Università degli Studi di Padova

DSO built-in measurement functions

Lecture #4

Electronic measurements

Alessandro Pozzebon

Measurement algorithms

- **Digital Signal Processing (DSP)**

↓

- **Waveform measurement functions**

- True RMS voltmeter
- Frequency meter
- Peak voltmeter

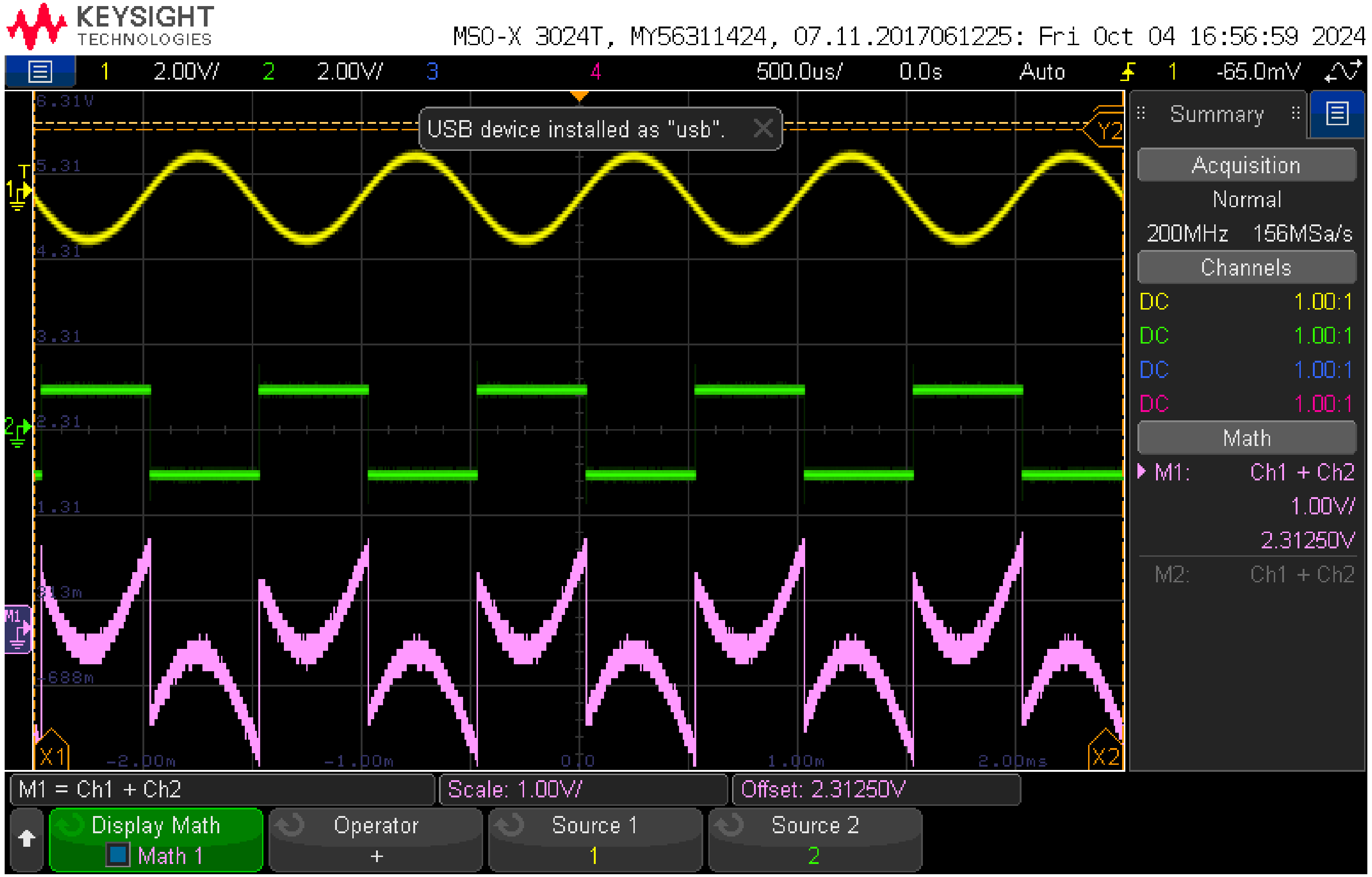
} Numerical indications provided by different measuring instruments

Common measurement algorithms

- **Common measurement functions**
  - Minimum value
  - Maximum value
  - RMS value
  - Average value
  - Peak-to-peak amplitude
  - Rise time
  - Frequency and period
  - Phase shift
  - Mathematical operations
  - Etc…

