MODELLI MECCANICI

1) $\sum$ Forze $= 0$

2) Tipologie delle forze meccaniche

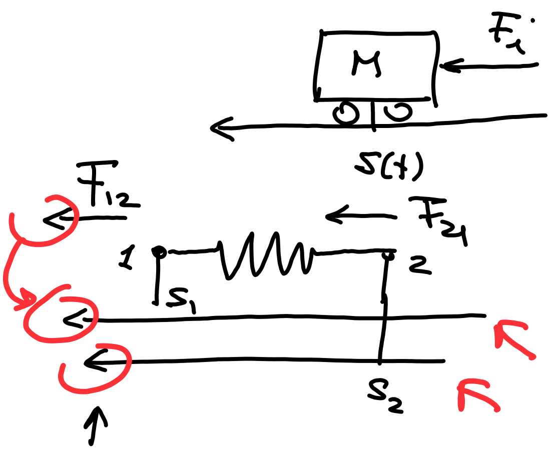
2A) Forza inerziale

$$F_i(t) = -M\,\ddot{S}(t)$$

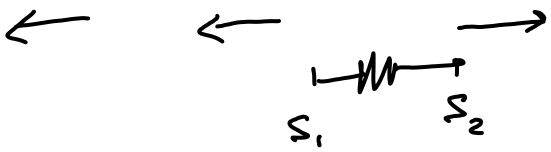
2B) Forza elastica

$$F_{12} = -k(S_1 - S_2)$$

$$F_{21} = -F_{12} = -k(S_2 - S_1)$$



Esempio: molla compressa

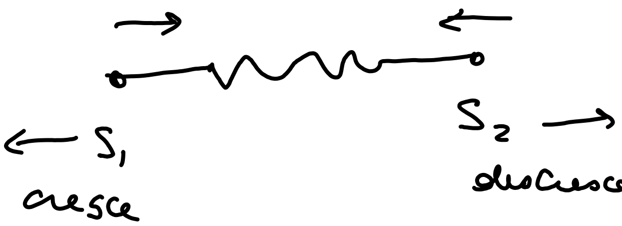


S_1 decresce

S_2 cresce

$\boxed{S_2 = S_1 \text{ molla a riposo}}$

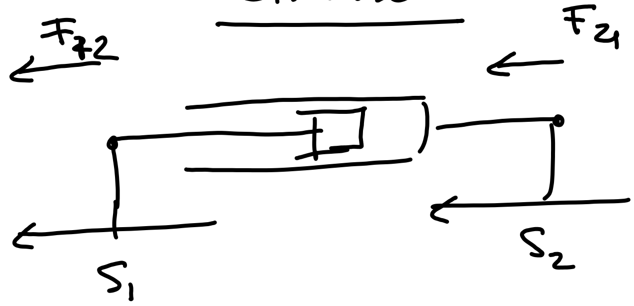
$S_2 > S_1$ $\quad F_{12} > 0$ $\quad F_{21} < 0$



$S_2 < S_1$ $\quad F_{12} < 0$

$F_{21} > 0$

2C) Smorzatore

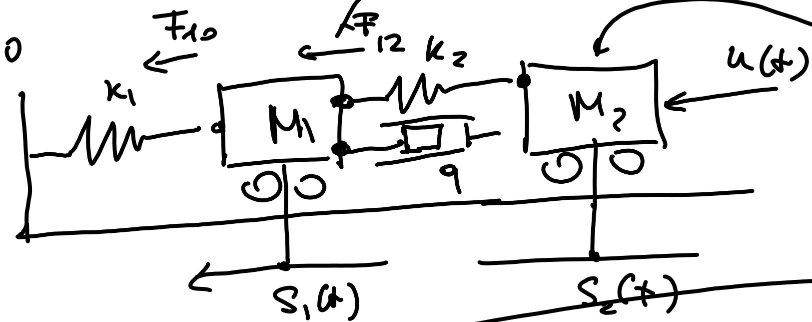


$$F_{12} = -q(\dot{s}_1 - \dot{s}_2)$$

$$F_{21} = -q(\dot{s}_2 - \dot{s}_1)$$

2D) Forze esterne

Esempio



$\sum$ Forze$_1$ = 0

$\sum$ forze$_2$ = 0

$$-M_1\ddot{s}_1 - k_1(s_1 - 0) - k_2(s_1 - s_2) - q(\dot{s}_1 - \dot{s}_2) = 0$$

