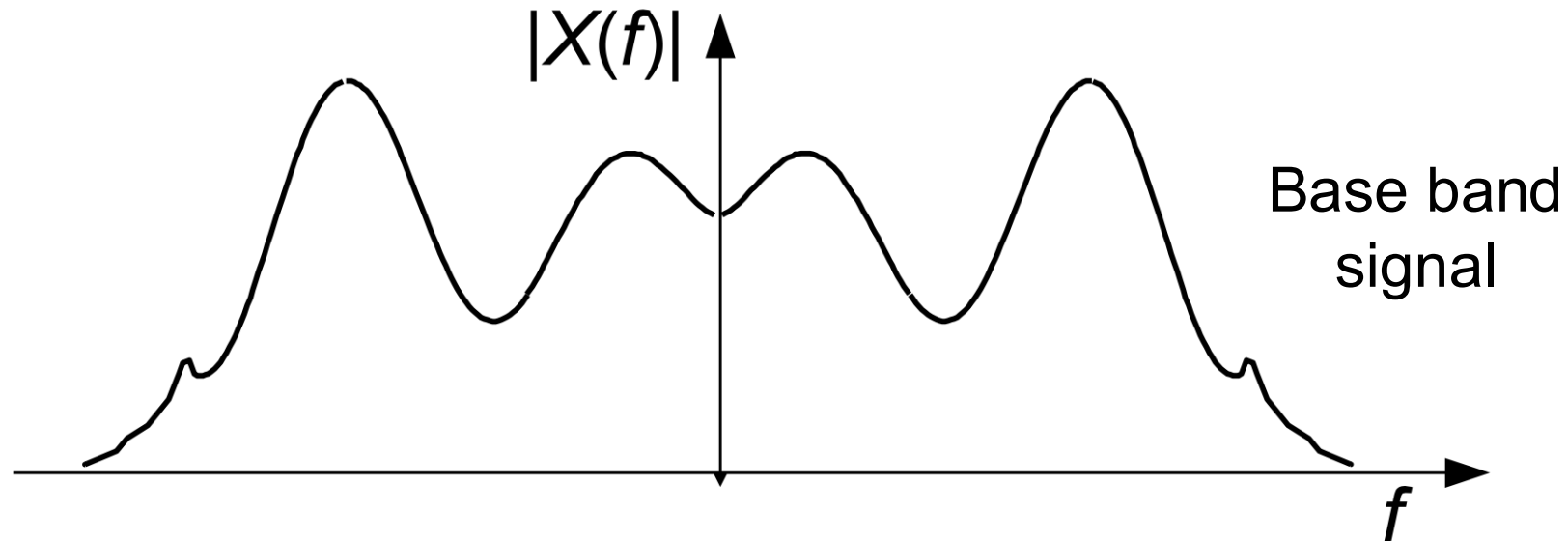
$$x(t) = |X(0)| + \sum_{k=1}^{+\infty} 2|X(f_k)| \cos(2\pi f_k t + \arg[X(f_k)])$$

- f_1, \dots, f_k, \dots signal components frequencies
- $\arg[X(f_k)] = \arctan \text{Im}[X(f)] / \text{Re}[X(f)]$ phase characteristic
- The full information about the signal spectral composition is already contained in **half the function** $X(f) \Rightarrow$ Positive frequencies
- $|X(0)|$ DC component \Rightarrow Not accounted in spectrum analyzers
- The full information is provided by $2 \cdot |X(f_1)|, \dots, 2 \cdot |X(f_k)|, \dots$



Continuous-spectrum signals

- No localized components at some specific frequencies
- Frequency bands



- $X(f) \cdot X^*(f) = |X(f)|^2 \Rightarrow$ Power Spectral density (PSD)
- Parseval theorem

$$\int_{-\infty}^{+\infty} x^2(t) dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$$



Spectrum measurements

- **Horizontal axis** \Rightarrow Calibrated frequency scale
- **Vertical axis** \Rightarrow Amplitude (RMS value [V_{RMS}]) or Power [W]

- **Amplitude values:**

$$\frac{2 \cdot |X(f_1)|}{\sqrt{2}}, \frac{2 \cdot |X(f_2)|}{\sqrt{2}}, \dots, \frac{2 \cdot |X(f_k)|}{\sqrt{2}}, \dots$$