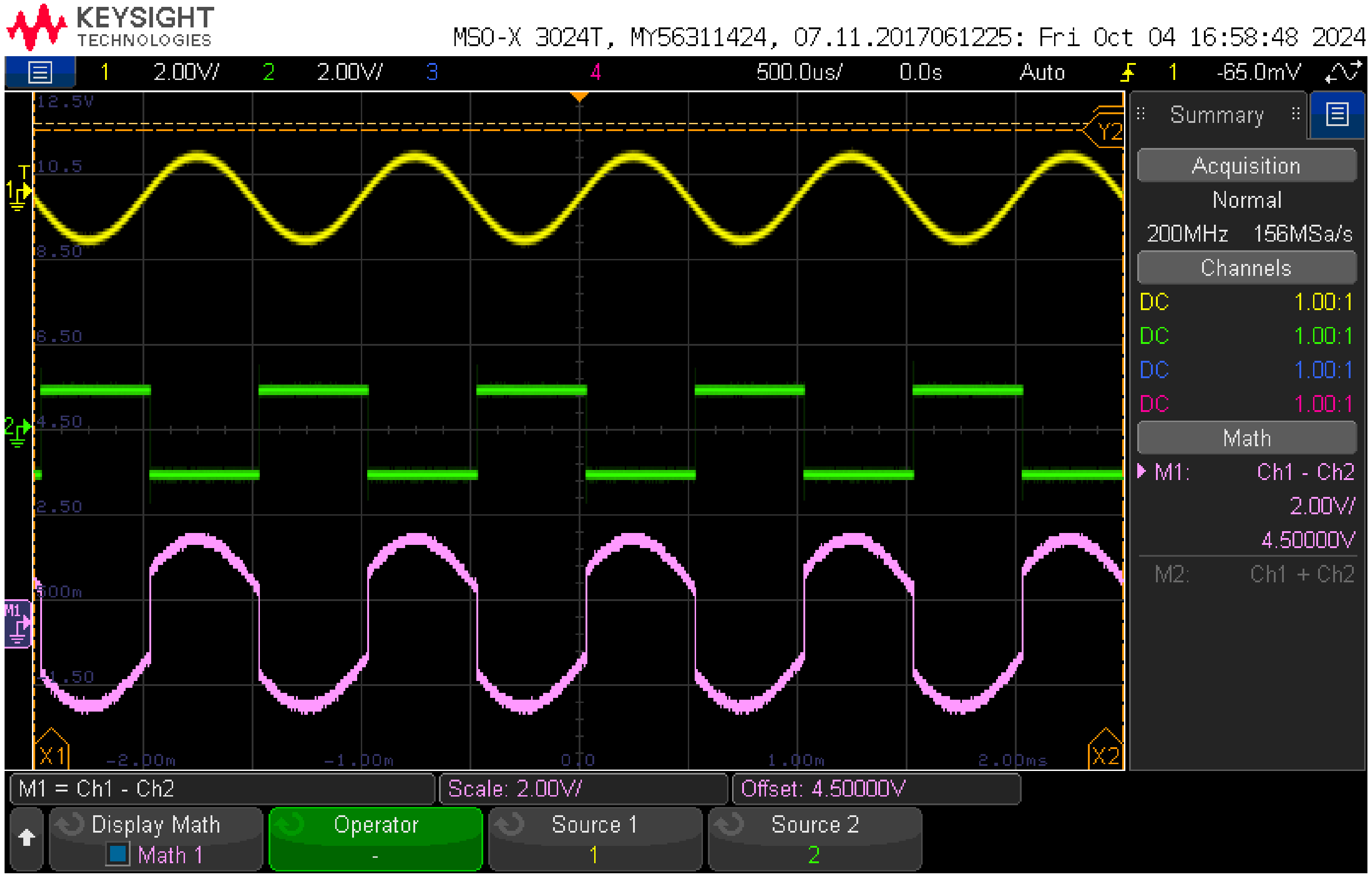
Mathematical functions

- Sum



Mathematical functions

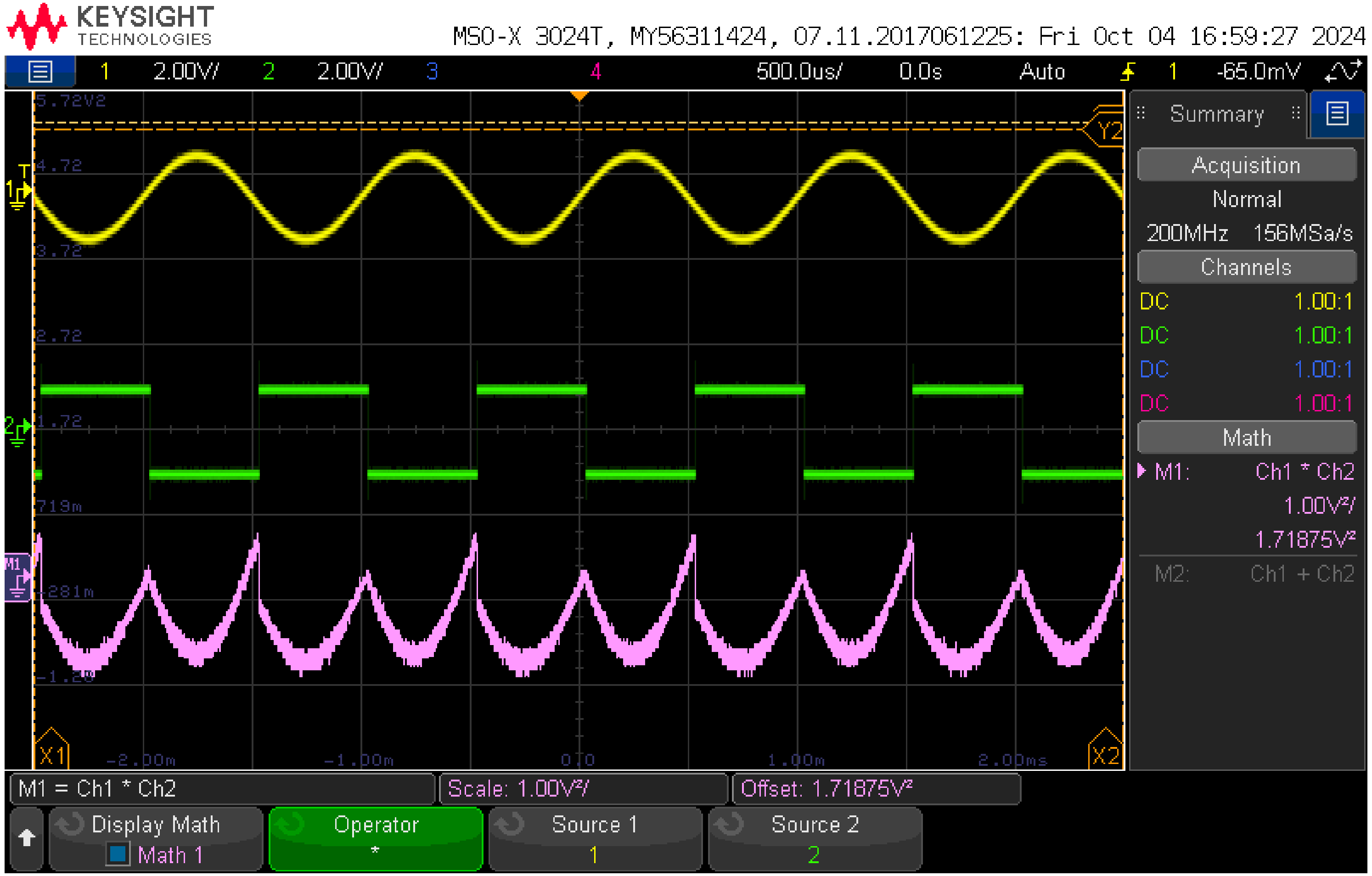
- **Difference**



Università degli Studi di Padova

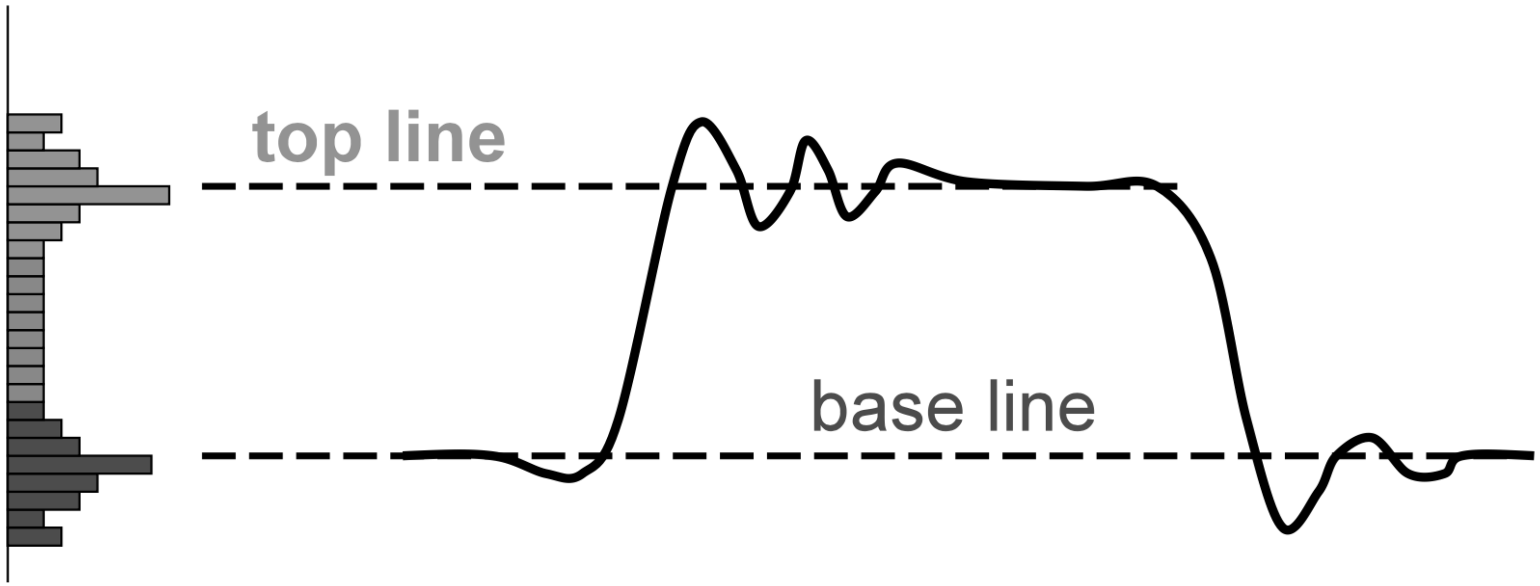
Mathematical functions

- **Product**



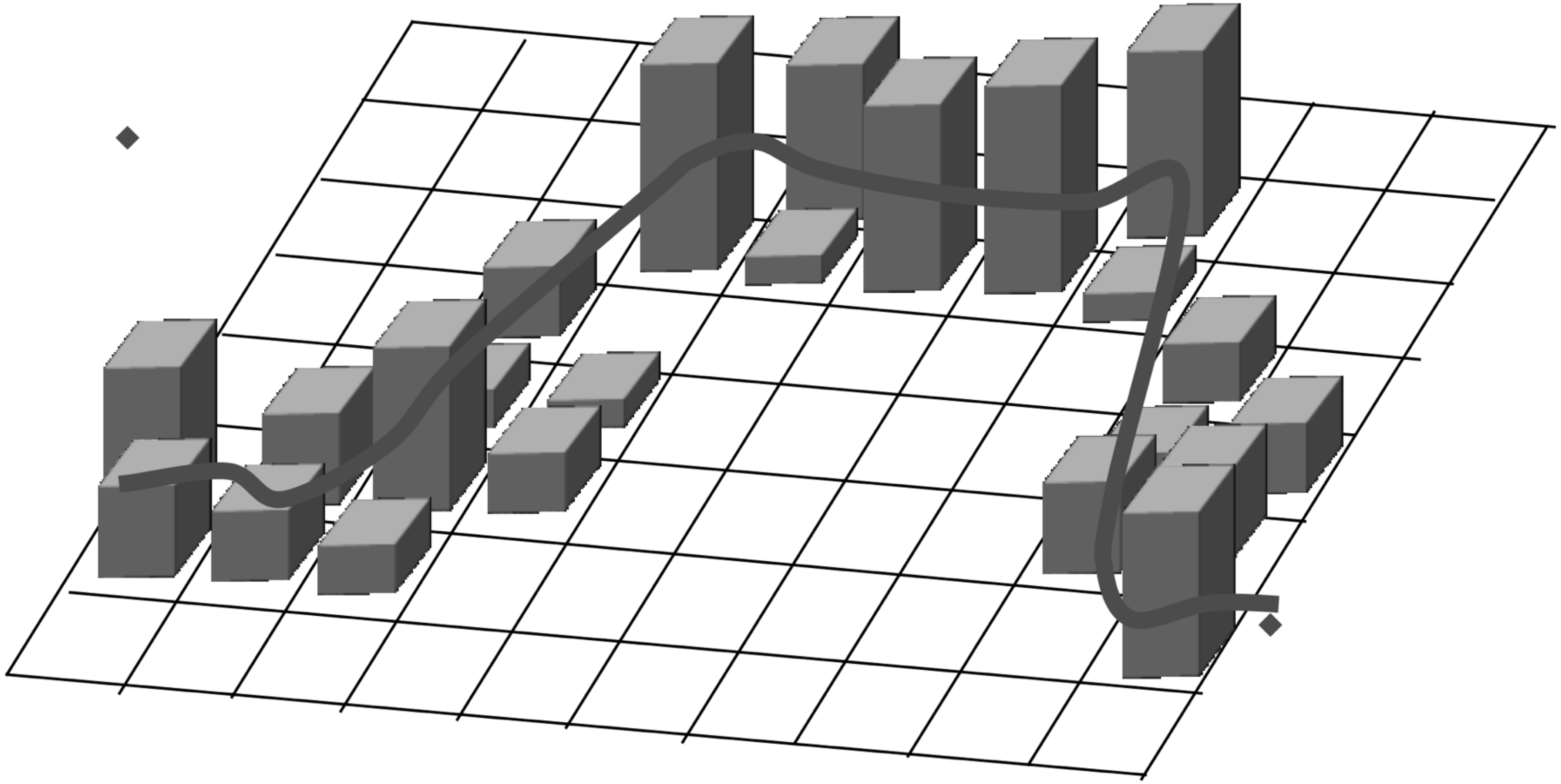
Statistics-based-measurement

- **Amplitude distribution of the measured signal**
  - Statistical sample of signal amplitude values
  - Signal probability density function



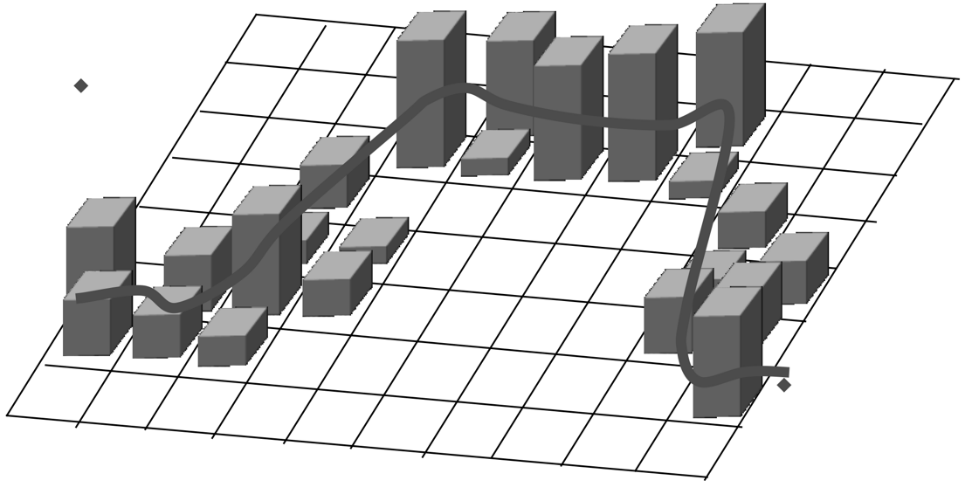
Statistics-based-measurement

- **Amplitude distribution of the measured signal**
  - More consecutive traces
  - Two-dimensional statistical sample



Statistics-based-measurement

- The display memory is **a matrix of cells**
- Each memory cell can be associated to a **counter**
- **Different colours**: discrete probability level associated to different colours
- **Trace brightness**



Università degli Studi di Padova

Root-mean-square value

- **Periodic waveform RMS (effective) value**

$$X_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

- **Digital RMS estimation** (on a finite sequence of N sample values $x(nT_S)$)

$$\hat{X}_{RMS} = \sqrt{\frac{1}{N}\sum_{n=n_0}^{n_0+N-1} x^2(nT_S)}$$

Root-mean-square value

- **RMS value of the AC component** $(x_{(AC)}(nT_S)) = x(nT_S) - X_M)$

$$\hat{X}_{RMS} = \sqrt{\frac{1}{N}\sum_{n=n_0}^{n_0+N-1} x_{(AC)}^2(nT_S)} = \sqrt{\frac{1}{N}\sum_{n=n_0}^{n_0+N-1} [x(nT_S) - X_M]^2}$$

