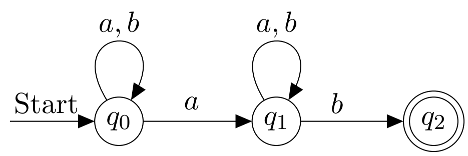
Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

September 11th, 2025

1. **[6 points]** Assume the NFA N whose transition function δ_N is graphically represented below.



Answer the following questions.

(a) The textbook defines the extended transition function $\hat{\delta}_N$ as

i. base: $\hat{\delta}_N(q, \epsilon) = \{q\}$

ii. induction: $\hat{\delta}_N(q, xa) = \cup_{p \in \hat{\delta}_N(q,x)} \delta_N(p, a)$

Assess whether the string $baba$ belongs to the language $L(N)$ by computing the value of $\hat{\delta}_N(q_0, baba)$. Report all of the **intermediate steps**.

(b) Transform N into an equivalent deterministic finite automaton D, with transition function δ_D, by applying the subset construction together with the lazy evaluation. Depict the graphical representation of the function δ_D.

2. **[8 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b\}$:

$$\begin{aligned} L_1 &= \{ba^n ba^n b \mid n \geq 1\} \\ L_2 &= \{ba^n a^n b \mid n \geq 1\} \\ L_3 &= L_2 \cdot L_2 \end{aligned}$$

For each of the above languages, state whether it belongs to REG, to CFL∖REG, or else whether it is outside of CFL. Provide a mathematical proof for all of your answers.

(please turn to the next page)

3. **[6 points]** With reference to the membership problem for context-free languages, answer the following two questions.

   (a) Specify the recursive relation underlying the dynamic programming algorithm reported in the textbook for the solution of this problem.

   (b) Consider the CFG G in Chomsky normal form defined by the following rules:

$$
\begin{aligned}
S &\to AB \\
A &\to AA \mid AB \mid a \\
B &\to BB \mid b
\end{aligned}
$$

   Assuming as input the CFG G and the string $w = aaabbbb$, trace the application of the algorithm in (a) and assess whether $w \in L(G)$.

4. **[7 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) For every languages L_1 in REG and L_2 in REC, we have that $L_1 \cap L_2$ is in REG.

   (b) For every languages L_1 in REG and L_2 in REC, we have that $L_1 \cap L_2$ is in REC.

   (c) There exists languages L_1, L_2 in CFL$\smallsetminus$REG such that the language $L_1 \cap L_2$ is in REG.

   (d) Let $\mathcal{P}$ be the class of languages that can be recognized in polynomial time by a TM. For every languages L_1 in CFL and L_2 in $\mathcal{P}$, we have that $L_1 \cdot L_2$ is in $\mathcal{P}$.

5. **[6 points]** Define the notion of property of the languages generated by TMs and state Rice's theorem. Provide the proof of Rice's theorem that we have developed in class.