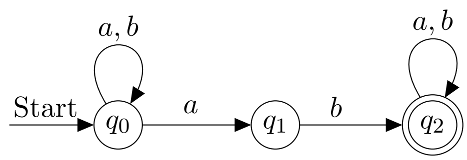
Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

July 3rd, 2025

1. **[5 points]** Consider the NFA N whose transition function is graphically represented below, and answer the following questions.



   (a) Describe in words the language $L(N)$ accepted by N.

   (b) Apply to N the subset construction presented in the textbook, together with the lazy evaluation, to produce a DFA D equivalent to N, and report all of the intermediate steps.

   (c) Depict the graphical representation of the transition function of D.

2. **[9 points]** Let $\Sigma = \{a, b\}$. Consider the following languages

$$\begin{aligned} L_1 &= \{a^n b^n a^n \mid n \geq 1\} \\ L_2 &= \{a^n a^n b^n \mid n \geq 1\} \\ L_3 &= \{a^n a^n a^n \mid n \geq 1\} \end{aligned}$$

   For each of the above languages, state whether it belongs to REG, to CFL$\setminus$REG, or else whether it is outside of CFL. Provide a mathematical proof for all of your answers.

3. **[6 points]** With reference to context-free languages, answer the following questions.

   (a) Prove that context-free languages are not closed with respect to the operation of intersection.

   (b) Prove that context-free languages are closed under the operation of intersection with regular languages, presenting the construction of the corresponding PDA reported in the textbook.

(please turn to the next page)

4. **[7 points]** Assess whether the following statements are true or false, providing mathematical proofs for all of your answers.

   (a) There exist a language L_1 in REC and a language L_2 in REG, such that $L_1 \cup L_2$ is in REG.

   (b) There exist a language L_1 in REC and a language L_2 in REG, such that $L_1 \cup L_2$ is not in REG.

   (c) The class REG is closed under intersection with the class REC.

   (d) The class REC is closed under intersection with the class REG.

   (e) The class REC is closed under intersection with the class $\mathcal{P}$ of languages that can be recognized in polynomial time by a TM.

5. **[6 points]** Let Σ be some alphabet. Recall that for $w \in \Sigma^*$ and $X \in \Sigma$, we write $\#_X(w)$ to denote the number of occurrences of X in w. Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} \quad = \quad \{L \mid L \in \text{RE}, \ \text{ for every string } w \text{ in } L \text{ we have } \#_0(w) < \#_1(w)\}$$

   and define $L_{\mathcal{P}} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$.

   (a) Use Rice's theorem to show that $L_{\mathcal{P}}$ is not in REC.

   (b) Prove that $L_{\mathcal{P}}$ is not in RE.