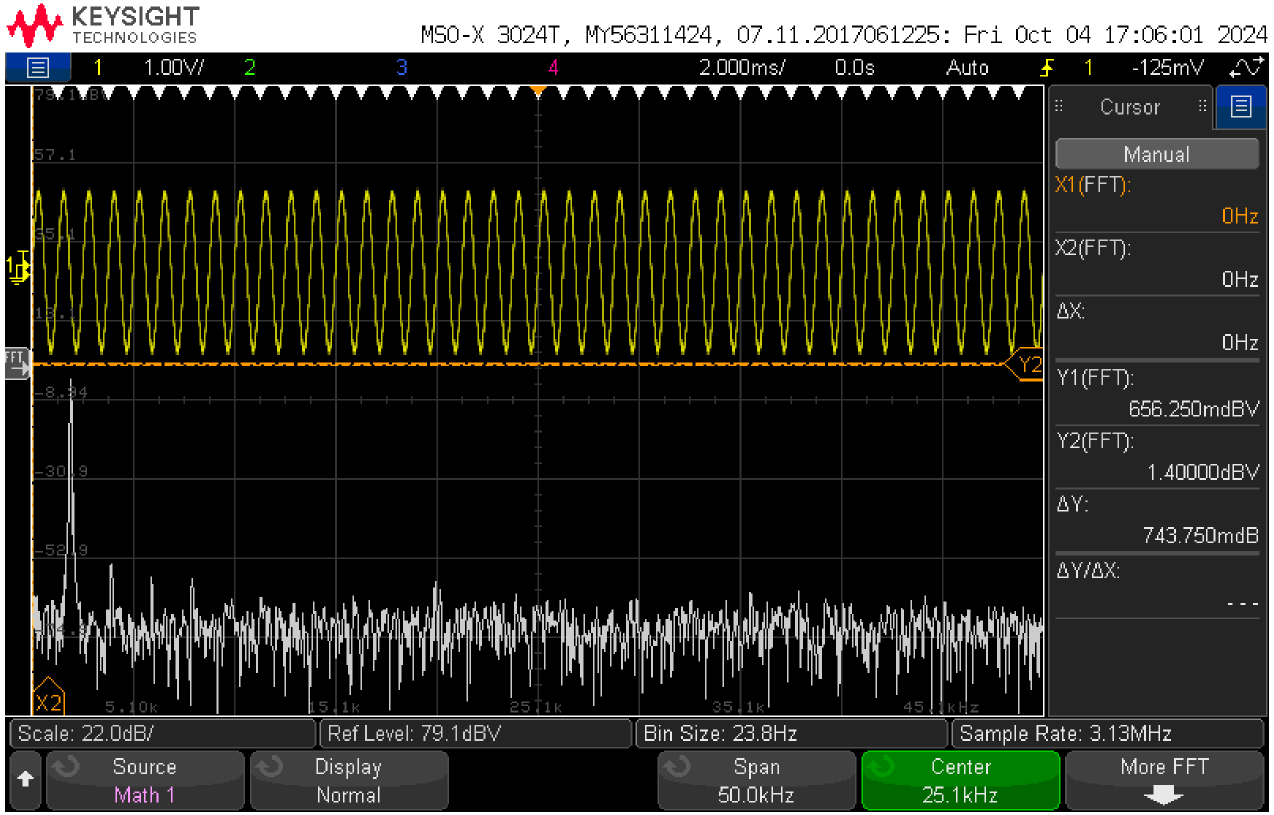
- $\frac{nT_S}{T}$ ratio between the sampling interval and the signal period:
  - $\frac{nT_S}{T}$ **integer** $\rightarrow x(nT_S)$ periodic $\rightarrow$ **EXACT ESTIMATE**
  - $\frac{nT_S}{T}$ **rational** $\rightarrow \frac{T_S}{T} = \frac{M}{P} \rightarrow PT_S = MT$
  - $\frac{nT_S}{T}$ **irrational** $\rightarrow x(nT_S)$ not periodic $\rightarrow$ **MOST REALISTIC CASE**

Root-mean-square value

- **Two versions of the algorithm**
- **1.**
  - Preliminary stage: estimate of the signal period
  - N samples $\rightarrow N_P$ samples. $N_P < N : (N_P - 1)T_S \leq T \leq (N_P + 1)T_S$
  - Time resolution T_S
  - **Quasi-coherent sampling**
- **2.**
  - N samples regardless of the signal features
  - **Asynchronous sampling**

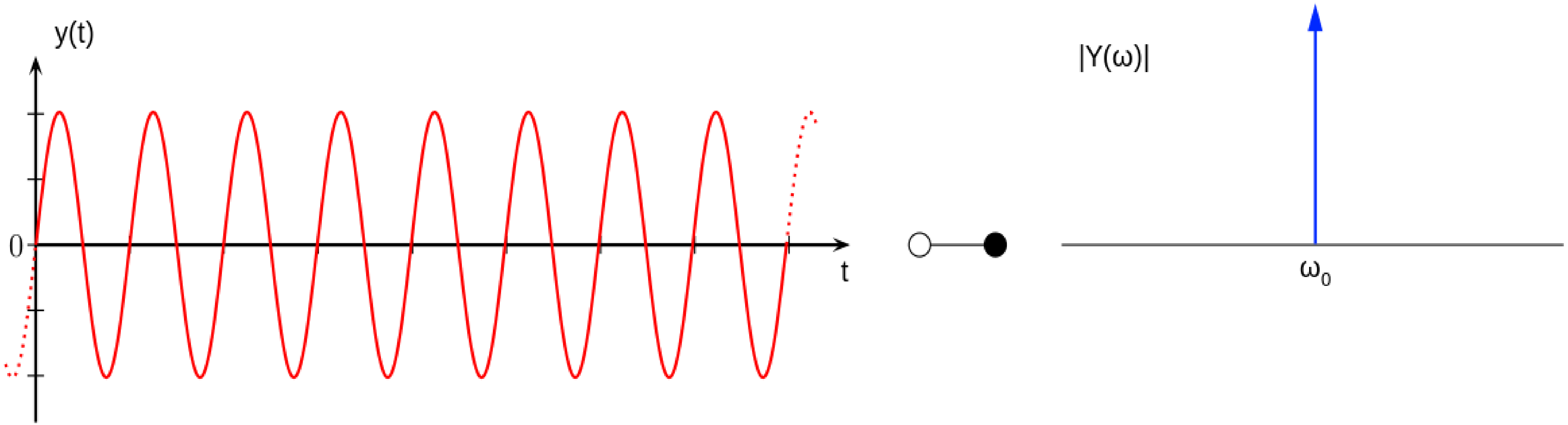
Fourier analysis function

- Convert **a sequence of samples** acquired from a signal into information about its **frequency spectrum**
- **Fast Fourier Transform (FFT)**



Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

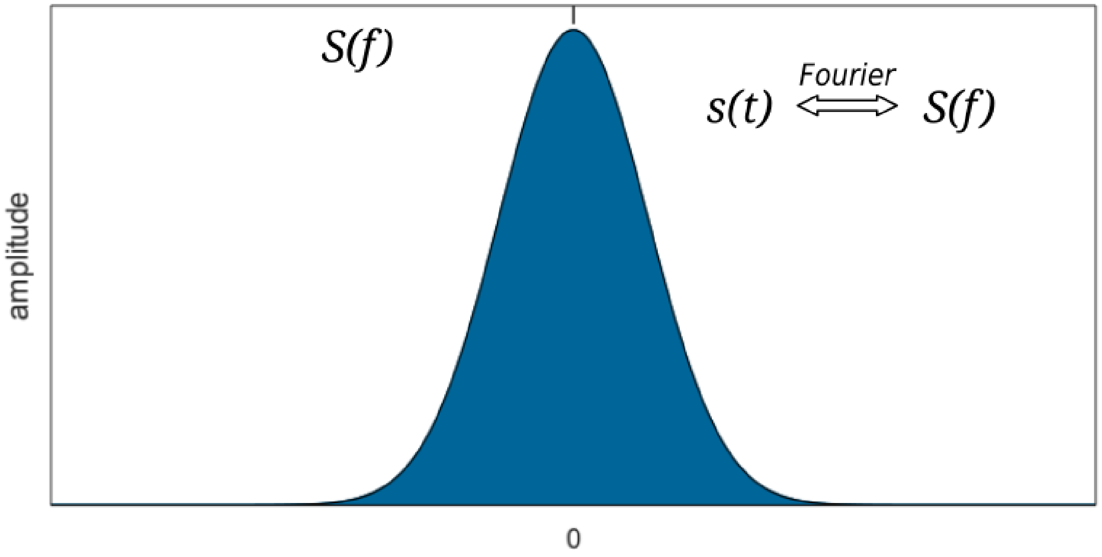


UNIVERSITÀ DEGLI STUDI DI PADOVA

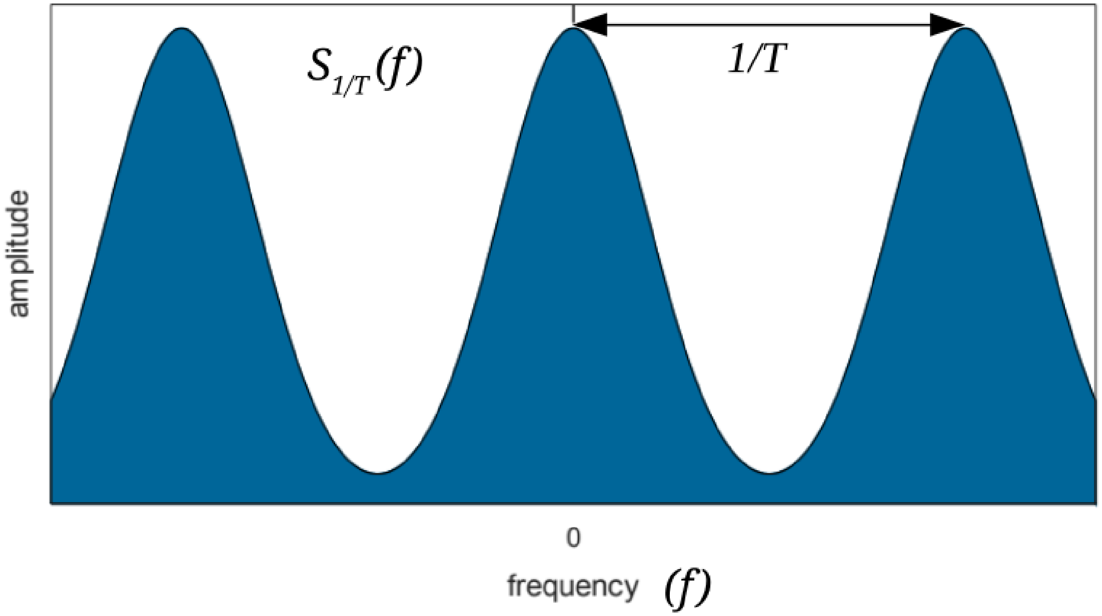
Sampled signal transform

$$\hat{X}(t) = X(t) \cdot \sum_{n=1}^{N} \delta(t - T_n)$$

Fourier transform of a function s(t) (which is not shown)



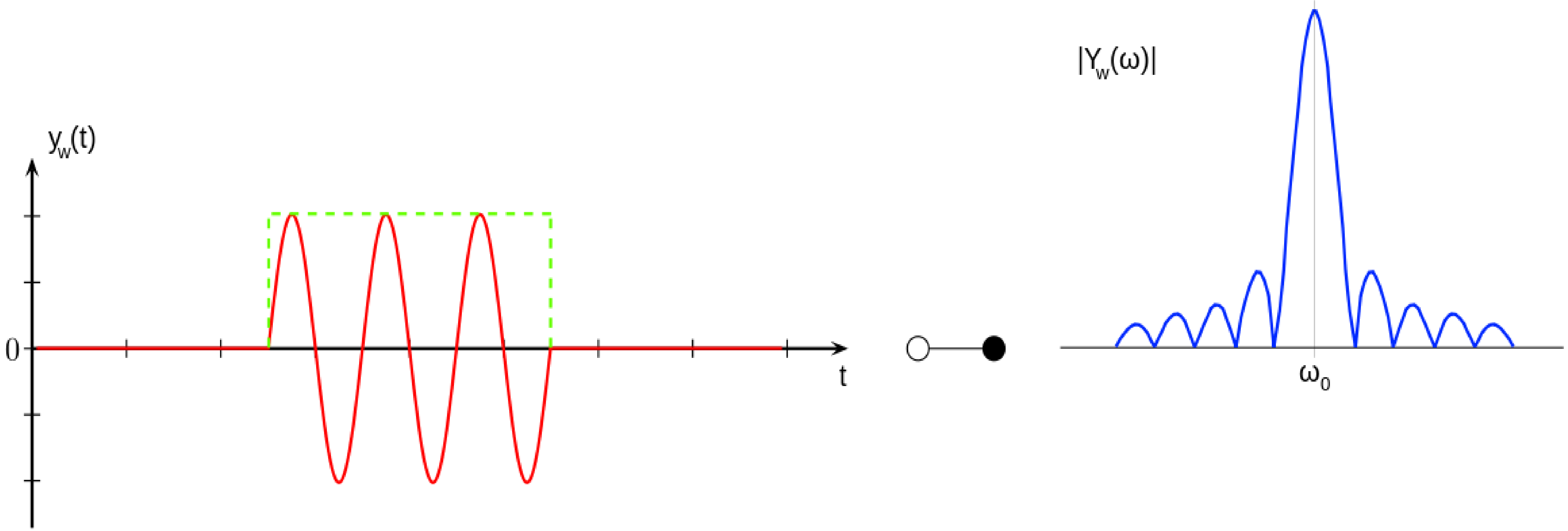
Transform of periodically sampled s(t)
aka "Discrete-time Fourier transform"