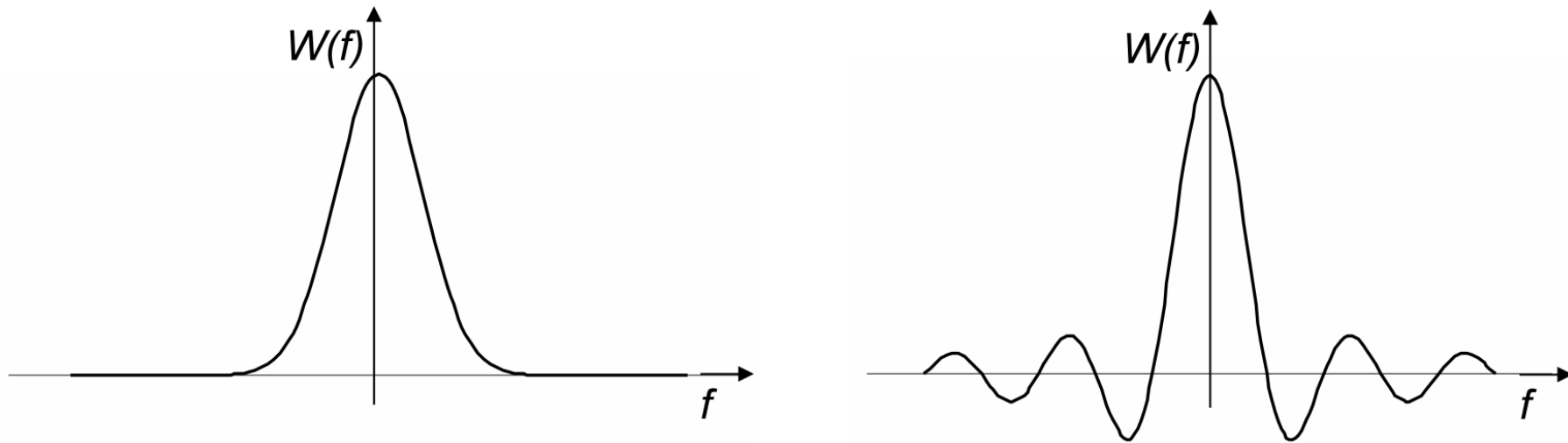
- **Power values** (referred to instrument input impedance $Z_i = R_i = 50 \Omega$):

$$\frac{2 \cdot |X(f_1)|^2}{R_i}, \frac{2 \cdot |X(f_2)|^2}{R_i}, \dots, \frac{2 \cdot |X(f_k)|^2}{R_i}, \dots$$



Spectrum measurements

- The spectrum analyzer output is **not an exact measurement** of the observed signal spectrum $X(f)$
- **Resolution function** $W(f) \Rightarrow$ capability to discriminate individual discrete frequency components



- **Measured trace** $G(f) \Rightarrow$ frequency-domain convolution of $X(f)$ and $W(f)$



Spectrum measurements

$$G(f) = \int_{-\infty}^{+\infty} X(\nu) \cdot W(f - \nu) d\nu$$

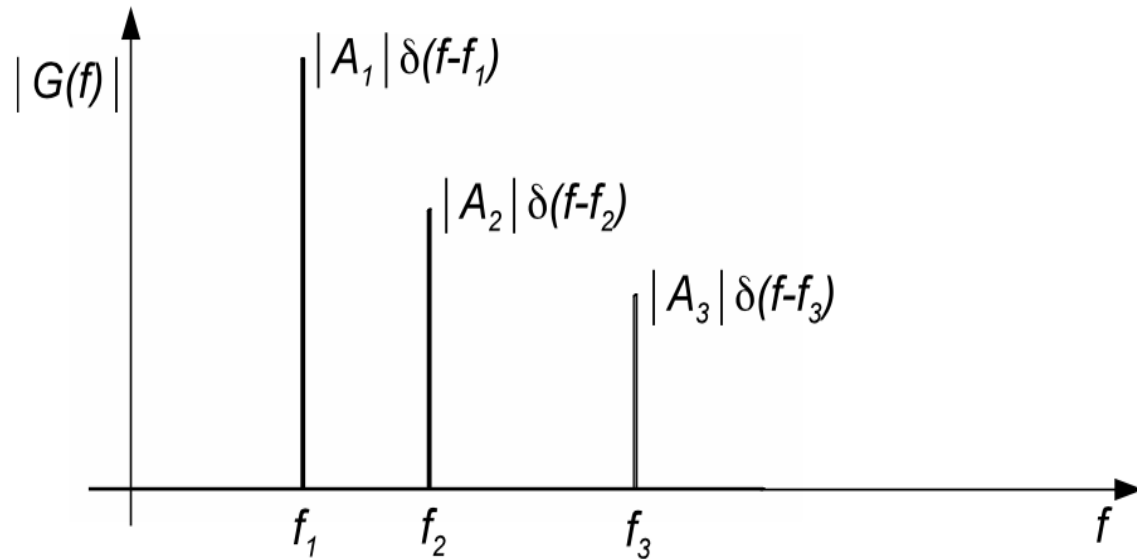
- Discrete spectrum:

$$G(f) = \sum_{k=-\infty}^{+\infty} A_k \cdot W(f - f_k)$$

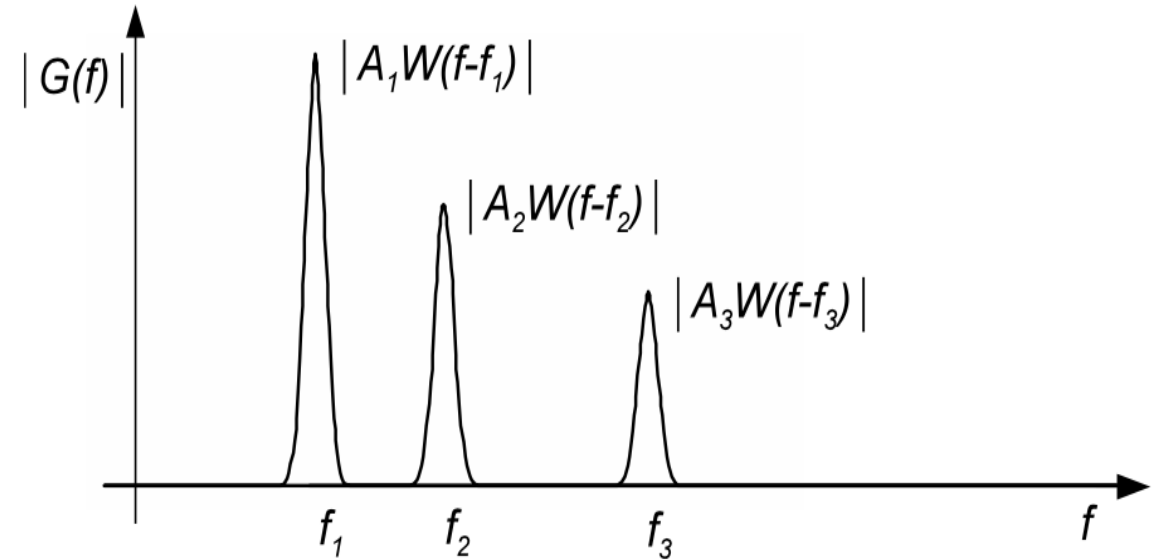
- A_k and f_k **amplitude and frequency** of the signal components
- The spectrum would remain unaltered only if $W(f) \cong \delta(f)$
- In practice, each signal component is associated to a peak whose shape is described by $W(f)$
- $W(f)$ should be a **narrow-band function**
- $G(f_k) = A_k \cdot W(0), \forall k$



