LCD (02/03)

* Calculus of communicating systems

toy example

$$CS = \overline{pub}.CS1 \qquad CS1 = \overline{coin}.CS2 \qquad CS2 = coffee.CS$$

$$CM = coin.CM1 \qquad CM1 = \overline{coffee}.CM$$

$$Off = (CS \mid CM) \setminus L \qquad L = \{coin, coffee\}$$

$$\begin{tikzcd}
(CS \mid CM) \setminus L \arrow[d, "\overline{pub}"] \\
(CS1 \mid CM) \setminus L \arrow[d, "\tau"] \\
(CS2 \mid CM1) \arrow[uu, bend left=60, "\tau"]
\end{tikzcd}$$

$$Spec = \overline{pub}.Spec$$

$$Off \approx Spec$$

→ Syntax

→ Operational behaviour

→ Program equivalence

SYNTAX

set of channels / ports (infinite denumerable)

$\mathcal{A}$ $\quad a, b, c$

$\overline{\mathcal{A}} = \{\overline{a} \mid a \in \mathcal{A}\}$

$\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$

$Act = \mathcal{L} \cup \{\tau\} \qquad \tau \notin \mathcal{L}$

process constants

$\mathcal{K} \ni K$

(denumerable infinite)

CCS processes

$$P, Q ::= K \quad | \quad \alpha . P \quad | \quad \sum_{i \in I} P_i \quad | \quad P | Q$$

$K \in \mathcal{K}$

$K \stackrel{def}{=} P$

$\alpha \in \mathcal{A}$

(possibly $\alpha = \tau$)

I can be infinite

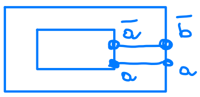
$$| \quad P[f] \quad | \quad P \setminus L$$

$f : Act \to Act$

$f(\tau) = \tau$

$f(a) = b \Rightarrow f(\bar{a}) = f(\bar{b})$

$L \subseteq \mathcal{A}$



Remarks

- → deadlock process missing ?

  0 identified with $\sum_{i \in \emptyset} P_i$

- → non deterministic binary choice

  $P_1 + P_2 \rightsquigarrow \sum_{i \in \{1,2\}} P_i$

  $\wr\wr$

  $P_2 + P_1$

- → relabeling

  $a_1 \quad \dots \quad a_m$

  $\downarrow \qquad\quad \downarrow$

  $b_1 \qquad\quad b_m$

  $P[b_1/a_1, \dots, b_m/a_m]$

  $f : Act \to Act$

  $f(a_i) = b_i \qquad i \in \{1, \dots, m\}$

  $f(\bar{a}_i) = \bar{b}_i \qquad i \in \{1, \dots, m\}$

  $f(c) = c \qquad \forall c \neq a_i, \bar{a}_i$

  $P[f]$

- restrict

$P \setminus \{a\}$ $\qquad\qquad$ $P \setminus a$

- priority

$\_ \setminus \_ \qquad \_[f] \qquad \alpha.\_ \qquad \_|\_ \qquad \_+\_$

higher $\qquad\qquad\qquad\qquad$ lower

$$a.0 + (b.0 \mid (c.0\,[d/c]))$$

* <u>Operational Behavioural</u>

$$P \xrightarrow{\alpha} P'$$

syntax driven rules (structural operational semantics , Plotkin '81)

$$\frac{P_1 \xrightarrow{\alpha_1} P_1' \qquad\qquad P_2 \xrightarrow{\alpha_2} P_2'}{P_1 \ op \ P_2 \xrightarrow{\alpha} P'}$$

depends on α_1, α_2 $\qquad$ depends on P_1', P_2'

CCS Rules

* <u>ACT</u>

$$\frac{}{\alpha.P \xrightarrow{\alpha} P}$$

example

$$\frac{}{\overline{pub}.CS1 \xrightarrow{\overline{pub}} CS1} \ \text{ACT}$$

* NONDETERMINISM

$$\text{SUM}\quad \frac{P_j \xrightarrow{\alpha} P_j'}{\sum_{i\in I} P_i \xrightarrow{\alpha} P_j'}\quad j\in I \qquad \left(\frac{P_1 \xrightarrow{\alpha} P_1'}{P_1+P_2 \xrightarrow{\alpha} P_1'} \qquad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_1+P_2 \xrightarrow{\alpha} P_2'} \right)$$

NOT

$$\frac{}{\sum_{i\in I} P_i \xrightarrow{\tau} P_j}\quad j\in I$$

Example

$$\text{SUM}\quad \frac{\text{ACT}\quad \dfrac{}{\overline{coffee}.CTM \xrightarrow{\overline{coffee}} CTM}}{\overline{coffee}.CTM + \overline{tea}.CTM \xrightarrow{\overline{coffee}} CTM}$$

$$\text{SUM}\quad \frac{\text{ACT}\quad \dfrac{}{\overline{tea}.CTM \xrightarrow{\overline{tea}} CTM}}{\overline{coffee}.CTM + \overline{tea}.CTM \xrightarrow{\overline{tea}} CTM}$$

Example :

$$BC = tick\ (BC + 0) \qquad\qquad \frac{0 \not\rightarrow 0}{BC+0 \not\rightarrow 0}$$

$$BC = tick.BC + tick.0 \qquad\qquad \text{SUM}\quad \frac{\text{ACT}\quad \dfrac{}{tick.0 \xrightarrow{tick} 0}}{tick.BC + tick.0 \xrightarrow{tick} 0}$$