Truncated-signal Fourier Transform

$$x(t) \cdot rect\left(\frac{t}{T}\right) = X(f) * Tsinc(Tf)$$

- If $x(t) = \sin 2\pi f_0 t \rightarrow$ Amplitude modulation



Discrete Fourier Transform

- $x[n] = x(nT_s)$ with $n_0 \leq n \leq n_0 + N - 1$

$$X_{DFT}[k] = \frac{1}{N} \sum_{n=n_0}^{n_0+N-1} x[n] e^{-j2\pi\frac{kn}{N}}$$

- $X_{DFT}[k]$ is **periodic** with period N

$$X_{DFT}[kF] = \frac{1}{NT_s} \sum_{n=n_0}^{n_0+N-1} T_s x(nT_s) e^{-j2\pi\frac{k}{NT_s}nT_s}$$

where $F = 1/NT_s$

- Frequency values are quantized
- **Frequency granularity** $F = 1/T_w$ where $T_W = NT_s$ signal **observation interval**

Discrete Time Fourier Transform

$$\tilde{X}(f) = \sum_{n=-\infty}^{+\infty} T_s \cdot x(nT_s) e^{-j2\pi f n T_s}$$

- **Unlimited** sample sequence $x(nT_s)$
- Frequency-**continuous**

- **TRUNCATION** (finite-length samples sequency):

$$x_W(nT_s) = x(nT_s) \cdot w_R(nT_s)$$

$$w_R(nT_s) = \begin{cases} 1 \quad for \quad n_0 \le n \le n_0 + N - 1 \\ 0 \quad elsewhere \end{cases}$$ → **Window** function

DTFT vs DFT

- **DFT**: time-discrete, **limited** sequence, frequency-**discrete**
- **DTFT**: time-discrete, **unlimited** sequence, frequency-**continuous**

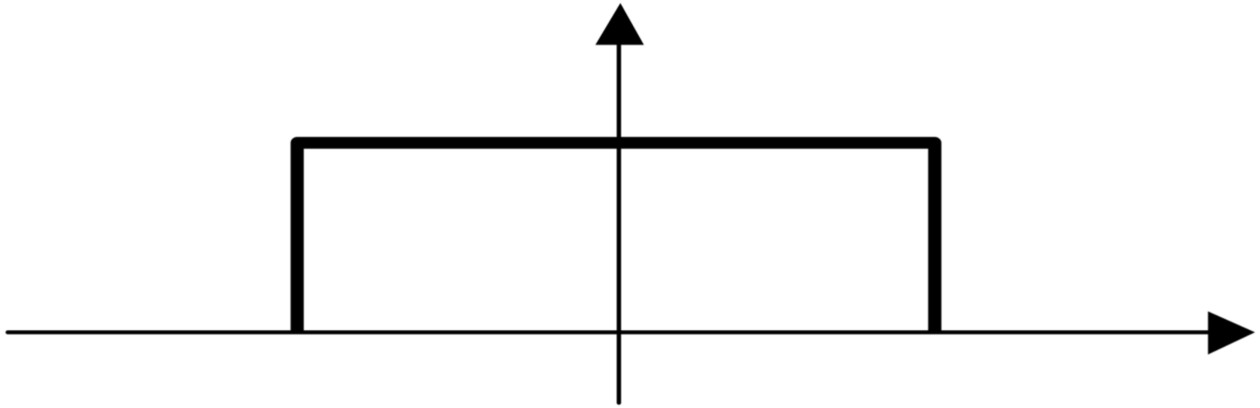
$$X_{DFT}(kF) = \frac{1}{NT_S}\left[\tilde{X}(f) * \widetilde{W}_R(f)\right]_{f=kF}$$

$$X_{DFT}(kF) = \frac{1}{NT_S}\tilde{X}_W(f)\Big|_{f=kF}$$

- **TRUNCATION** (Convolution in frequency)
- **QUANTIZATION** of frequency

Uniform Window

- **Rectangular window** → Same weight (1) to all the samples



$$\tilde{W}_R(f) = \sum_{n=0}^{N-1} T_s e^{-j2\pi f n T_S} = T_S \frac{\sin \pi f N T_S}{\sin \frac{\pi}{N} f N T_S} e^{-j\frac{2\pi}{N} f N T_S \left(n_0 + \frac{N-1}{2}\right)}$$

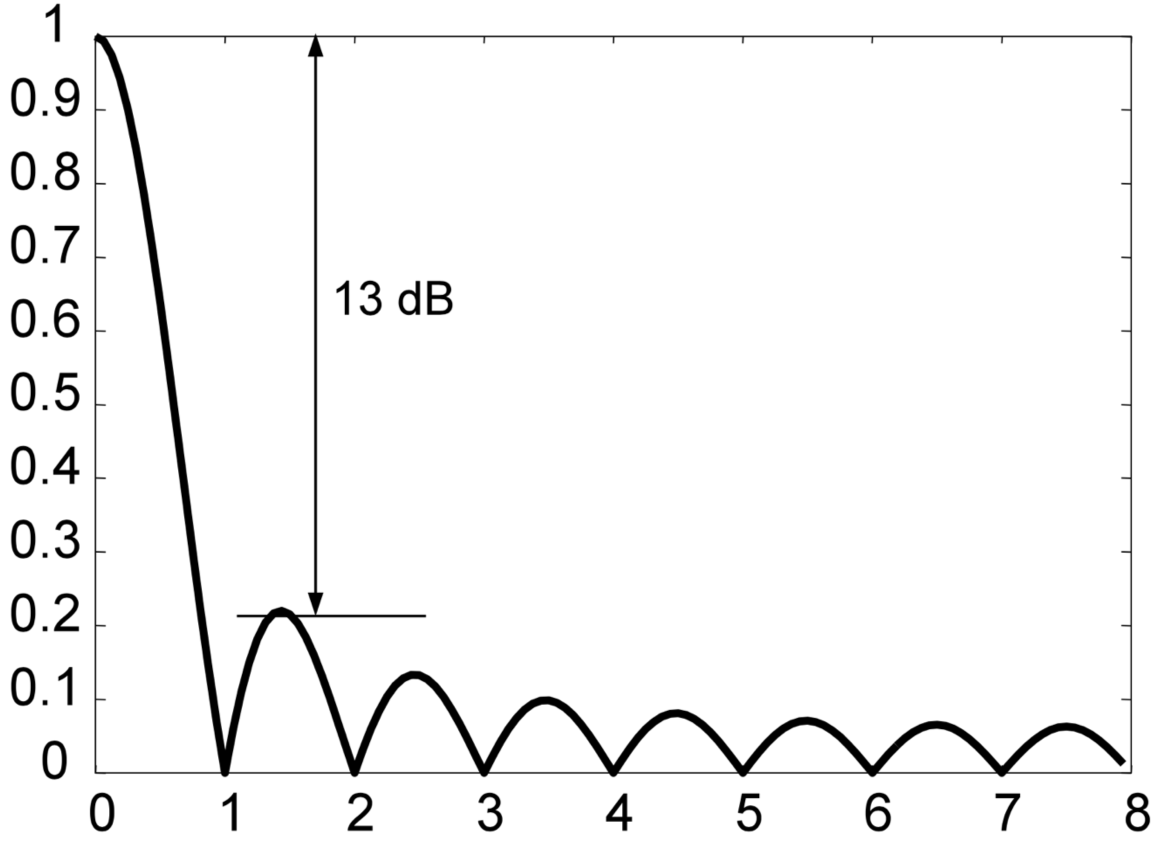
Normalized frequency $\lambda = f N T_S$

$$\tilde{W}_R(\lambda) = \frac{\sin \pi\lambda}{N \sin \frac{\pi}{N} \lambda} e^{-j\frac{2\pi}{N}\lambda\left(n_0 + \frac{N-1}{2}\right)}$$

Uniform Window

Amplitude spectrum

$$\left|\widetilde{W}_{R(DFT)}(\lambda)\right|$$

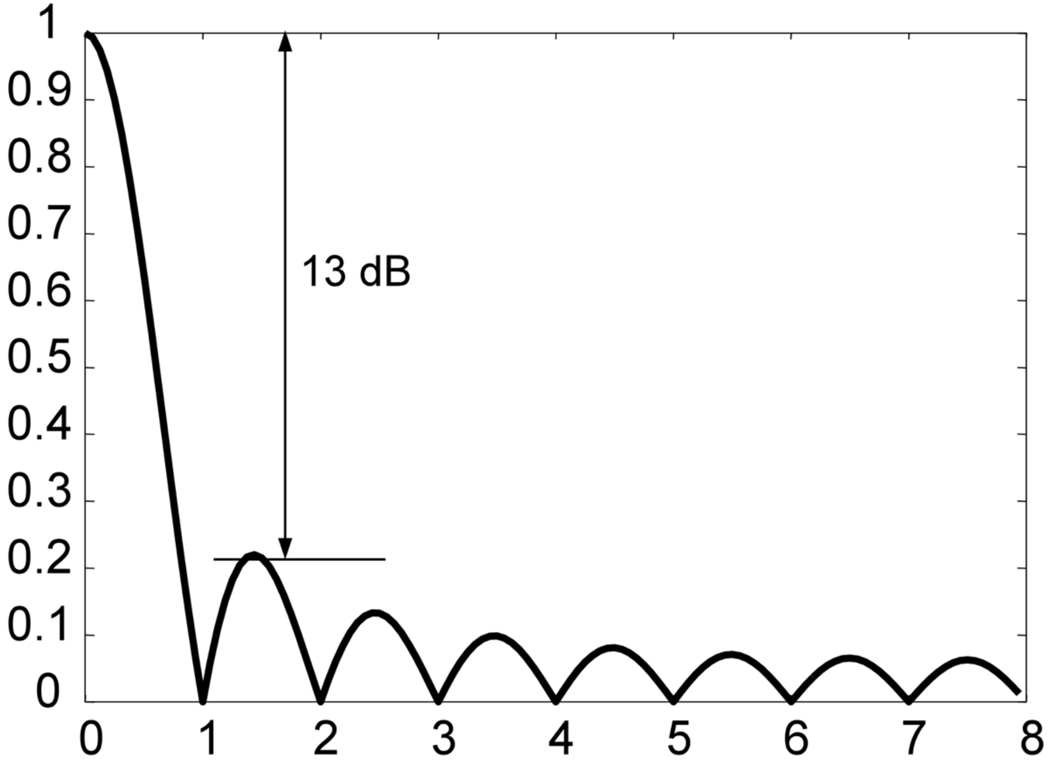


DTFT magnitude, normalized with respect to NT_S, for a uniform window with $N = 16$

Uniform Window

- $w_R(nT_s)$ real-valued → Hermitian symmetry
- Only $0 \leq \lambda \leq N/2$ values need to be presented

- **Main lobe**: height 1 and width 2
- **Several side lobes**: decreasing magnitude, width equal to 1



DFT-based sinewave spectrum

- **Sampled sinewave values**

$$x(nT_S) = A_0 \sin(2\pi f_0 nT_S + \phi_0), \quad n = n_0, \dots, n_0 + N - 1$$

