Definizione: $T \leq \mathbb{R}^n$ il suo ortogonale

$$T^\perp = \{ v \in \mathbb{R}^n \mid v \cdot t = 0 \quad \forall\, t \in T \}$$

Domanda 15: dimostrare che se $T \leq \mathbb{R}^n$ allora anche $T^\perp \leq \mathbb{R}^n$.

Dim:

① $0_{\mathbb{R}^n} \in T^\perp$ perché $0_{\mathbb{R}^n} \cdot t = 0 \quad \forall t \in T$

② Chiuso per la somma

$v_1 \in T^\perp \qquad v_1 \cdot t = 0 \quad \forall t \in T$

$v_2 \in T^\perp \qquad v_2 \cdot t = 0 \quad \forall t \in T$

$v_1 + v_2 \in T^\perp$ perché $(v_1 + v_2) \cdot t = v_1 \cdot t + v_2 \cdot t = 0 + 0 = 0 \quad \forall t \in T$

③ Chiuso per prodotto per scalari

$v \in T^\perp \qquad v \cdot t = 0 \quad \forall t \in T$

$a \in \mathbb{R}$

$a v \in T^\perp$ perché $(a v) \cdot t = a (v \cdot t) = a \cdot 0 = 0 \quad \forall t \in T.$ $\square$

Domanda 16: dato $T \leq \mathbb{R}^n$ si ha

$$T \oplus T^\perp = \mathbb{R}^n.$$

Dim:

$T \cap T^\perp = \{0_{\mathbb{R}^n}\}$ perché se $v \in T \cap T^\perp \Rightarrow v \perp v$ cioè

$v \cdot v = 0 \qquad \|v\|^2 = 0 \Rightarrow v = 0_{\mathbb{R}^n}.$

$T \qquad B_T = \{ t_1, \dots, t_k \} \qquad \dim T = k$ per dimostrare che

$T \oplus T^\perp = \mathbb{R}^n$ è nec. e suff. dimostrare che $\dim T^\perp = n - k$

$T = \langle t_1, \dots, t_k \rangle$

$$T^\perp = \{ v \in \mathbb{R}^n \mid v \cdot t_1 = 0,\ v \cdot t_2 = 0 \dots v \cdot t_k = 0 \}$$
$$t_1 \cdot v = 0 \quad \dots \quad t_k \cdot v = 0$$
$$v = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\begin{cases} t_1 \cdot v = 0 \\ \vdots \\ t_k \cdot v = 0 \end{cases}$$

sistema lineare di k equazioni in n incognite $x_1, \dots, x_n$ $\quad A\underline{x} = \underline{0}$

$\dim T^\perp = n - \mathrm{rg}A = n - k$ perché $\mathrm{rg}A = k \leq n$

$$A = \begin{pmatrix} t_1^t \\ \vdots \\ t_k^t \end{pmatrix}$$

cioè t_1 messo in riga è la 1° riga di A
t_2 messo in riga è la 2° riga di A
$\vdots$
t_k messo in riga è la k° riga di A

$\Rightarrow$ le righe di A sono lin. indip. $\Rightarrow \mathrm{rg}A = k$,
$\Rightarrow T \oplus T^\perp = \mathbb{R}^n$. $\square$

Esempio:

1) Dato $T = \left\langle \begin{pmatrix} 1 \\ 0 \\ -2 \\ 3 \end{pmatrix} \right\rangle$ determinare $T^\perp$.

2) Determinare un sottospazio $U \leq \mathbb{R}^4$: $U \oplus T = \mathbb{R}^4$.