* Parallel composition

$$\frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'} \qquad \frac{P \xrightarrow{\alpha} P' \quad Q \xrightarrow{\bar{\alpha}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

convention

$$\alpha = \bar{a} \qquad \bar{\alpha} = \bar{\bar{a}} = a$$

Example

$$\dfrac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM}\ \text{ACT} \qquad \dfrac{}{coffee.CS \xrightarrow{coffee} CS}\ \text{ACT}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\tau} CM \mid CS}$$

$$\dfrac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM}\ \text{ACT}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\overline{coffee}} CM \mid coffee.CS}$$

* **RESTRICTION**

$$\dfrac{P \xrightarrow{\alpha} P'}{P \setminus L \xrightarrow{\alpha} P' \setminus L} \qquad \alpha, \overline{\alpha} \notin L$$

$$\dfrac{\dfrac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM} \qquad \dfrac{}{coffee.CS \xrightarrow{coffee} CS}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\tau} CM \mid CS}}{(\overline{coffee}.CM \mid coffee.CS) \setminus L \xrightarrow{\tau} (CM \mid CS) \setminus L} \qquad L = \{coin, coffee\}$$

$$\dfrac{\dfrac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\overline{coffee}} CM \mid coffee.CS}}{(\overline{coffee}.CM \mid coffee.CS) \setminus L \cancel{\xrightarrow{\overline{coffee}}} (CM \mid CS) \setminus L} \qquad \cancel{coffee \notin L}$$

Example

$$(\overline{coffee}.CTM + \overline{tea}.CTM \mid coffee.CS) \setminus \{coffee, tea\}$$

$$\xrightarrow{\tau} (CTM \mid CS) \setminus \{coffee, tea\}$$

external choice

$$(\tau . \overline{coffee} . CTM + \tau . \overline{tea} . CTM \mid coffee . CS) \setminus \{ coffee , tea \}$$

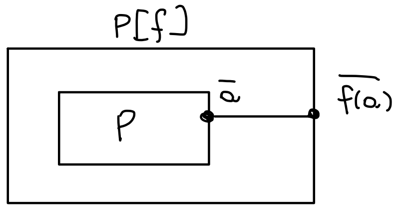
$$\xrightarrow{\tau} (\overline{tea} . CTM \mid coffee . CS) \setminus \{ coin , coffee \}$$

internal choice

$$BC = tick (BC + \tau . 0)$$

* <u>Redirection</u>

$$\frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$



* <u>Constant</u>

$$\frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'} \qquad K \overset{def}{=} P$$

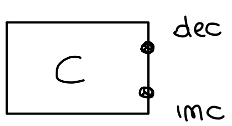
NOT

$$\frac{}{K \xrightarrow{\tau} P}$$

<u>Interpreter</u>

$$\frac{\quad}{P \longrightarrow}$$

<u>EXERCISE</u> : Define a counter



→ increment always possible

→ decrement possible if > 0

constants

$C_i \quad i \in \mathbb{N}$

$$\begin{tikzcd} C_0 \arrow[d, shift right, "inc"'] \\ C_1 \arrow[u, shift right, bend right, "dec"'] \arrow[d, shift right, "inc"'] \\ C_2 \arrow[u, shift right, bend right, "dec"'] \arrow[d] \\ \vdots \end{tikzcd}$$

$$\begin{cases} C_0 = inc.\, C_1 \\ C_{i+1} = inc.\, C_{i+2} + dec.\, C_i \qquad i \in \mathbb{N} \end{cases}$$

infinite program

$$C = inc.\,(C \mid dec.0)$$

$$\begin{tikzcd}
(C|0)|0 & \overset{?}{\sim} & C \arrow[d, "inc"] & \overset{?}{\sim} & C|0 & & (C|0)|0 \\
(C|0)\,|\,dec.0 \arrow[u, "dec"] & \overset{?}{\sim} & C \mid dec.0 \arrow[d, "inc"] \arrow[urr, "dec"'] & & & \overset{?}{\sim} & (C \mid dec.0)\,|\,0 \arrow[u, "dec"'] \\
& & (C \mid dec.0) \mid dec.0 \arrow[ull, "dec"] \arrow[urrrr, "dec"'] & & & & \\
& & \vdots & & & &
\end{tikzcd}$$

$$C \overset{?}{\sim} C_0$$

Example:

$A = (a.A)\setminus b$

$A \xrightarrow{a} A\setminus b$

$A\setminus b \xrightarrow{a} (A\setminus b)\setminus b$

$$\dfrac{\dfrac{\dfrac{}{a.A \xrightarrow{a} A}}{(a.A)\setminus b \xrightarrow{a} A\setminus b}}{A \xrightarrow{a} A\setminus b}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a.A \xrightarrow{a} A}}{(a.A)\setminus b \xrightarrow{a} A\setminus b}}{A \xrightarrow{a} A\setminus b}}{A\setminus b \xrightarrow{a} (A\setminus b)\setminus b}$$

$$A \xrightarrow{a} A \backslash b \xrightarrow{a} (A \backslash b) \backslash b \xrightarrow{a} ((A \backslash b) \backslash b) \backslash b \;\; \ldots .$$

$$A \backslash b \overset{?}{\sim} A \backslash b \backslash b$$