$$-M_2\ddot{s}_2 - k_2(s_2 - s_1) - q(\dot{s}_2 - \dot{s}_1) + u(t) = 0$$

Modello di stato

$x_1(t) = s_1(t)$ $\quad x_2(t) = \dot{s}_1(t)$

$x_3(t) = s_2(t)$ $\quad x_4(t) = \dot{s}_2(t)$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{s}_1 = \frac{1}{M_1}\left(-k_1x_1 - k_2x_1 + k_2x_3 - qx_2 + qx_4\right)$$

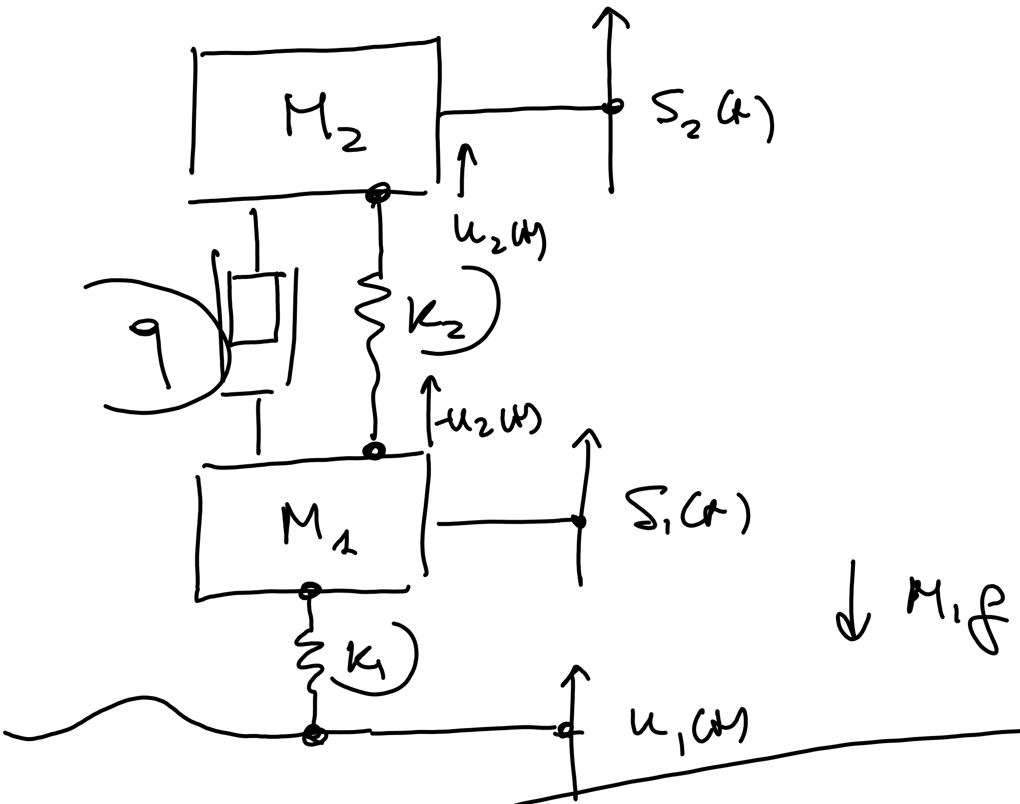
$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \ddot{s}_2 = \frac{1}{M_2}\left(k_2x_1 - k_2x_3 + qx_2 - qx_4\right) + \frac{1}{M_2}u$$

4×4

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{M_1} & -\frac{q}{M_1} & \frac{k_2}{M_1} & \frac{q}{M_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{M_2} & \frac{q}{M_2} & -\frac{k_2}{M_2} & -\frac{q}{M_2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix}$$

Esempio: modello di una sospensione



$$-M_1 \ddot{S}_1 - k_1(S_1 - u_1) - k_2(S_1 - S_2) - q(\dot{S}_1 - \dot{S}_2) - u_2(t) - \cancel{M_1 g} = 0$$

$$-M_2 \ddot{S}_2 - k_2(S_2 - S_1) - q(\dot{S}_2 - \dot{S}_1) + u_2 - \cancel{M_2 g} = 0$$