Spectrum measurements



spectrum, from theory

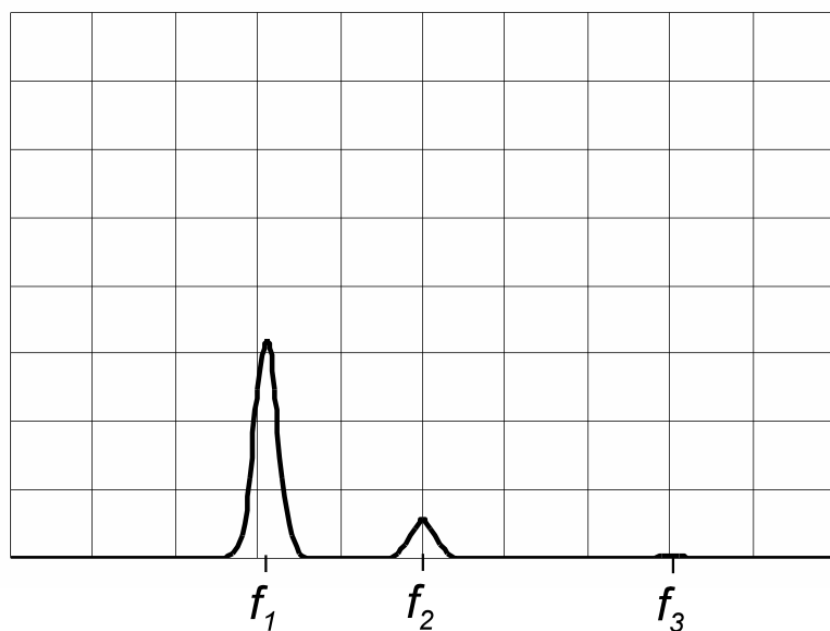


convolution with $W(f)$.

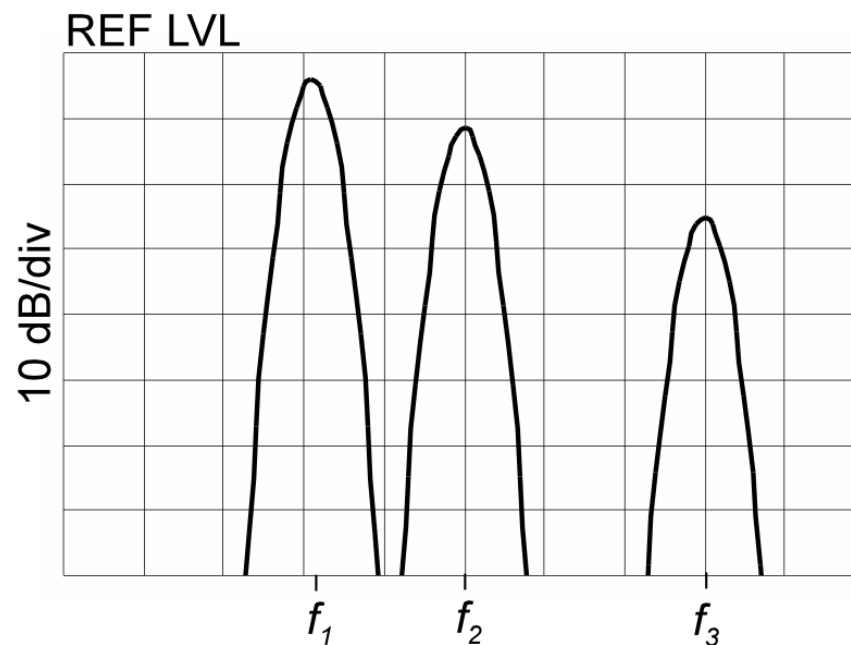


Logarithmic scale display

- **Measurements** \Rightarrow grid and cursors
- Simultaneous detection of **very small and large signal components**
- Typical values of 80 dB **amplitude dynamics**
- **Logarithmic** vertical scale



Linear scale



Logarithmic scale



Logarithmic scale display

- The displayed trace is always positive, being proportional either to $|X(f)|$, or to $|X(f)|^2$
- **Linear scale** \Rightarrow 0 at the bottom of the scale, upper bound determined by the vertical scale factor
- **Logarithmic scale** $\Rightarrow \log_x(0) = -\infty$. Reference level defined for the maximum value, corresponding to the top of the grid
- Units:
 - **Amplitude** measurements: dBV $\Rightarrow 20 \log_{10} \frac{V_X}{1[V_{RMS}]}$
 - **Power** measurements: dBm $\Rightarrow 10 \log_{10} \frac{P_X}{1[mW]}$



Frequency specifications

- **Wide frequency range:** several decades with a given level of accuracy (e.g., from 10 kHz to 100 GHz, or from 0.1 Hz to 100 kHz)
 - Measurable frequency range vs measured frequency span
- **Good frequency resolution:** ability to distinguish the components of a signal even when their frequency separation is small



Amplitude specifications

- **High amplitude dynamics:** accurate measurement of low-level components even when much larger ones are simultaneously present
 - Dynamics is specified as the **ratio, in dB**, of the largest to the smallest levels that can be measured on the instrument display
- **Good sensitivity:** low-level signal components should be accurately measured even when much larger ones are present
- **Uniformity:** the instrument behaviour should be **the same over the full range** of analyzed frequencies



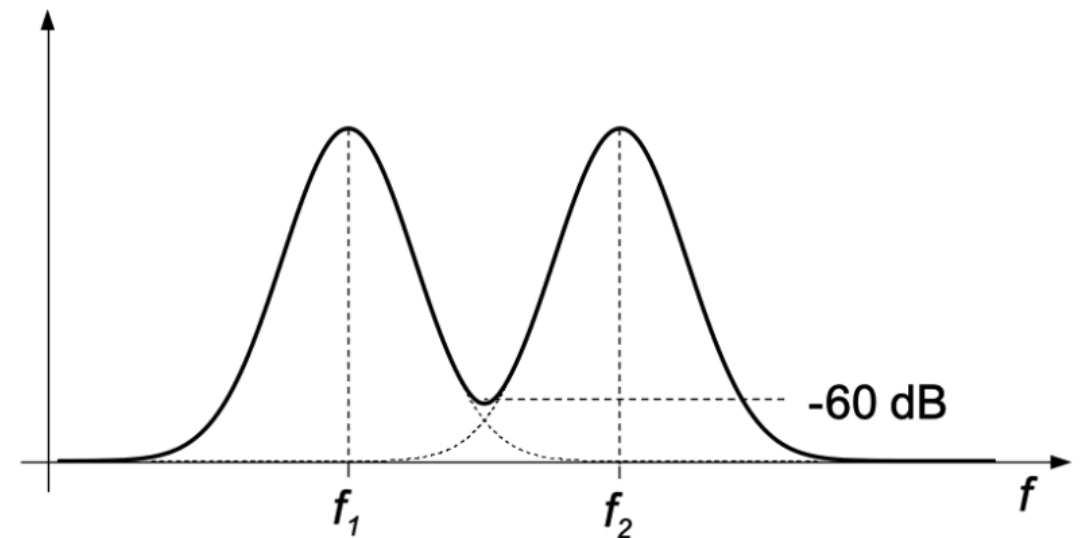
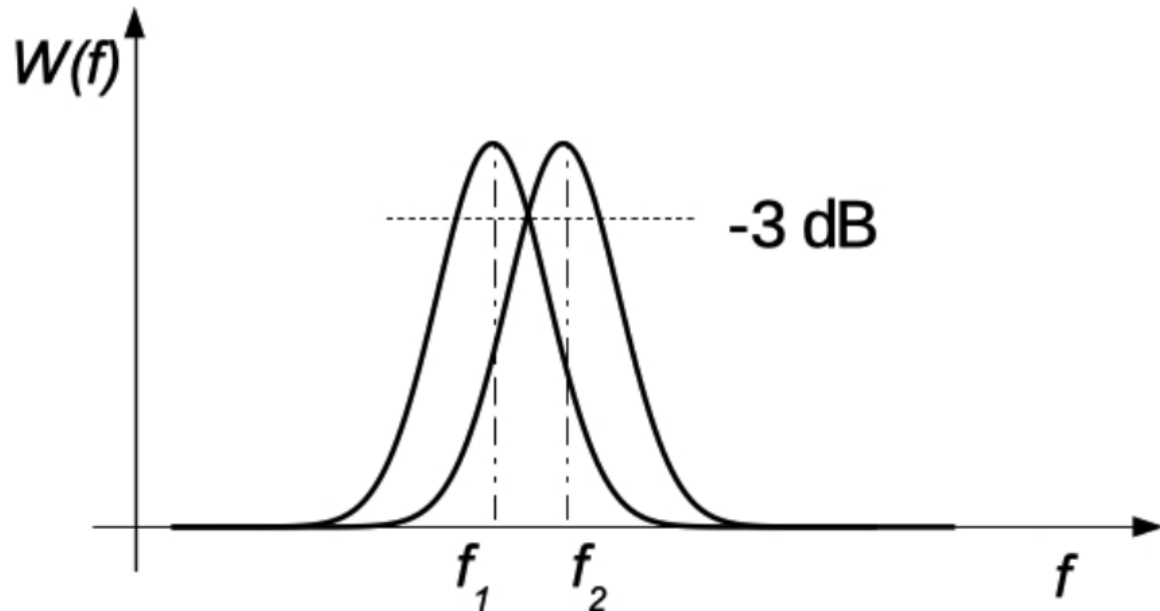
Frequency resolution

- Instrument resolution vs frequency resolution
- *“the minimum frequency separation at which two sinusoidal components having the same amplitude can be distinguished as two individual spectral components”* ⇒ **Equal amplitude**
- **Resolution bandwidth** $B_R = 2 \cdot B_{-3dB}$
- Accurate measurement requires greater separation ⇒ spectral interference needs to be avoided