Svolg:

1) $T^\perp$: $x_1 - 2x_3 + 3x_4 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -2 \\ 3 \end{pmatrix} = 0$$

$\dim T^\perp = 3$

$T \oplus T^\perp = \mathbb{R}^4$

$$\mathrm{rg}\begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 4$$

$U = \langle e_2, e_3, e_4 \rangle$

$T \oplus T^{\perp} = \mathbb{R}^n \qquad (T^{\perp})^{\perp} = T$

$t \cdot s = 0 \qquad t \perp s$

$s \perp t$

T	$T^{\perp}$
$\left\langle \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \right\rangle$	$x + 3y + 5z = 0$
$x + 3y + 5z = 0$	$\left\langle \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \right\rangle$
$\pi: \left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} \right\rangle$ 2	$r: \begin{cases} x + 2y + 3z = 0 \\ 2x + 5z = 0 \end{cases}$ $3-2=1$
$r: \begin{cases} x + y + z = 0 \\ 2x - 8y + 11z = 0 \end{cases}$	$\pi: \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -8 \\ 11 \end{pmatrix} \right\rangle$
$\pi: \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \\ 5 \end{pmatrix} \right\rangle$	$\sigma: \begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + 3x_3 + 5x_4 = 0 \end{cases}$

Sia $U = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \\ 5 \end{pmatrix} \right\rangle$ determinare $W \leq \mathbb{R}^4$: $U \oplus W = \mathbb{R}^4$

eq. cartesiane

$W = U^{\perp} = \begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + 3x_3 + 5x_4 = 0 \end{cases} \qquad \boxed{U \oplus U^{\perp} = \mathbb{R}^4}$

Coordinate di un vettore in una base ortonormale.

$B = \{u_1, \dots, u_n\}$ base ortonormale di $\mathbb{R}^n$ allora le coordinate di un vettore $v \in \mathbb{R}^n$ rispetto alla base B sono

$$v = \sum_{i=1}^{n} a_i u_i \qquad a_i = \boxed{v \cdot u_i}$$

Dim:

$$v = \sum_{i=1}^{n} a_i u_i$$

$$v \cdot u_1 = \left(\sum_{i=1}^{n} a_i u_i\right) \cdot u_1 = \sum_{i=1}^{n} a_i u_i \cdot u_1 = a_1 \underbrace{u_1 \cdot u_1}_{1} = a_1$$

$$\|u_1\|^2 = u_1 \cdot u_1 = 1$$

perché u_1 è un versore

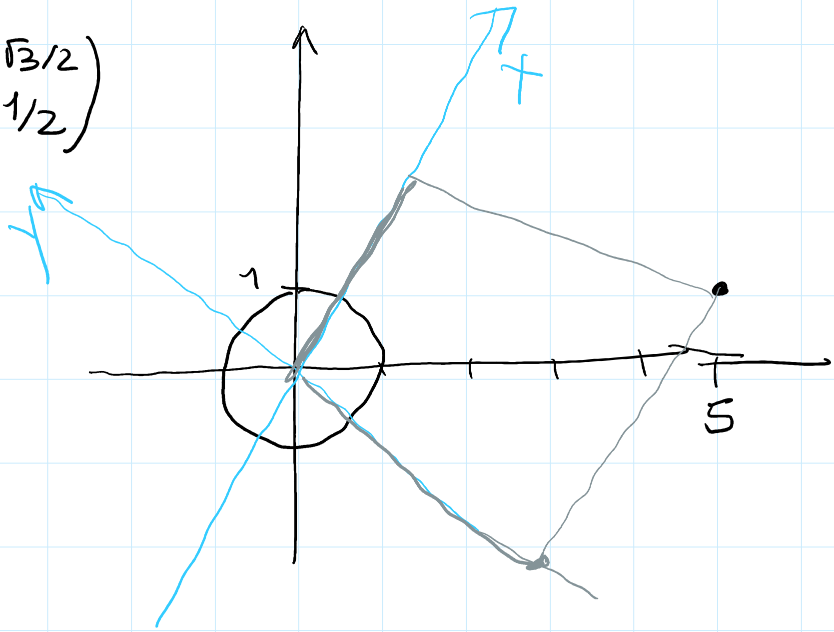
$$v \cdot u_h = \left(\sum_{i=1}^{n} a_i u_i\right) \cdot u_h = a_h u_h \cdot u_h = a_h$$

$\forall h = 1, \dots, n$

Esempio:

$\mathbb{R}^2$ $\quad u_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \qquad u_2 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \qquad \theta = \frac{\pi}{3}$

$$u_1 = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix} \qquad u_2 = \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \end{pmatrix}$$



$$v = \begin{pmatrix} 5 \\ 1 \end{pmatrix} = a u_1 + b u_2$$

$$a = v \cdot u_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix} =$$

$$= \frac{5}{2} + \frac{\sqrt{3}}{2} = \frac{5+\sqrt{3}}{2}$$

$$b = v \cdot u_2 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \end{pmatrix} = -\frac{5\sqrt{3}}{2} + \frac{1}{2} = \frac{1-5\sqrt{3}}{2}$$

$$B = \left\{ \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}, \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \end{pmatrix} \right\} \qquad T_B^{\mathcal{E}} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} \frac{5+\sqrt{3}}{2} \\ \frac{1-5\sqrt{3}}{2} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{5+\sqrt{3}}{4} - \frac{\sqrt{3}(1-5\sqrt{3})}{4} \\ \frac{\sqrt{3}(5+\sqrt{3})+1-5\sqrt{3}}{4} \end{pmatrix} = \begin{pmatrix} \frac{5+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}+15}{4} \\ \frac{\cancel{5\sqrt{3}}+3+1-\cancel{5\sqrt{3}}}{4} \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} = v$$

$$H = \begin{array}{c} \begin{array}{cc} u_1 & u_2 \end{array} \\ \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \end{array} \qquad (u_1 \; u_2)$$

$$H^{-1} = H^t = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \qquad \begin{pmatrix} u_1^t \\ u_2^t \end{pmatrix}$$

$$\begin{pmatrix} u_1^t \\ u_2^t \end{pmatrix} (u_1 \; u_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Esempio:

$$B = \left\{ \underset{u_1}{\begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}}, \underset{u_2}{\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}}, \underset{u_3}{\begin{pmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{pmatrix}} \right\}$$ è base ortonormale di $\mathbb{R}^3$

$$H=\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \end{pmatrix}$$

$$H^{-1}=H^{t}=\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{pmatrix}$$

Esercizi:

1) Determinare l'angolo compreso tra i vettori v e w di $\mathbb{R}^3$

$$v=\begin{pmatrix} \sqrt{3} \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \qquad w=\begin{pmatrix} 2 \\ \sqrt{6} \\ \sqrt{6} \end{pmatrix}$$

$$\cos\theta=\frac{v\cdot w}{\|v\|\,\|w\|} \qquad v\cdot w=\begin{pmatrix} \sqrt{3} \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\cdot\begin{pmatrix} 2 \\ \sqrt{6} \\ \sqrt{6} \end{pmatrix}=$$

$$=2\sqrt{3}+\sqrt{3}+\sqrt{3}=4\sqrt{3}$$

$$\|v\|=\sqrt{3+\frac{1}{2}+\frac{1}{2}}=\sqrt{4}=2$$

$$\cos\theta=\frac{\cancel{4}\sqrt{3}}{2\cdot\cancel{4}}=\frac{\sqrt{3}}{2} \qquad \theta=\frac{\pi}{6}$$

$$\|w\|=\sqrt{4+6+6}=\sqrt{16}=4$$

$-v \qquad -w \qquad \cos\theta=\dfrac{(-v)\cdot(-w)}{\|-v\|\,\|-w\|}=\dfrac{v\cdot w}{\|v\|\,\|w\|}=\dfrac{\sqrt{3}}{2}$

2) Determinare $K\in\mathbb{R}$ tale che i vettori

$v=\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad w=\begin{pmatrix} K \\ 3 \end{pmatrix}$ siano ortogonali fra loro.