Modello in forma di stato

Poniamo forza gravità = 0

$x_1(t) = s_1(t)$ $\quad$ $x_2(t) = \dot{s}_1(t)$ $\quad$ $x_3(t) = s_2$

$x_4 = \dot{s}_2$

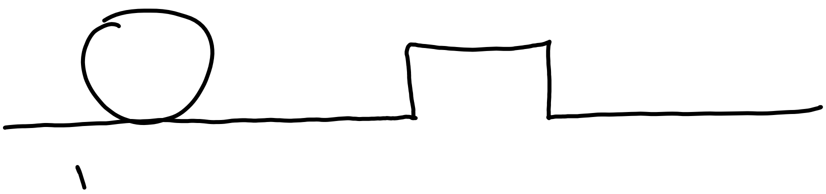
$\dot{x}_1 = x_2$ $\quad$ $\dot{x}_3 = x_4$

$$\dot{x}_2 = \ddot{s}_1 = \frac{1}{M_1}\left(-k_1 x_1 - k_2 x_1 - q x_2 + k_2 x_3 + q x_4\right) - \frac{1}{M_1} u_2 + \frac{k_1}{M_1} u_1$$

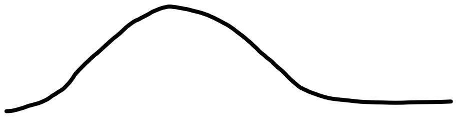
$$\dot{x}_4 = \ddot{s}_2 = \frac{1}{M_2}\left(k_2 x_1 + q x_2 - k_2 x_3 - q x_4\right) + \frac{1}{M_2} u_2$$

$$A = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad B = \begin{array}{c c} & \begin{array}{cc} u_1 & u_2 \end{array} \\ & \begin{bmatrix} 0 & 0 \\ k_1/M_1 & -\frac{1}{M_1} \\ 0 & 0 \\ 0 & 1/M_2 \end{bmatrix} \end{array}$$

$\dot{x}_4$



S_2

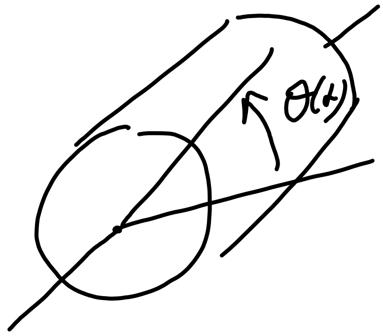


$u_1(t)$

Sistemi meccanici in moto rotatorio

1) Σ coppie $= 0$

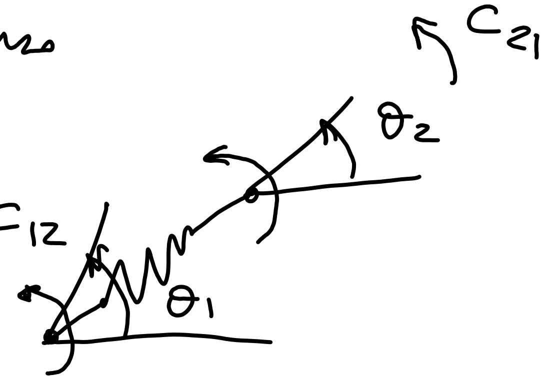
2) 2A) Coppia inerziale



$$C_1(t) = -J\,\ddot{\theta}(t)$$

$\uparrow$

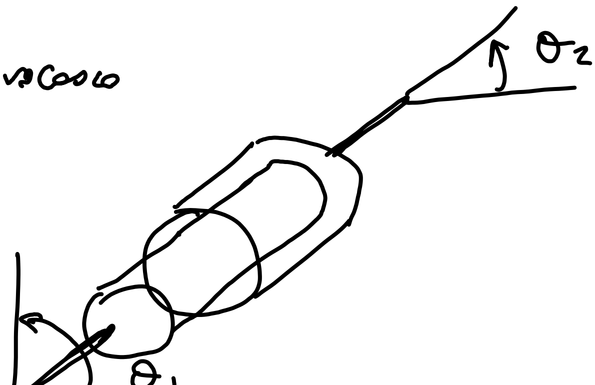
momento di inerzia



2B) Coppia elastica

$$C_{12} = -k(\theta_1 - \theta_2)$$
$$C_{21} = -k(\theta_2 - \theta_1)$$

3C) coppia di attrito viscoso

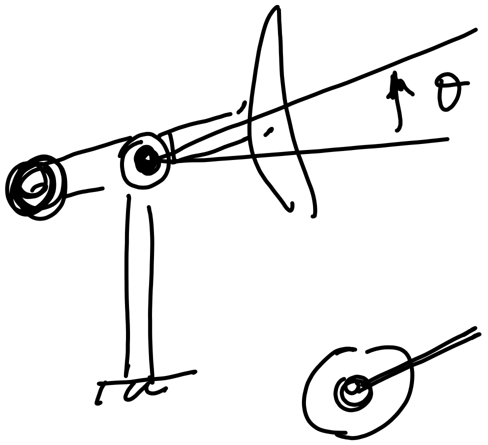


$$C_{12} = -q(\dot{\theta}_1 - \dot{\theta}_2)$$

$$C_{21} = -q(\dot{\theta}_2 - \dot{\theta}_1)$$

3D) Coppie esterne

Esempio Puntamento di un radar



$$-J\ddot{\theta} - q(\dot{\theta} - 0) + \overset{u_1}{C_M} + \overset{u_2}{C_V} = 0$$

C_M coppia generata dal motore elettrico

C_V coppia generata dal vento

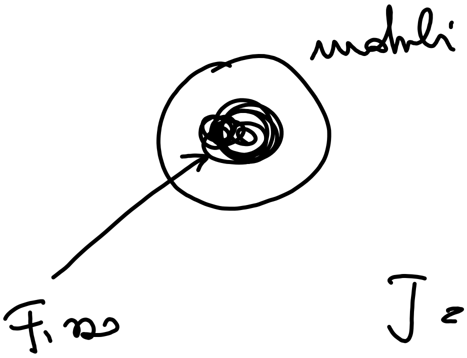
$$x_1 = \theta \qquad x_2 = \dot{\theta}$$

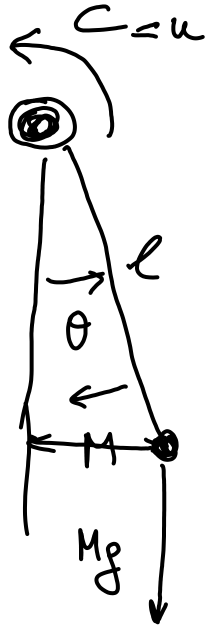
$$\dot{x}_1 = x_2 \leftarrow$$

$$\dot{x}_2 = \ddot{\theta} = \frac{1}{J}\left(-q x_2 + u_1 + u_2\right)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -q/J \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1/J & 1/J \end{bmatrix}$$

Pendolo





$$J = \ell^2 M$$

$$\boxed{-J\ddot{\theta} - q\dot{\theta} - Mg\ell\sin\theta + u = 0}$$

$x_1 = \theta \qquad x_2 = \dot{\theta}$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{J}\left(-Mg\ell\sin(x_1) - q x_2 + u\right) \end{cases}$$

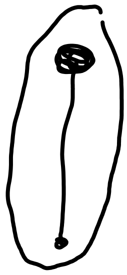
Se x_1 piccolo $\qquad \sin(x_1) \simeq x_1$

$\theta\,\uparrow$

$$\boxed{\dot{x}_2 \simeq -\frac{Mg\ell}{J}x_1 - \frac{q}{J}x_2 + \frac{1}{J}u}$$

Equilibrio = soluzioni costanti

$u(t) = \bar{u} = 0 \qquad \theta(t) = \bar{\theta}$ incognito

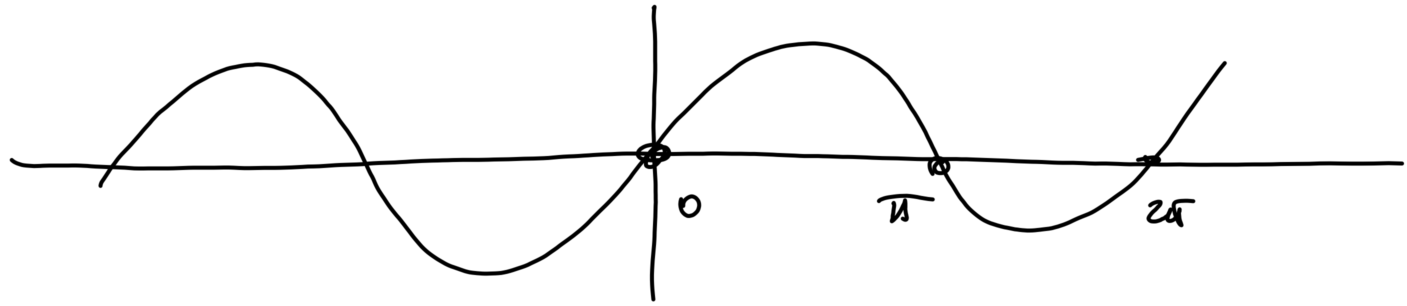


$$-J0 - q0 - Mg\ell\sin\bar{\theta} + \cancel{\bar{u}} = 0$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{Mg\ell}{J} & -\frac{q}{J} \end{bmatrix} \qquad \sin\bar{\theta} = 0$$

$\bar{\theta} = 0$

$\bar{\theta} = \pi$



Scelgo $\bar{\theta} = \pi$ e considero una nuova variabile $\tilde{\theta}(t) \overset{def}{=} \theta(t) - \bar{\theta} = \theta(t) - \pi$

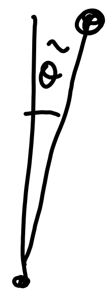
$$-J\ddot{\theta} - b\dot{\theta} - Mgl \sin\theta + u = 0$$

$$\theta(t) = \boxed{\bar{\theta} + \tilde{\theta}(t)}$$

$$\frac{d^2}{dt^2}\left(\bar{\theta} + \tilde{\theta}(t)\right) = \cancel{\frac{d^2}{dt^2}\bar{\theta}} + \frac{d^2}{dt^2}\tilde{\theta} = \ddot{\tilde{\theta}}$$

θ piccolo $\sin\theta \simeq \theta$ $\simeq -\tilde{\theta}$

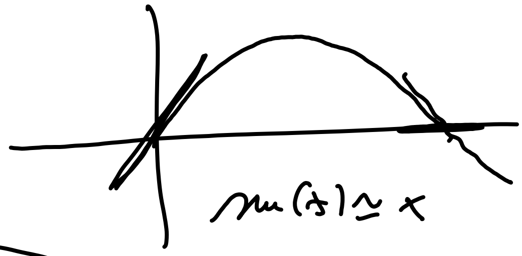
$$-J\ddot{\tilde{\theta}} - b\dot{\tilde{\theta}} - Mgl \boxed{\sin(\pi + \tilde{\theta})} + u = 0$$



$\tilde{\theta} \simeq 0$ $\quad f(x) \simeq f(\bar{x}) + f'(\bar{x})\,\tilde{x}$

$\sin(\pi + \tilde{\theta})$

$\simeq \cancel{\sin(\pi)} + \cos(\pi)\,\tilde{\theta} = -\tilde{\theta}$



$$\boxed{-J\ddot{\tilde{\theta}} - b\dot{\tilde{\theta}} + Mgl\,\tilde{\theta} + u = 0}$$

$x_1 = \tilde{\theta}$ $\quad x_2 = \dot{\tilde{\theta}}$ $\quad \dot{x}_1 = x_2$

$$A = \left[\begin{array}{c|c} 0 & 1 \\ \hline \cancel{0}\, Mg\ell/J & -b/J \end{array}\right] \qquad B = \begin{bmatrix} 0 \\ 1/J \end{bmatrix}$$

$$\dot{x}_2 = \ddot{\tilde{\theta}} = \frac{1}{J}\left(M\underline{g}\ell\, x_1 - \underline{b}\, x_2 + u \right)$$