Spectrum analyzer specifications

- -60 dB bandwidth of the resolution function B_{-60dB}
- Distance at which interference caused by one component is equal to 0.1% of its amplitude
- **Selectivity:** B_{-60dB}/B_{-3dB} → Depends on the shape of the resolution function $W(f)$





Linearity specifications

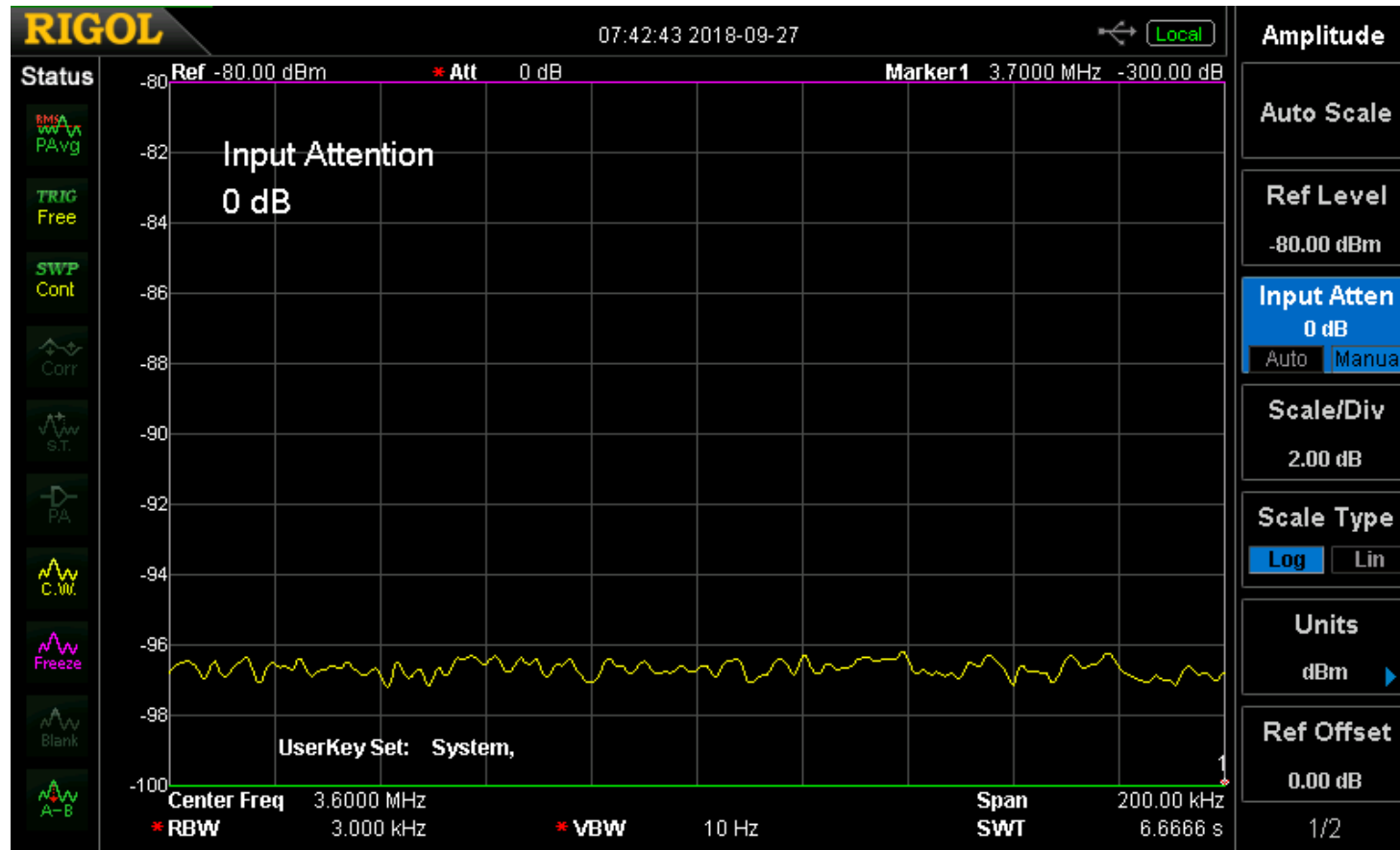
- Any distortion caused by non-linearity within a spectrum analyzer might introduce **new frequency components**
- Extremely good linearity is required with respect to the measured signal
- Spurious responses to be avoided



Spectrum analyzer specifications

Sensitivity

- The minimum power level at which a sinusoidal component can be distinguished from the instrument **noise floor**





Sensitivity

- **Thermal noise power spectral density** $\Rightarrow S_{nn}(f) = kT$
 - Boltzmann constant $k = 1.38 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$
 - Absolute temperature in Kelvin T
- Reference temperature $T = 290 \text{ K} \Rightarrow S_{nn}(f) = 4 \times 10^{-21} \text{ W/Hz} \Rightarrow -174 \text{ dBm/Hz}$
- **Noise Figure (NF)**: noise added by the instrument circuitry \Rightarrow between 10 and 20 dB
- $B_R = 1 \text{ kHz}$, $NF = 15 \text{ dB} \Rightarrow$
$$-174 \frac{\text{dBm}}{\text{Hz}} + NF + 10 \log_{10} B_R = -174 + 15 + 30 = -129 \text{ dBm}$$



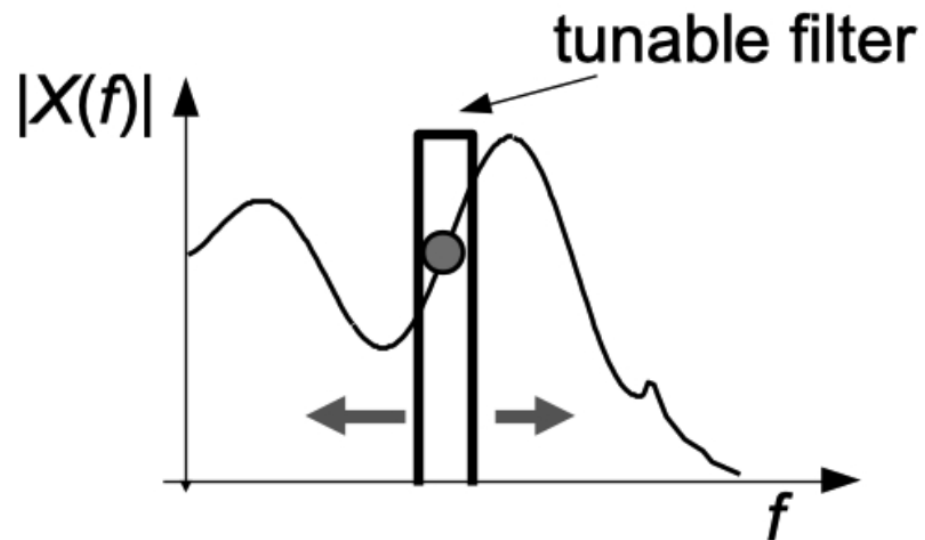
Spectrum analyzers

- Frequency composition \Rightarrow frequency selective narrow-band filters
- **Scanning spectrum analyzers:** The measuring instrument automatically scans the frequency range of interest and sequentially measures signal component strengths at each frequency \Rightarrow equivalent to **tunable filter**
- **“real-time” spectrum analyzers:** The measuring instrument digitizes the signal within the frequency range of interest and determines the spectrum by algorithm computation of the Fourier transform \Rightarrow equivalent to the use of **parallel analogue filter banks**