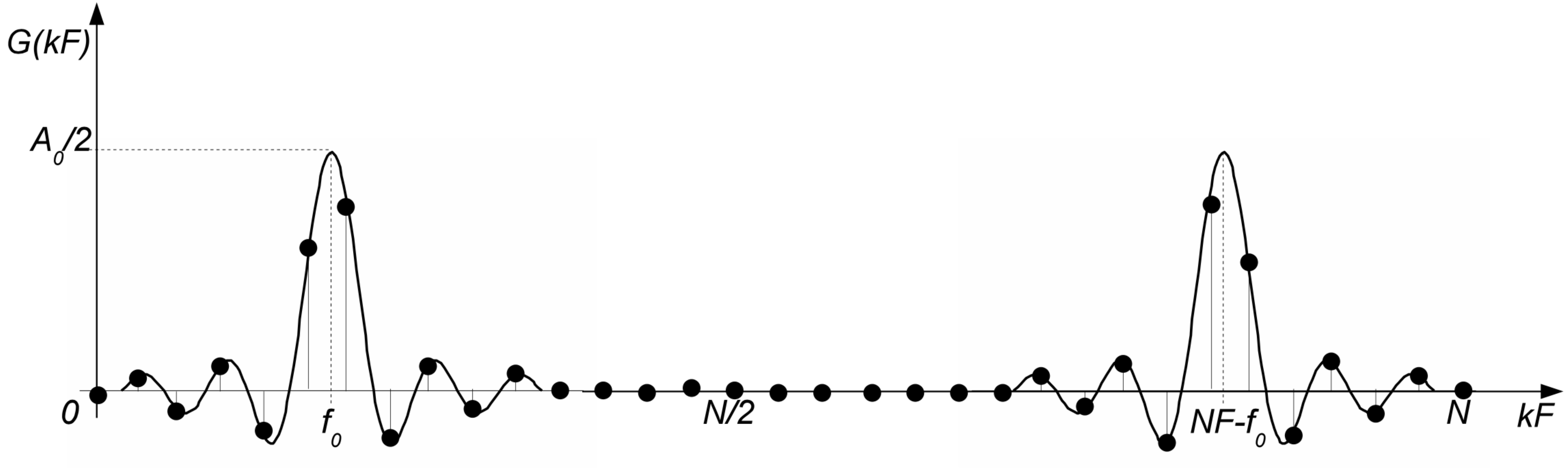
- Fourier coefficients

$$X_{DFT}(kF) = \frac{A_0}{2j} e^{j\phi_0} \cdot \frac{\widetilde{W}_R(kF - f_0)}{NT_S} - \frac{A_0}{2j} e^{-j\phi_0} \cdot \frac{\widetilde{W}_R(kF + f_0)}{NT_S} =$$

$$= \frac{A_0}{2j} e^{j\phi_0} \cdot W_{R(DFT)}(k - \lambda_0) - \frac{A_0}{2j} e^{j-\phi_0} \cdot W_{R(DFT)}(k + \lambda_0)$$

with $k = 0, \dots, N - 1$ and $\lambda_0 = f_0 N T_0$

DFT-based sinewave spectrum



- Two $\frac{\sin x}{x}$
- Coefficients with $k \geq N/2$ not displayed → Spectrum image components

DFT-based sinewave spectrum

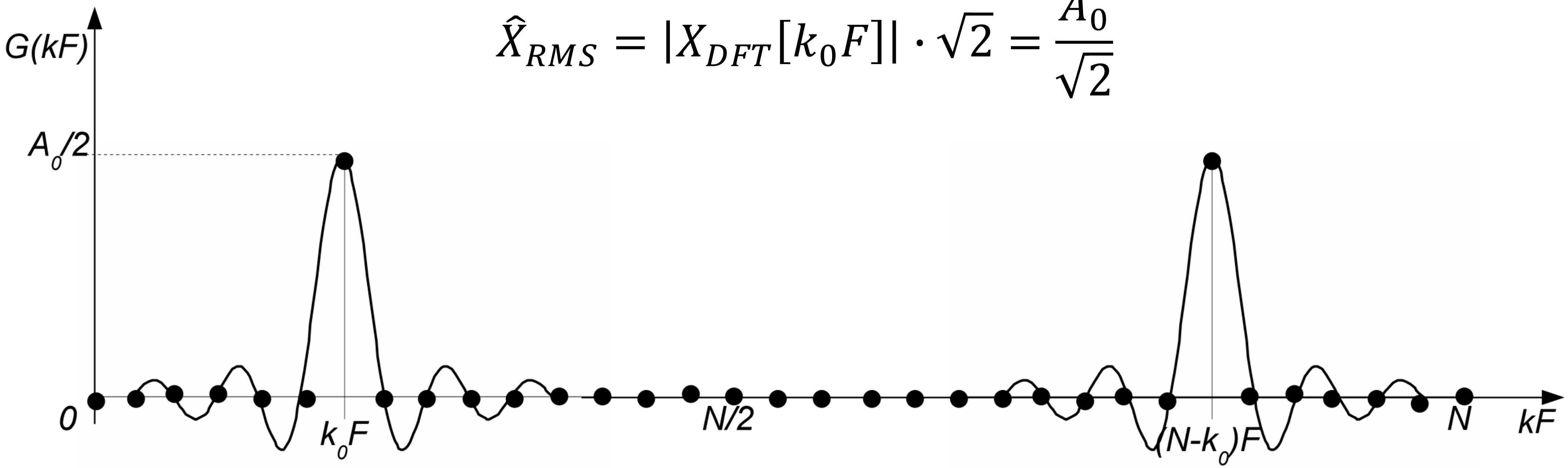
- T_W corresponds to an in integer number of periods

$$NT_S = k_0 \cdot \frac{1}{f_0} \Rightarrow f_0 = \frac{k_0}{NT_S} = k_0 F$$

$$|X_{DFT}(k_0 F)| = \frac{A_0}{2}$$

$$|X_{DFT}[kF]| = 0 \text{ for all other values of } k$$

$$\hat{X}_{RMS} = |X_{DFT}[k_0 F]| \cdot \sqrt{2} = \frac{A_0}{\sqrt{2}}$$

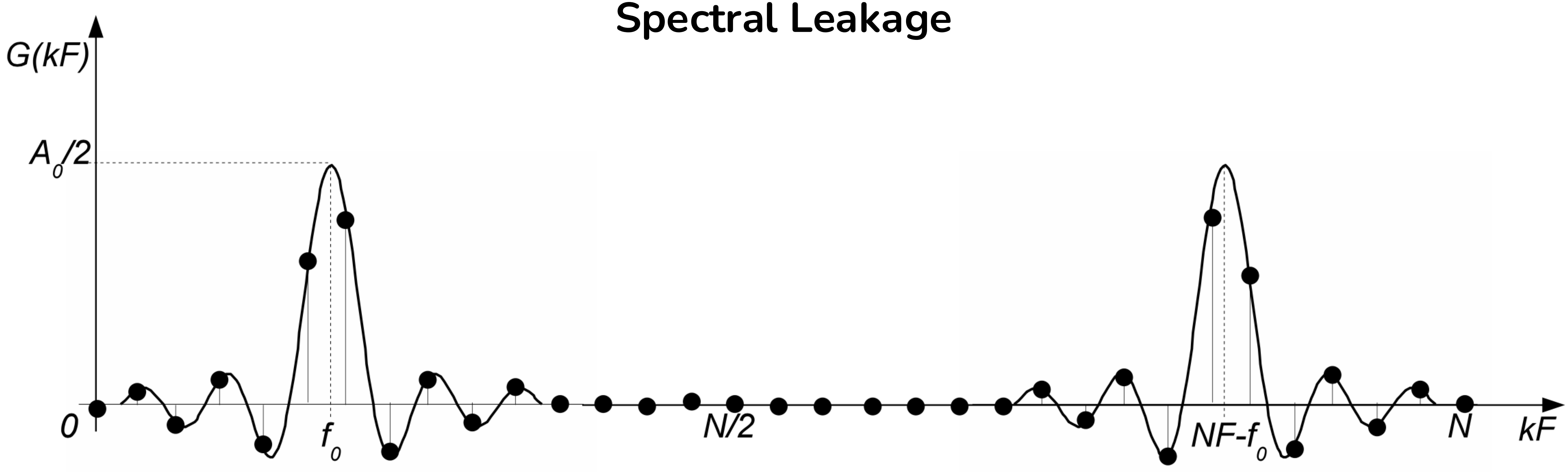


DFT-based sinewave spectrum

- f_0 is not an integer multiple of F
- There will be an index k_0 so that k_0F is the closest estimate of f_0

$$\delta = \frac{f_0 - kF}{F} \text{ with } |\delta| \le \frac{1}{2}$$

Spectral Leakage



Università degli Studi di Padova

Spectral Leakage

$$\hat{X}_{RMS} = |X_{DFT}[k_0 F]| \cdot \sqrt{2} = \frac{A_0}{\sqrt{2}} W_{R(DFT)}(\delta)$$

$$f_0 = k_0 F \quad \text{where} \quad k_0 = \arg\left[\max_{0 \le k \le N/2} |X_{DFT}(kF)|\right]$$

- Worst case frequency deviation $\frac{\Delta_F}{2} = \frac{F}{2}$
- Amplitude accuracy depends on $W_{R(DFT)}(\delta)$
- For $0 \le |\delta| \le \frac{1}{2} \Rightarrow W_{R(DFT)}(\delta) < 1 \Rightarrow \hat{X}_{RMS} < \frac{A_0}{\sqrt{2}}$

Scalloping Loss

- For $|\delta| = \frac{1}{2} \Rightarrow$ **Worst Case Scalloping Loss** (WCSL)

Frequency Resolution

- Signal composed of multiple sinusoidal components
- Multiple peaks
- Close peaks ⇒ Interference due to spectral leakage

Peak detection

Peak measurement

- Peak detection ⇒ recognize the presence of distinct components

Frequency Resolution

Minimum separation at which two equal amplitude sinusoidal components create distinct peaks in the spectrum trace

Frequency Resolution

- Two sinewaves of equal amplitude A at frequencies f_1 and f_2

$$|X_{DFT}[kF]| = \frac{A}{2}\left[e^{j\phi_1} \cdot W_{R(DFT)}(k - \lambda_1) + e^{j\phi_2} \cdot W_{R(DFT)}(k + \lambda_2)\right]$$

- Two distinct peaks if $|X_{DFT}[kF]| < \frac{A}{2}$ at $f = \frac{f_1+f_2}{2}$

$$\left|W_{R(DFT)}\left(\frac{\lambda_2-\lambda_1}{2}\right)\right| \cdot \left|e^{j\phi_1} + e^{j\phi_2}\right| < 1$$

$$\left|W_{R(DFT)}\left(\frac{\lambda_2-\lambda_1}{2}\right)\right| < \frac{1}{2} \quad \text{where } \frac{1}{2} \rightarrow -6\text{ dB}$$

Normalized 6 dB Bandwidth $\rightarrow 2 \cdot B_{-6dB}$

$$|f_2 - f_1| > \frac{2 \cdot B_{-6dB}}{NT_S} = 2 \cdot B_{-6dB} \cdot F$$

UNIVERSITÀ DEGLI STUDI DI PADOVA

Masking

- Sinewaves of **different amplitude**

  Masking (hidden smaller components)

- Information about the shape of the window
  - **Normalized main lobe width**
  - **Attenuation of the largest side lobe with respect to the main one**
  - **Side lobe fall-off with frequency**

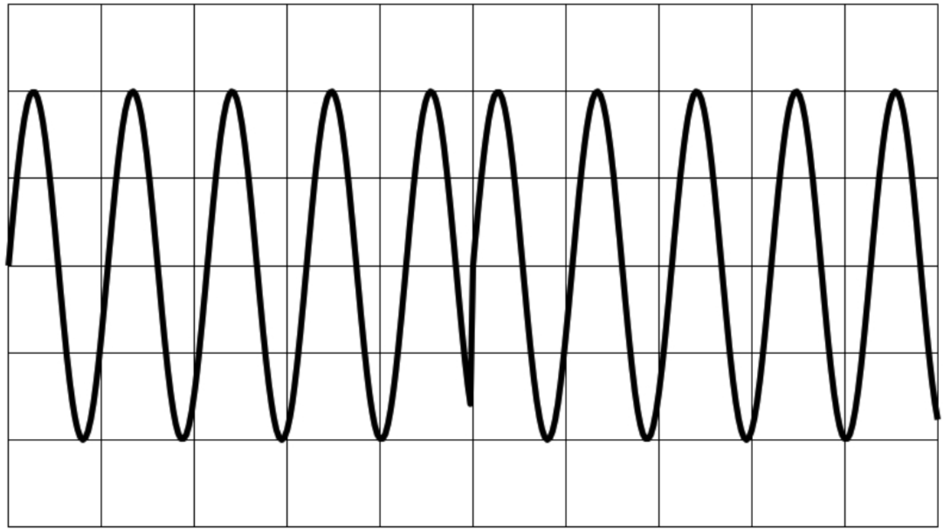
Window Functions

- **Uniform window**: poor behaviour with regards to both spectral interference and scalloping loss
- Different types of windows:
  - flat main lobe to minimize scalloping loss;
  - very low, or fast-decaying side lobes to reduce interference.

