Esercizio 1 Foglio 5

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d) $\begin{cases} 4x + y + 2z - 3t = 0 \\ 3x - y + t = 1 \\ y - 2z - t = -4 \\ 3x + z - t = 0 \end{cases}$

[R. (1, 8, 3, 6).]

Sistema lineare di 4 equazioni

$$\underline{x} = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \qquad (A|b) = \left(\begin{array}{cccc|c} 4 & 1 & 2 & -3 & 0 \\ 3 & -1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -1 & -4 \\ 3 & 0 & 1 & -1 & 0 \end{array}\right)$$

Riduciamo a scala: 1°−2°

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 3 & -1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -1 & -4 \\ 3 & 0 & 1 & -1 & 0 \end{array}\right) \quad \begin{array}{l} 2°R - 3\,1°R \\ 4°R - 2°R \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 0 & -7 & -6 & 13 & 4 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & 1 & 1 & -2 & -1 \end{array}\right) \quad \text{Scambio } 2°-3°$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & -7 & -6 & 13 & 4 \\ 0 & 1 & 1 & -2 & -1 \end{array}\right) \quad \begin{array}{l} 3°R + 7 \cdot 2°R \\ 4°R - 2°R \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & 0 & -20 & 6 & -24 \\ 0 & 0 & 3 & -1 & 3 \end{array}\right) \quad \begin{array}{l} \text{Moltiplico la } 3°R \text{ per } 1/2 \\ \text{Moltiplico la } 4°R \text{ per } 1/3 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & 0 & -10 & 3 & -12 \\ 0 & 0 & 1 & -1/3 & 1 \end{array}\right) \quad \text{scambio } 3°R \text{ e } 4°R$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & 0 & 1 & -1/3 & 1 \\ 0 & 0 & -10 & 3 & -12 \end{array}\right) \quad 4°R + 10 \cdot 3°R \qquad \left(\begin{array}{cccc|c} 1 & 2 & 2 & -4 & -1 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & 0 & 1 & -1/3 & 1 \\ 0 & 0 & 0 & -1/3 & -2 \end{array}\right)$$

rg $A = 4 = $ rg$(A|b)$ quindi il sistema ha soluzioni per il Teorema di Rouché-Capelli e le soluzioni dipendono da $n°$ parametri $= n°$ incognite $-$ rg$(A) = 4-4=0 \Rightarrow$ la soluzione è unica.

$$\begin{cases} -\frac{1}{3}t = -2 \\ z - \frac{1}{3}t = 1 \\ y - 2z - t = -4 \\ x + 2y + 2z - 4t = -1 \end{cases} \qquad \begin{cases} t = 6 \\ z = \frac{t}{3} + 1 = \frac{6}{3} + 1 = 3 \\ y = 2z + t - 4 = 6 + 6 - 4 = 8 \\ x = -2y - 2z + 4t - 1 = -16 - 6 + 24 - 1 = 1 \end{cases}$$

$$Sol = \left\{ \begin{pmatrix} 1 \\ 8 \\ 3 \\ 6 \end{pmatrix} \right\}$$

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e) $$\begin{cases} 2x + y - 2t = 0 \\ 3x + z = 1 \\ y - 2z + t = 2 \\ 2x - 7y + 4z + 4t = -1 \end{cases}$$

~~[R. Il sistema non ha so~~luzioni.]

Sistema lineare di 4 equazioni nelle incognite $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$(A|b) = \left(\begin{array}{cccc|c} 2 & 1 & 0 & -2 & 0 \\ 3 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 2 \\ 2 & -7 & 4 & 4 & -1 \end{array}\right)$$

$$\begin{array}{r} 2°R - 1°R \\ 2°R \\ \\ \\ \end{array} \left(\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 1 \\ 2 & 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 1 & 2 \\ 2 & -7 & 4 & 4 & -1 \end{array}\right) \qquad \begin{array}{l} 2°R - 2 \cdot 1°R \\ 4°R - 2 \cdot 1°R \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 1 \\ 0 & 3 & -2 & -6 & -2 \\ 0 & 1 & -2 & 1 & 2 \\ 0 & -5 & 2 & 0 & -3 \end{array}\right)$$

Metto la 3°R come 2°R
come 3°R metto 2°R−3·3°R
come 4°R metto 4°R−5·3°R

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 1 \\ 0 & 1 & -2 & 1 & 2 \\ 0 & 0 & 4 & -9 & -8 \\ 0 & 0 & -8 & 5 & 7 \end{array}\right)$$

4°R−2 3°R

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 1 \\ 0 & 1 & -2 & 1 & 2 \\ 0 & 0 & 4 & -9 & -8 \\ 0 & 0 & 0 & -13 & -9 \end{array}\right) \qquad t = \frac{9}{13}$$

$$\begin{cases} t = \frac{9}{13} \\ 4z - 9t = -8 & z = \frac{9t-8}{4} = \frac{1}{4}\left(\frac{81}{13} - 8\right) = \frac{81-104}{52} = \frac{-23}{52} \\ y - 2z + t = 2 & y = 2z - t + 2 = -\frac{46}{52} - \frac{9}{13} + 2 = \frac{-46-36+104}{52} = \frac{22}{52} \\ x - y + z + 2t = 1 & x = y - z - 2t + 1 = \frac{22}{52} + \frac{23}{52} - \frac{18}{13} + 1 \end{cases}$$

$$= \frac{45-72+52}{52} = \frac{25}{52}$$

$$\text{Sol.} = \left\{ \begin{pmatrix} \frac{25}{52} \\ \frac{22}{52} \\ -\frac{23}{52} \\ \frac{9}{13} \end{pmatrix} \right\}$$

e) $\begin{cases} 2x + y - 2t = 0 \ ✓ \\ 3x + z = 1 \ ✓ \\ y - 2z + t = 2 \ ✓ \\ 2x - 7y + 4z + 4t = -1 \ ✓ \end{cases}$

~~[R. Il sistema non ha soluzioni.]~~

Verifica

$$2 \cdot \frac{25}{52} + \frac{22}{52} - \frac{18}{13} = \frac{50+22-72}{52} = \frac{72-72}{52} = 0 \quad ✓$$

$$3 \cdot \frac{25}{52} - \frac{23}{52} = \frac{75-23}{52} = \frac{52}{52} = 1 \quad ✓$$

$$\frac{22}{52} + \frac{46}{52} + \frac{9}{13} = \frac{11}{26} + \frac{23}{26} + \frac{18}{26} = \frac{52}{26} = 2 \quad ✓$$

$$\cancel{2} \cdot \frac{25}{\cancel{52}_{26}} - 7 \cdot \frac{\cancel{22}^{11}}{\cancel{52}_{26}} - \cancel{4}^{2} \cdot \frac{23}{\cancel{52}_{26}} + \frac{36}{13} = \frac{25-77-46+72}{26} = \frac{-26}{26} = -1 \quad ✓$$

f) $\begin{cases} 2x+y-z=3 \\ x+2z+t=2 \\ -x+y+2z=0 \\ 2y-z-t=1 \end{cases}$

[R. $(1,1,0,1)+\langle(1,-1,1,-3)\rangle$]

$$(A|b) = \left(\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 3 \\ 1 & 0 & 2 & 1 & 2 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & 2 & -1 & -1 & 1 \end{array}\right)$$ Scambio 1°R-2°R

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ 2 & 1 & -1 & 0 & 3 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & 2 & -1 & -1 & 1 \end{array}\right)$$ 2°R - 2·1°R
3°R + 1°R°

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & -5 & -2 & -1 \\ 0 & 1 & 4 & 1 & 2 \\ 0 & 2 & -1 & -1 & 1 \end{array}\right)$$ 3°R-2°R
4°R-2·3°R

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & -5 & -2 & -1 \\ 0 & 0 & 9 & 3 & 3 \\ 0 & 0 & 9 & 3 & 3 \end{array}\right)$$ 1/3 · 3°R
4°R-3°R

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & -5 & -2 & -1 \\ 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$ 1/2 · 3°R
4°R $-\frac{5}{2}$ · 3°R

$rg(A) = rg(A|b) = 3 \Rightarrow$ le soluzioni dipendono da $n°$ incognite $- rg(A) = 4-3=1$ parametro

$$\begin{cases} 3z+t=1 & t=1-3z \\ y-5z-2t=-1 & y=5z+2t-1=5z+2-6z-1=-z+1 \\ x+2z+t=2 & x=-2z-t+2=-2z-1+3z+2=z+1 \end{cases}$$

$$\text{Sol: } \left\{ \begin{pmatrix} z+1 \\ -z+1 \\ z \\ 1-3z \end{pmatrix} \middle| \, z \in \mathbb{R} \right\} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \\ -3 \end{pmatrix} \right\rangle$$