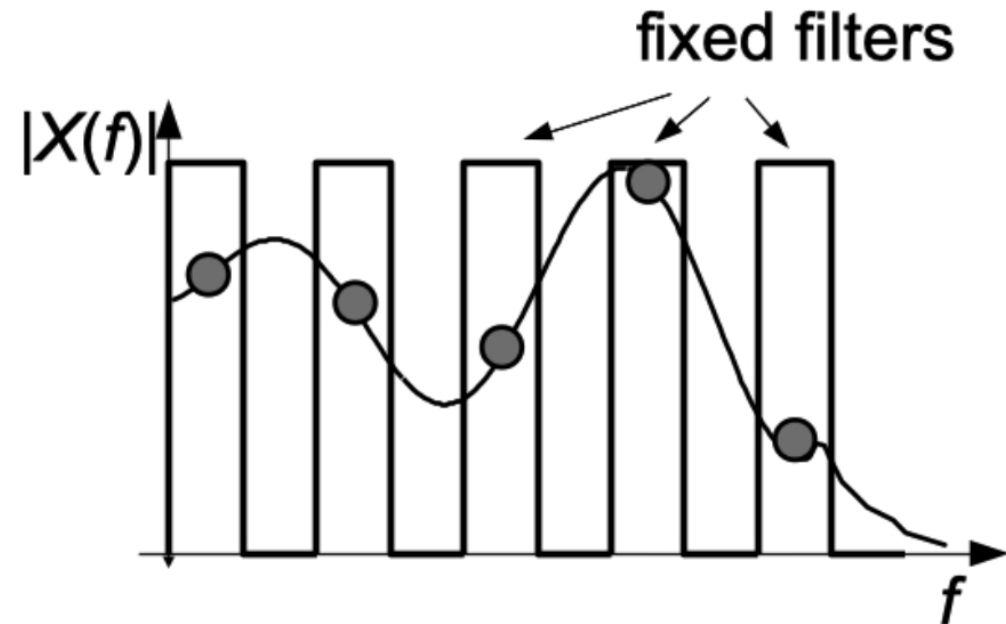


Spectrum analyzers

- Frequency composition \Rightarrow frequency selective narrow-band filters



tunable filter



selective filter bank



Swept frequency spectrum analyzer

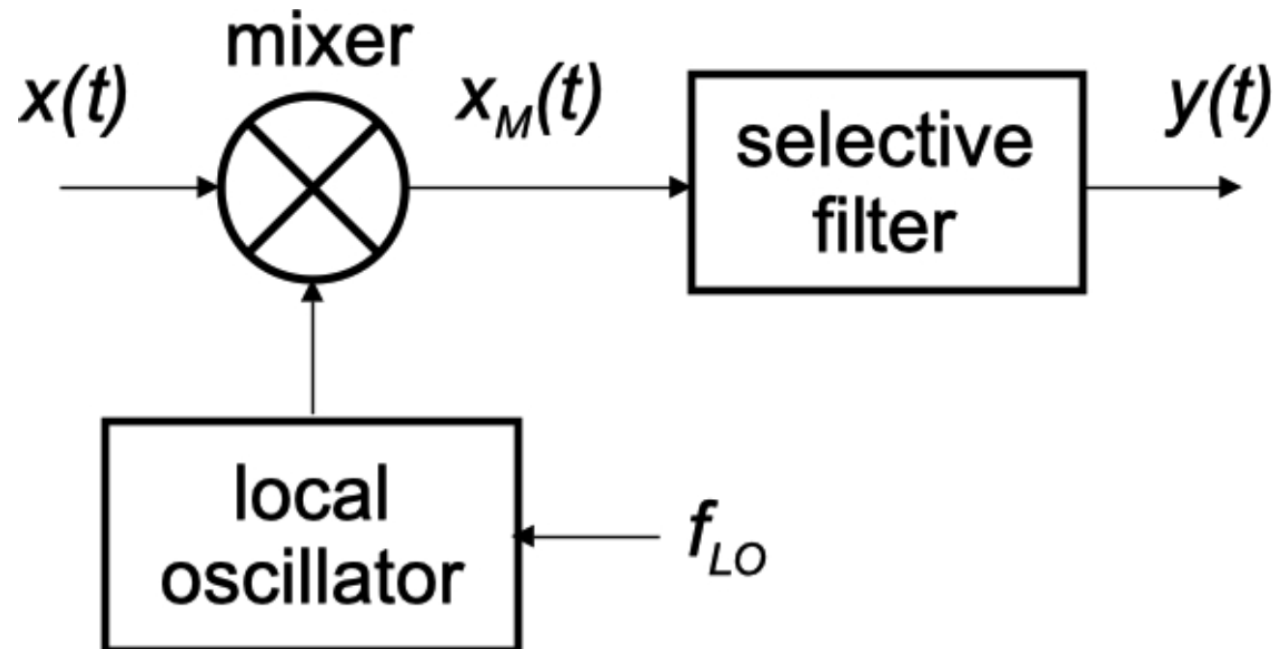
- Used in **radiofrequency** and microwave measurement
- Typical range from **few kHz to several GHz** (possible extension to few tens of Hz and 100-200 GHz)
- **Wide input range (up to 6 decades)** \Rightarrow hard to use tunable selective filters
- **Superheterodyne** (Frequency transposition)
 - Selectivity in frequency
 - Wide frequency intervals



Swept frequency spectrum analyzer

Basic elements:

- Mixer
- Variable-frequency oscillator
- Selective filter





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