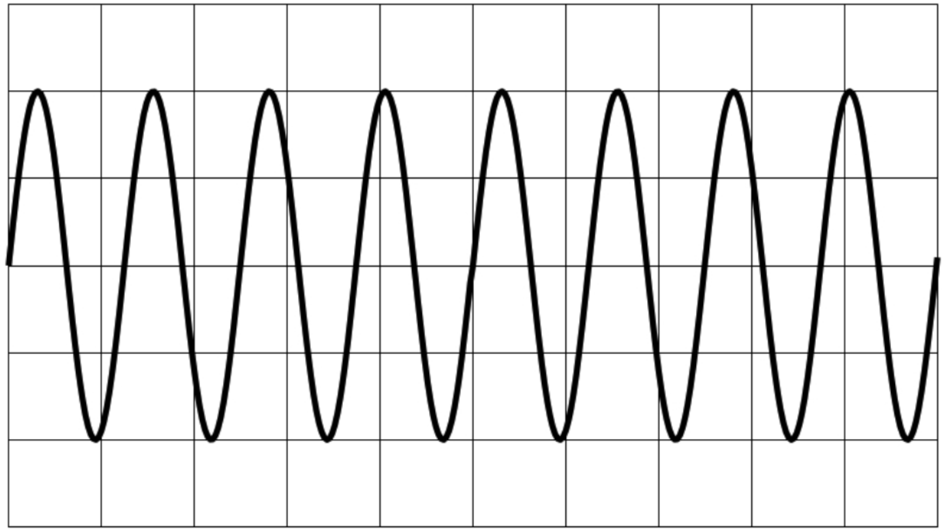
⬇

Requirements in contrast

Windowing effect



(a) $T_W \neq PT \rightarrow$ truncation



(b) $T_W = PT \rightarrow$ no leakage

- **Discontinuity** of the the segment edges
- Window function → **weight samples** according to their position in the segment

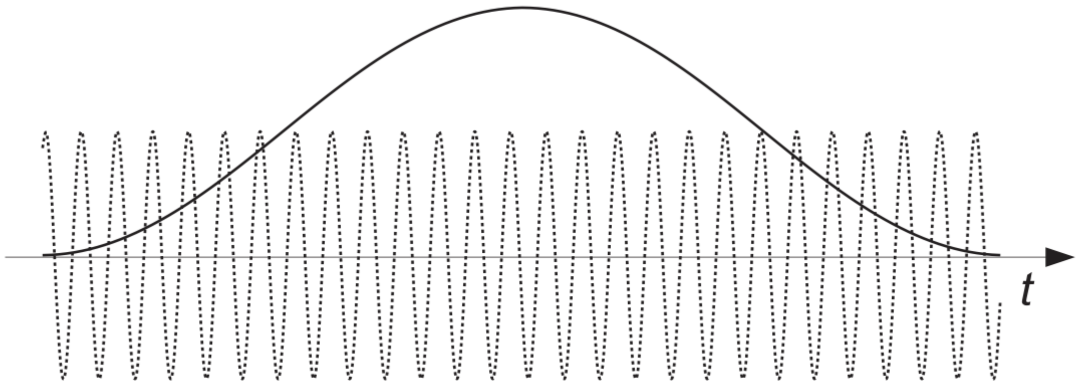
Hanning Window

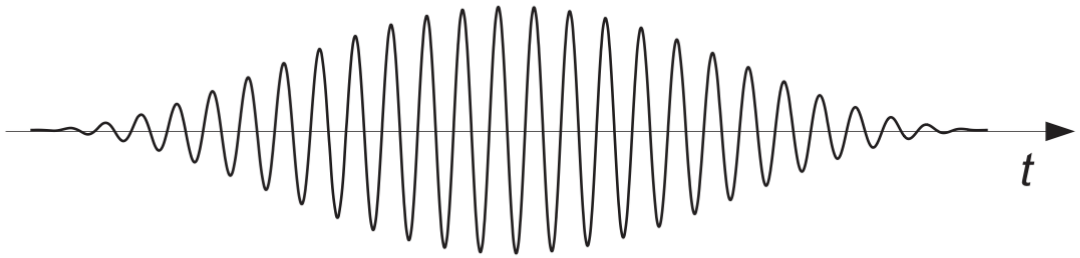
- Most windows are defined by a **mathematical expression** like this:

$$w(nT_S) = \sum_{m=0}^{M} a_m (-1)^m \cos\left(2\pi m \frac{n}{N}\right) \qquad n = 0, \dots, N-1$$

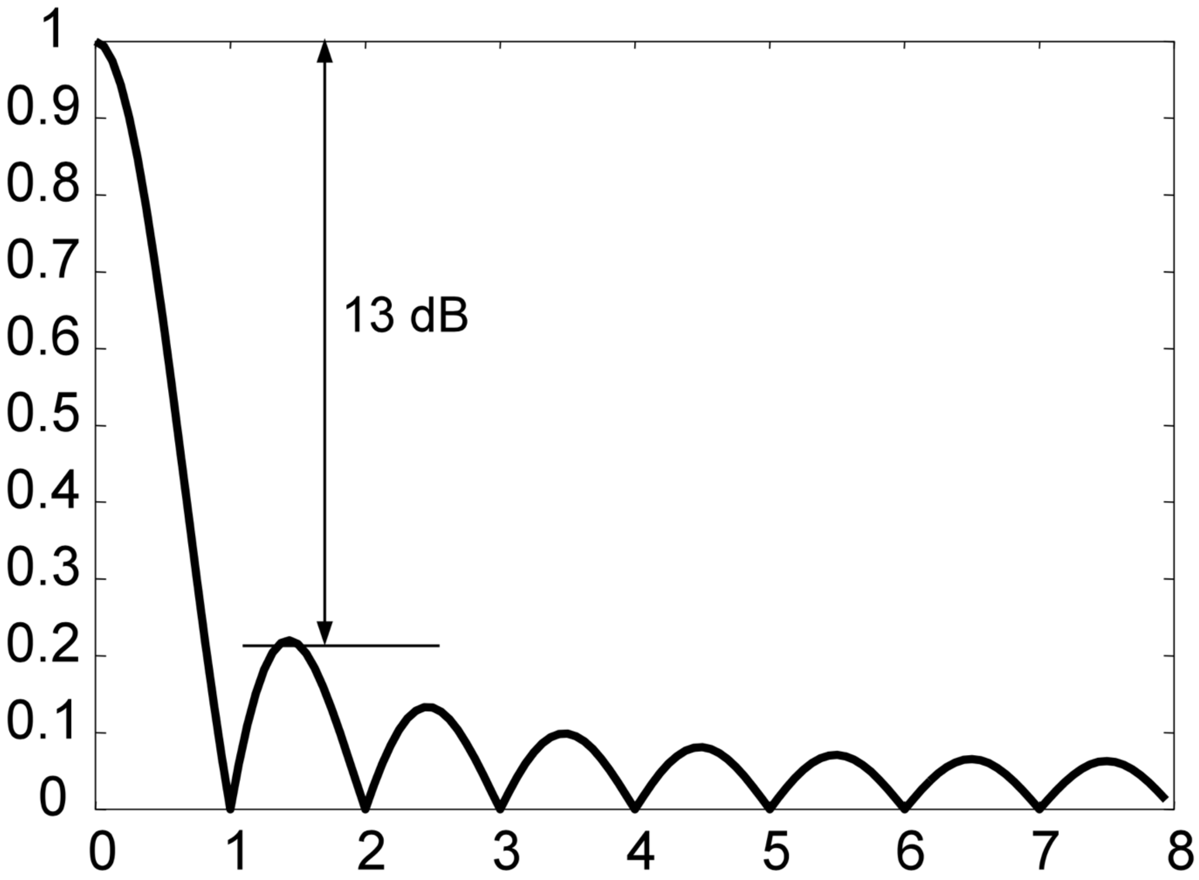
- **Hanning** window

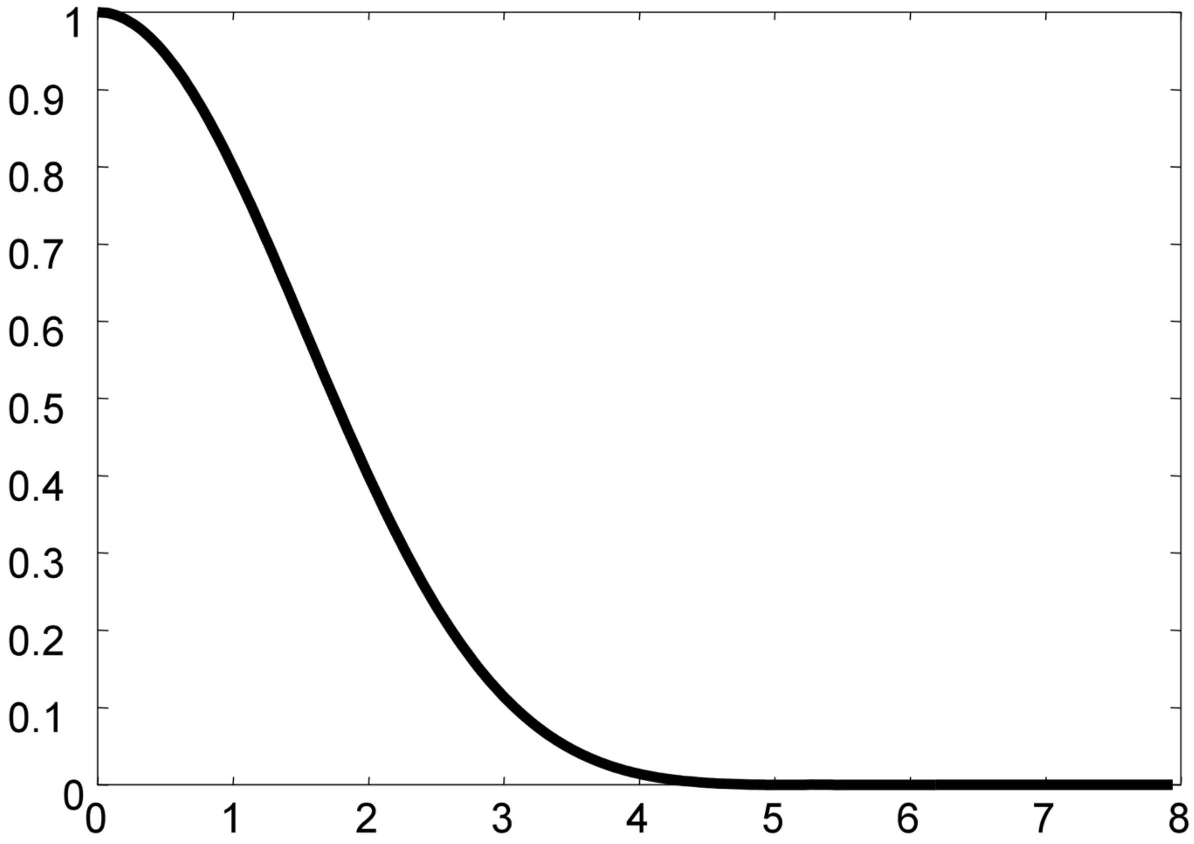
$$w(nT_S) = \frac{1}{2} - \frac{1}{2}\cos\left(2\pi \frac{n}{N}\right) \qquad n = 0, \dots, N-1$$