$v \cdot w = 0$ $\quad \begin{pmatrix}1\\-1\end{pmatrix}\cdot\begin{pmatrix}k\\3\end{pmatrix} = k-3=0$ $\quad \boxed{k=3}$

$v_1=\begin{pmatrix}1\\-1\\0\end{pmatrix}$ $\quad v_2=\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\quad v_3=\begin{pmatrix}k\\-1\\1\end{pmatrix}$ per quali valori di $k\in\mathbb{R}$

$v_3 \perp v_1$ e $v_3\perp v_2$

$v_3\perp v_1$: $\quad v_1\cdot v_3=0 \quad k+1+0=0$

$v_3\perp v_2$: $\quad v_2\cdot v_3=0 \quad k-1+1=0$

$\begin{cases}k=-1\\k=0\end{cases}$ impossibile

$$T\oplus T^\perp = \mathbb{R}^n$$

Proiezioni $\quad V=U\oplus W \quad p_U^W(v)=u$

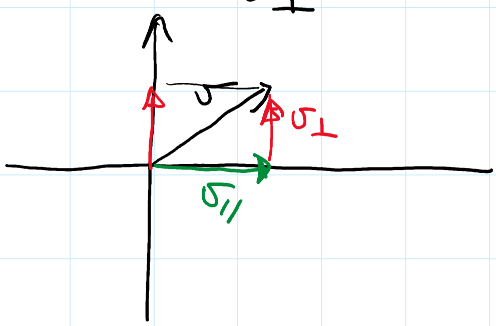
$v=u+w$

Proiezioni ortogonali: dato $T\le\mathbb{R}^n$ si dice proiezione ortogonale su T l'endomorfismo di $\mathbb{R}^n$

$P_T:\mathbb{R}^n\longrightarrow\mathbb{R}^n$ associato alla decomposizione $T\oplus T^\perp=\mathbb{R}^n$ cioè $v\in\mathbb{R}^n$

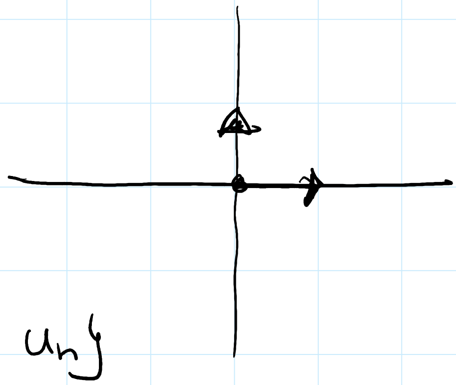
$v=v_\parallel+v_\perp$ con $v_\parallel\in T$, $v_\perp\in T^\perp$

$P_T(v)=v_\parallel$



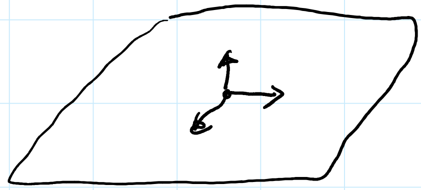
$T \oplus T^{\perp} = \mathbb{R}^n$ supponiamo di avere una base ortonormale di T $B_T = \{u_1, \dots, u_k\}$ e una base ortonormale di $T^{\perp}$

$$B_{T^{\perp}} = \{u_{k+1}, \dots, u_n\}$$



allora $v = \sum_{i=1}^{n} a_i u_i$

$$B = B_T \cup B_{T^{\perp}} = \{u_1, \dots, u_n\}$$



$$v = \sum_{i=1}^{n} (v \cdot u_i) u_i$$

$$= \underbrace{\boxed{\sum_{i=1}^{k} (v \cdot u_i) u_i}}_{T} + \underbrace{\boxed{\sum_{i=k+1}^{n} (v \cdot u_i) u_i}}_{T^{\perp}}$$

$$\boxed{P_T(v) = \sum_{i=1}^{k} (v \cdot u_i) u_i}$$

Proiezione ortogonale su T: trovare una base ortonormale di $T \Rightarrow B_T = \{u_1, \dots, u_k\}$

$$P_T(v) = \sum_{i=1}^{k} (v \cdot u_i) u_i$$

Esempio: $T = \left\langle \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix} \right\rangle$ $\quad T^{\perp} = \left\langle \begin{pmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{pmatrix} \right\rangle$

