Problem sheet 3: Distribution theory
Functions theory

Exercise 1. Let $N > 1$ and $E \subseteq \mathbb{R}^N$ be a bounded open set with boundary of class C^1 and consider the distribution

$$T_{\chi_E}(\phi) := \int_E \phi(x)dx.$$

Compute the distributional gradient of T_{χ_E}.

Exercise 2. Let $T \in \mathcal{D}'(I)$ for some interval $I \subseteq \mathbb{R}$. Show that $T' = 0$ in the sense of distribution (this means $T'\phi = -\int T\phi' dx = 0$) if and only if there exists a constant c and $T = c$ (this means $T\phi = c\int \phi dx$).
Hint: argue as in the fundamental lemma of the calculus of variations.