Uniform vs Hanning Window

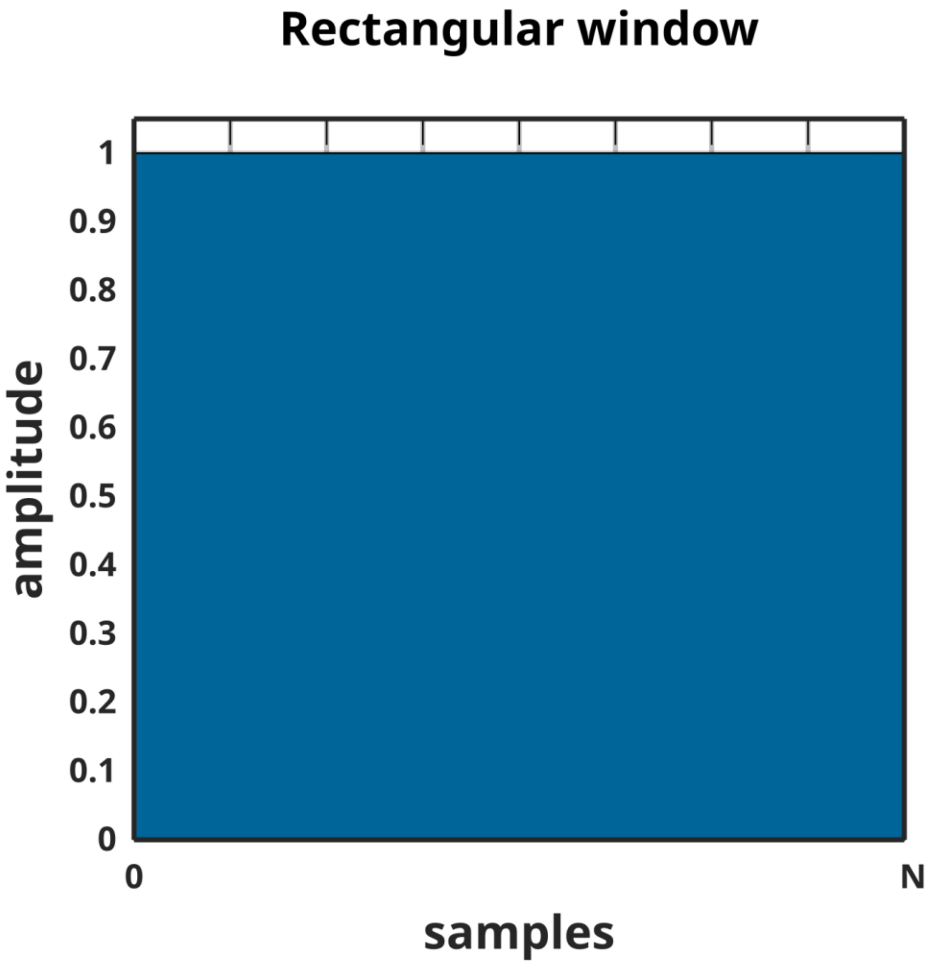


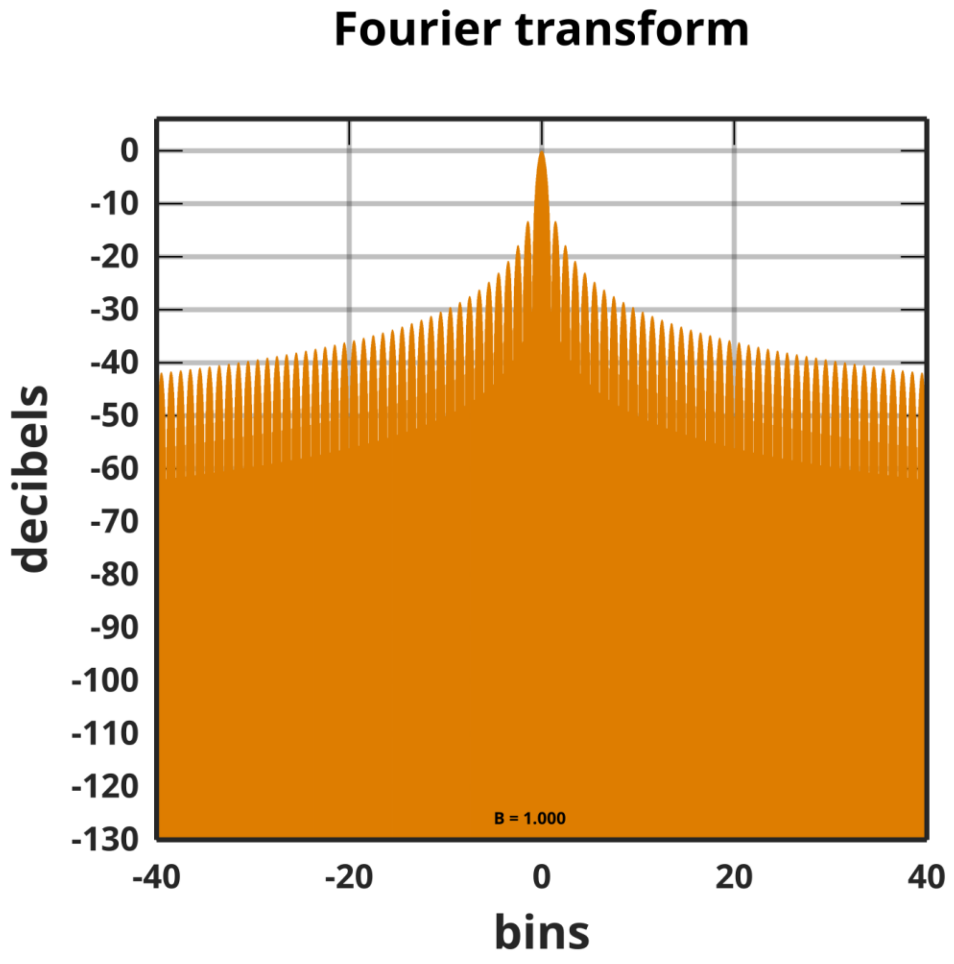


Window Functions

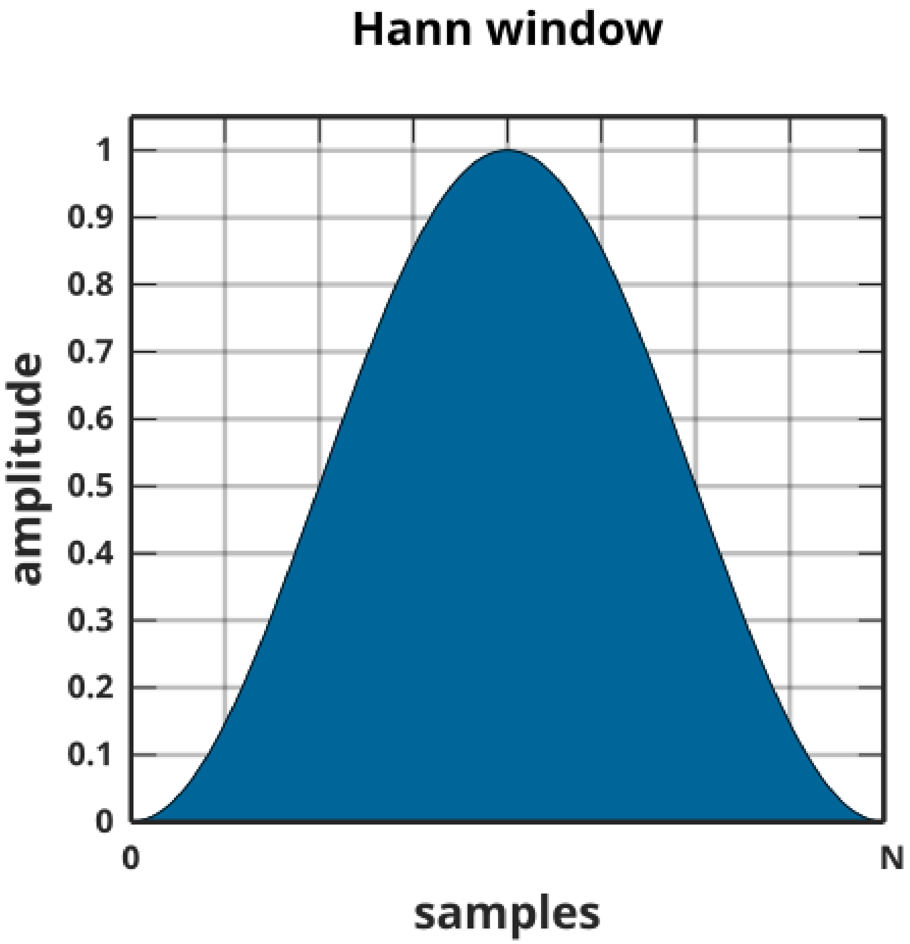
Window	WCSL [dB]	minimum side lobe attenuation [dB]	main lobe width [bin]	$2 \cdot B_{-6dB}$ [bin]	ENBW [bin]
uniform	3.92	13	2	1.21	1
Hann (*Hanning*)	1.42	32	4	2	1.5
Blackman-Harris	1.13	71	6	2.27	1.71
flat-top	< 0.01	93	10	4.58	3.77

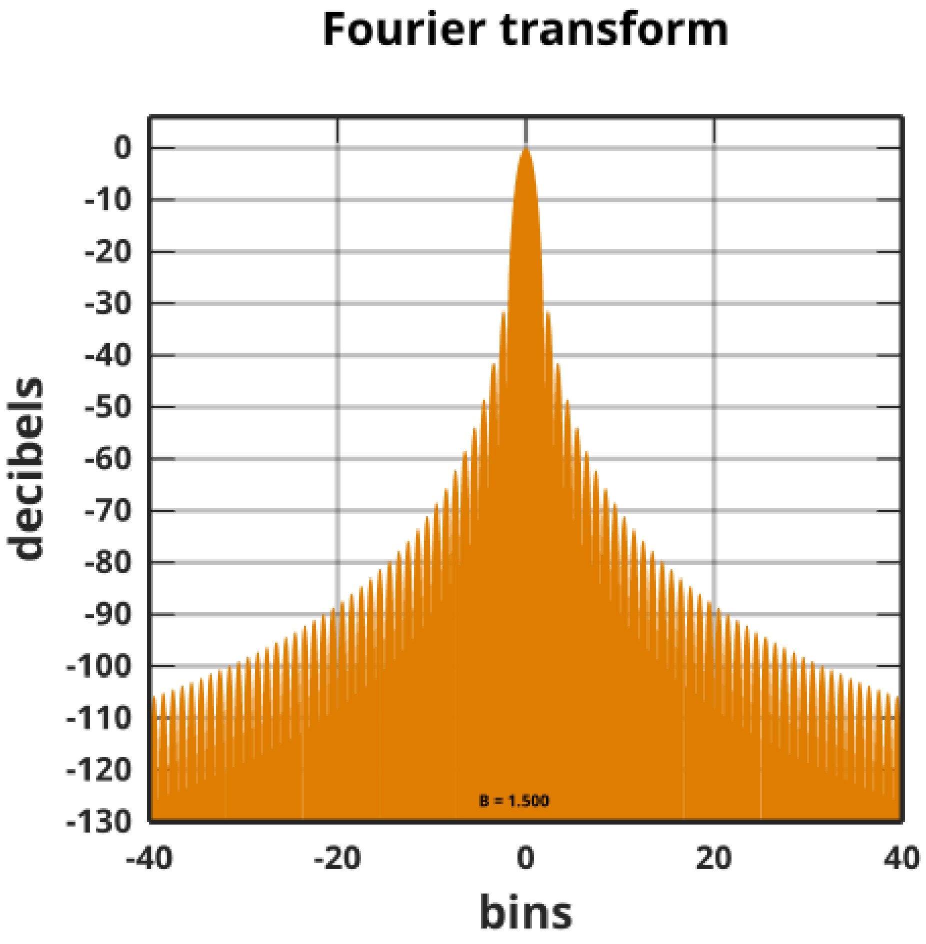
Uniform window



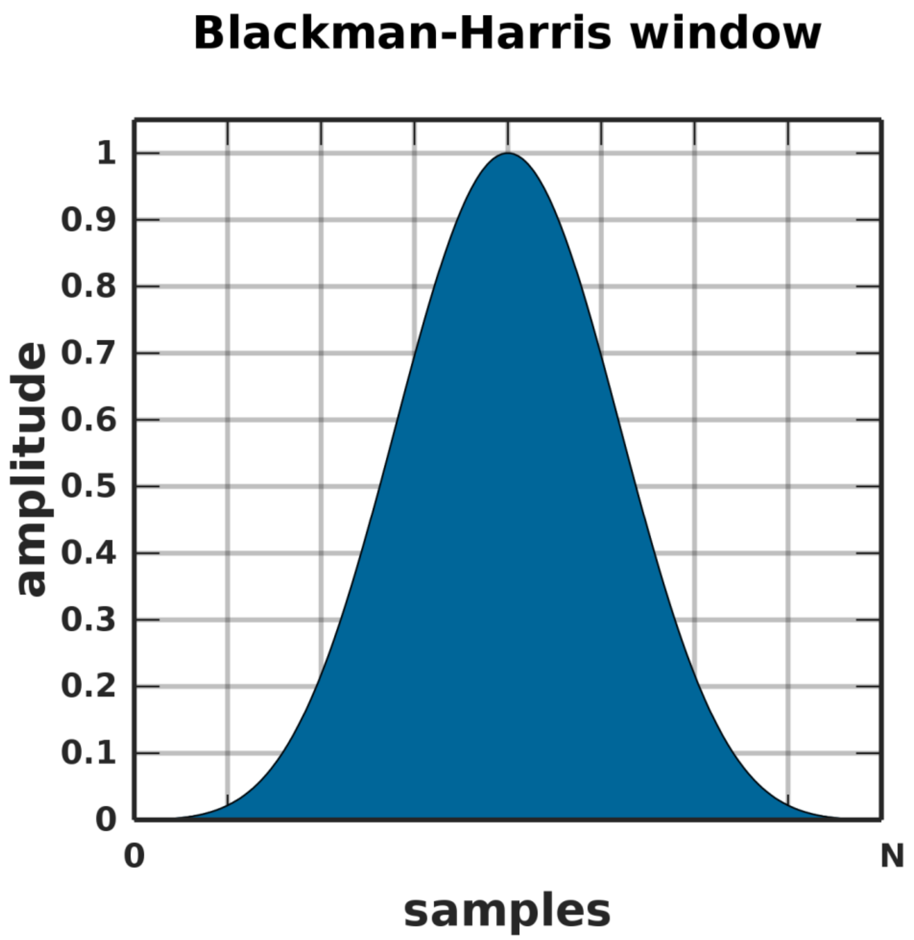


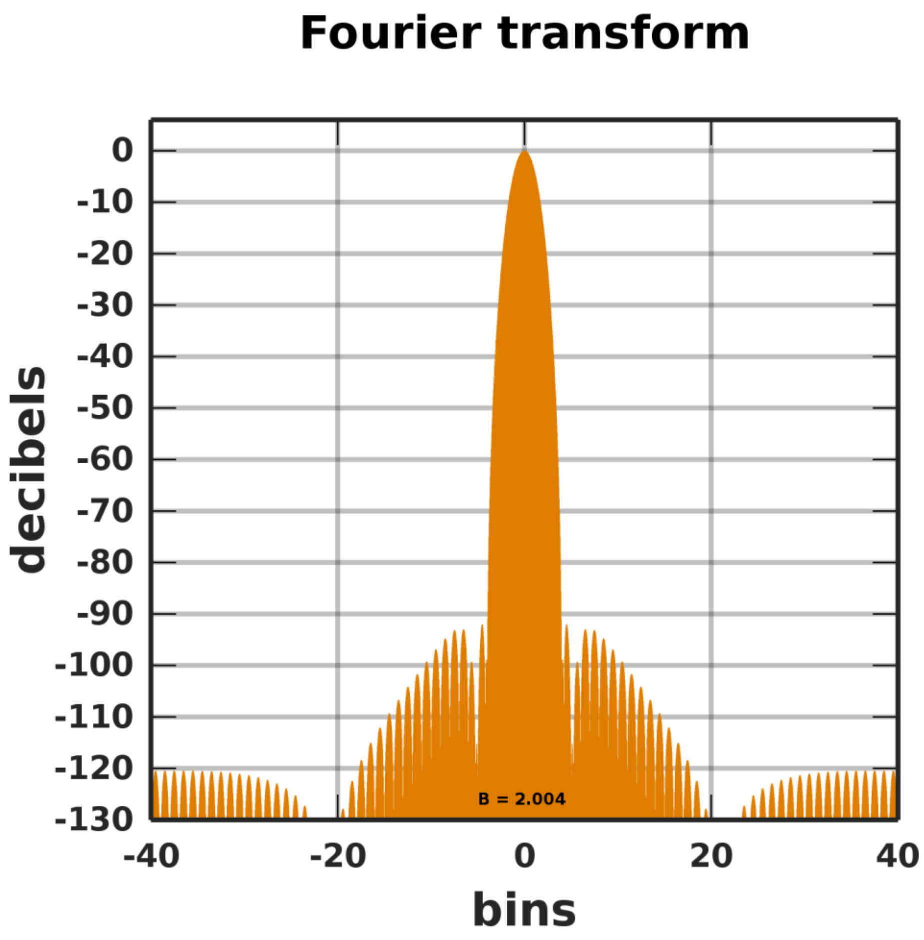
Hann window


Hann window
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
amplitude
0
N
samples

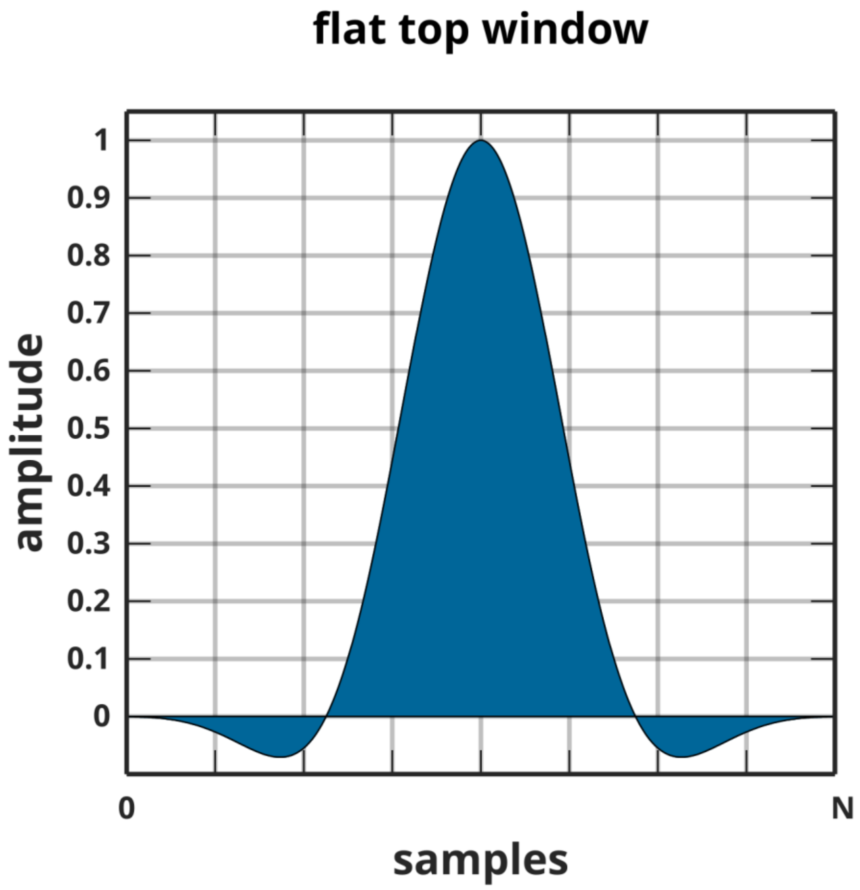

Fourier transform
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
decibels
B = 1.500
-40
-20
0
20
40
bins

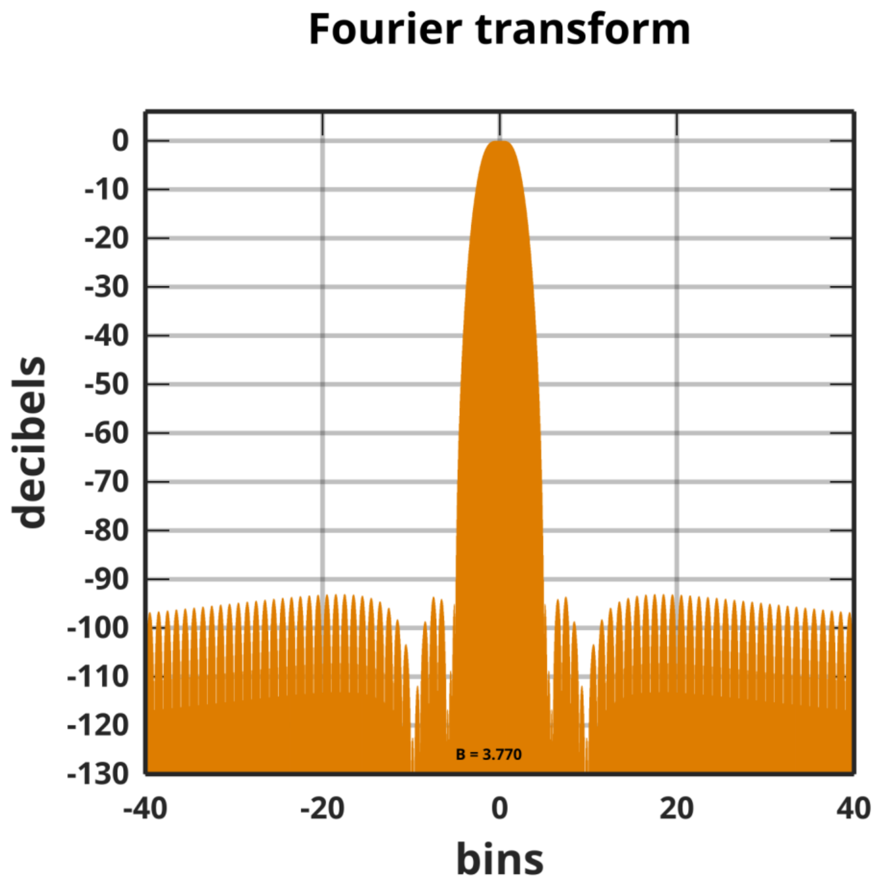
Blackman-Harris window





Flat-top window





UNIVERSITÀ
DEGLI STUDI
DI PADOVA