$\qquad\qquad\qquad u_1 \qquad\qquad u_2$

$B_T = \{u_1, u_2\}$ è base ortonormale di T

$$P_T(v) = (v \cdot u_1)u_1 + (v \cdot u_2)u_2$$

$$v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$P_T(v) = \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}\right] \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} + \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}\right] \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix} =$$

$$= \frac{x+y+z}{\sqrt{3}} \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} + \frac{x-y}{\sqrt{2}} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{x+y+z}{3} \\ \frac{x+y+z}{3} \\ \frac{x+y+z}{3} \end{pmatrix} + \begin{pmatrix} \frac{x-y}{2} \\ \frac{-x+y}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2x+2y+2z+3x-3y}{6} \\ \frac{2x+2y+2z-3x+3y}{6} \\ \frac{x+y+z}{3} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{5x-y+2z}{6} \\ \frac{-x+5y+2z}{6} \\ \frac{x+y+z}{3} \end{pmatrix} \qquad A_{\mathcal{E},\mathcal{E},P_T} = \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & \frac{5}{6} & 1/3 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} = A$$

$A = A^t \qquad \mathrm{rg}\, A = 2$

Per esercizio: Calcolare la matrice di proiezione ortogonale su $T = \left\langle \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \right\rangle$ $\qquad p_T(v) = (v \cdot u_1)\, u_1$

$T = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$

Se vogliamo proiettare su $T = \left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\rangle$ e dobbiamo cercare una base ortonormale di T

$T = \langle v \rangle \qquad v \neq 0_{\mathbb{R}^n} \qquad \|v\|$

$u = \dfrac{v}{\|v\|}$ è versore perché $\|u\| = \left\| \dfrac{v}{\|v\|} \right\| = \dfrac{1}{\cancel{\|v\|}} \cancel{\|v\|} = 1$

$B_T = \{u\} \qquad P_T(w) = (w \cdot u)(u) = \left(w \cdot \dfrac{v}{\|v\|}\right) \dfrac{v}{\|v\|} = \dfrac{1}{\|v\|^2} (w \cdot v) v$

$T = \left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\rangle \qquad \left\| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\| = \sqrt{1+4+9} = \sqrt{14}$

$u = \begin{pmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{pmatrix} \quad P_T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{1}{\sqrt{14}} \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right] \begin{pmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{pmatrix} =$

$= \dfrac{1}{14}(x+2y+3z) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \dfrac{1}{14} \begin{pmatrix} x+2y+3z \\ 2x+4y+6z \\ 3x+6y+9z \end{pmatrix}$

$$\frac{1}{14}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$$u = \frac{1}{\sqrt{14}}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

3×1

$$\underset{3\times 1\ \ 1\times 3}{u\, u^t} = \frac{1}{\sqrt{14}}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \frac{1}{\sqrt{14}}(1\ 2\ 3) = \frac{1}{14}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$\boxed{u u^t}$ $\quad$ $\boxed{(u u^t)^t = (u^t)^t \cdot u^t = u \cdot u^t}$

$$id = P_T + P_{T^\perp}$$

T $\dim T = 1$ $\quad T \leq \mathbb{R}^4$ $\quad P_T + P_{T^\perp} = id_{\mathbb{R}^4}$

$T^\perp$ $\dim T^\perp = 3$

$T = \langle u \rangle$ $\quad (u u^t) + (\mathbb{I}_4 - u u^t) = \mathbb{I}_4$

$T: x_1 + x_2 + x_3 + x_4 = 0$ $\quad T^\perp = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$ $\quad u = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$

$$P_T = \mathbb{I}_4 - u \cdot u^t =$$

$$= \mathbb{I}_4 - \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} (1/2\ \ 1/2\ \ 1/2\ \ 1/2) =$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} = \begin{pmatrix} 3/4 & -1/4 & -1/4 & -1/4 \\ -1/4 & 3/4 & -1/4 & -1/4 \\ -1/4 & -1/4 & 3/4 & -1/4 \\ -1/4 & -1/4 & -\frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

sono diagonalizzabili: