Esercizio:

Determinare tutti i numeri $z \in \mathbb{C}$ tali che

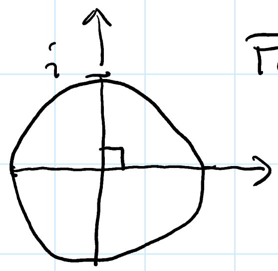
$$(1+2i)\,z^5 = -14+7i.$$

Soluzione:

$$z^5 = \frac{-14+7i}{1+2i} = \frac{-14+7i}{1+2i}\cdot\frac{1-2i}{1-2i} = \frac{-\cancel{14}+28i+7i\overset{+14}{\cancel{-14i^2}}}{1+4} =$$

$$= \frac{35i}{5} = 7i$$

$$\boxed{z^5 = 7i}$$



Formule di De Moivre

$7i = 7e^{i\pi/2}$ $\qquad z^5 = 7\left(\cos\frac{\pi}{2} + i\operatorname{sen}\frac{\pi}{2}\right)$ $\qquad m=5$

$$\frac{1}{5}\left(\frac{\pi+4k\pi}{2}\right)$$

$$z_k = \sqrt[5]{7}\left(\cos\left(\frac{\frac{\pi}{2}+2k\pi}{5}\right) + i\operatorname{sen}\left(\frac{\pi/2+2k\pi}{5}\right)\right) \qquad k=0,1,2,3,4$$

$$= \sqrt[5]{7}\left(\cos\left(\frac{\pi+4k\pi}{10}\right) + i\operatorname{sen}\left(\frac{\pi+4k\pi}{10}\right)\right)$$

$$z_0 = \sqrt[5]{7}\left(\cos\left(\frac{\pi}{10}\right) + i\operatorname{sen}\left(\frac{\pi}{10}\right)\right) = \sqrt[5]{7}\,e^{i\frac{\pi}{10}} \qquad k=0$$

$$\underline{z_1} = \sqrt[5]{7}\left(\cos\left(\frac{\pi+4\pi}{10}\right) + i \operatorname{sen}\left(\frac{\pi+4\pi}{10}\right)\right) = \sqrt[5]{7}\, e^{i\pi/2} = \sqrt[5]{7}\, i$$

$$z_1^5 = \left(\sqrt[5]{7}\right)^5 i^5 = 7i$$

$$z_2 = \sqrt[5]{7}\left(\cos\left(\frac{\pi+8\pi}{10}\right) + i \operatorname{sen}\left(\frac{\pi+8\pi}{10}\right)\right)$$

$$z_3 = \sqrt[5]{7}\left(\cos\left(\frac{13}{10}\pi\right) + i \operatorname{sen}\left(\frac{13}{10}\pi\right)\right)$$

$$z_4 = \sqrt[5]{7}\left(\cos\left(\frac{17\pi}{10}\right) + i \operatorname{sen}\left(\frac{17\pi}{10}\right)\right)$$

$\sqrt[5]{7}$

$\rho = \sqrt[5]{7}$