Esempio: Dinamica dei prezzi

p prezzo di bene

q quantità del bene programmato per la produzione

Comportamento di produttori

$$q = b\,p + Q \qquad b > 0$$

Comportamento di consumatori

$$q = -a\,p + D \qquad a > 0$$

Modello dinamico

$$\begin{cases} q(k) = -a\,p(k) + D & (1) \\ q(k+1) = b\,p(k) + Q & (2) \end{cases} \qquad \text{tempo discreto}$$

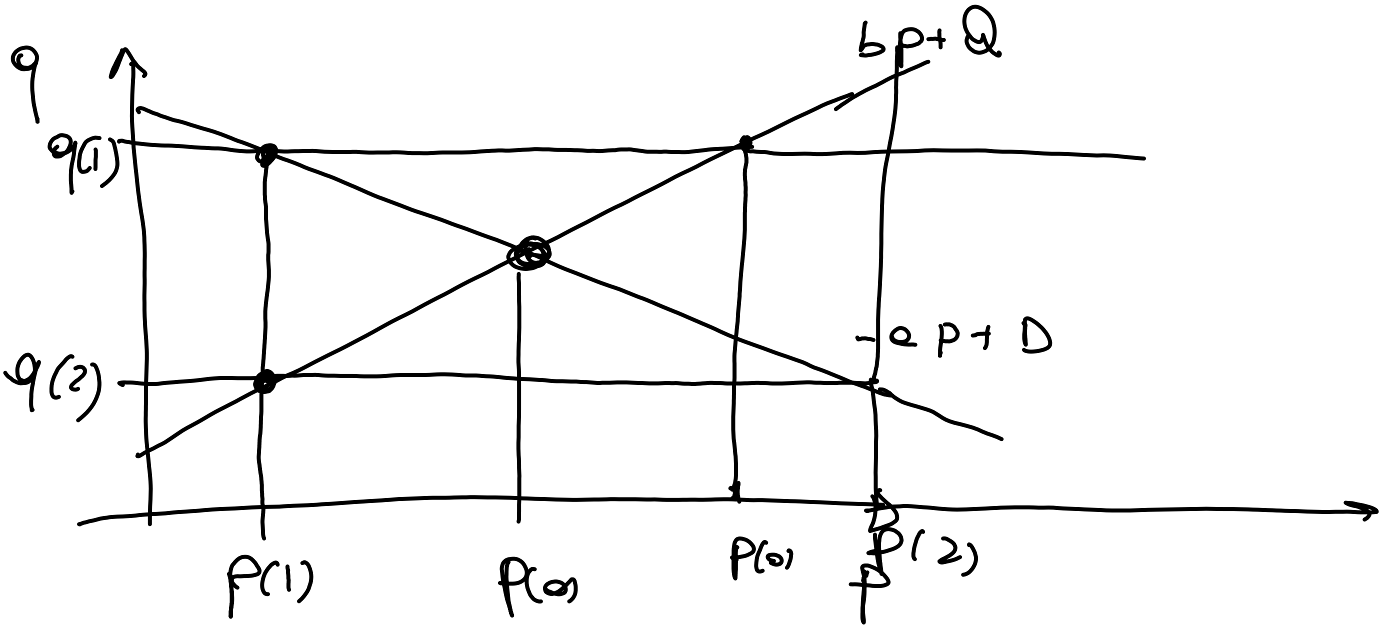
$$x_1(k) = p(k)$$

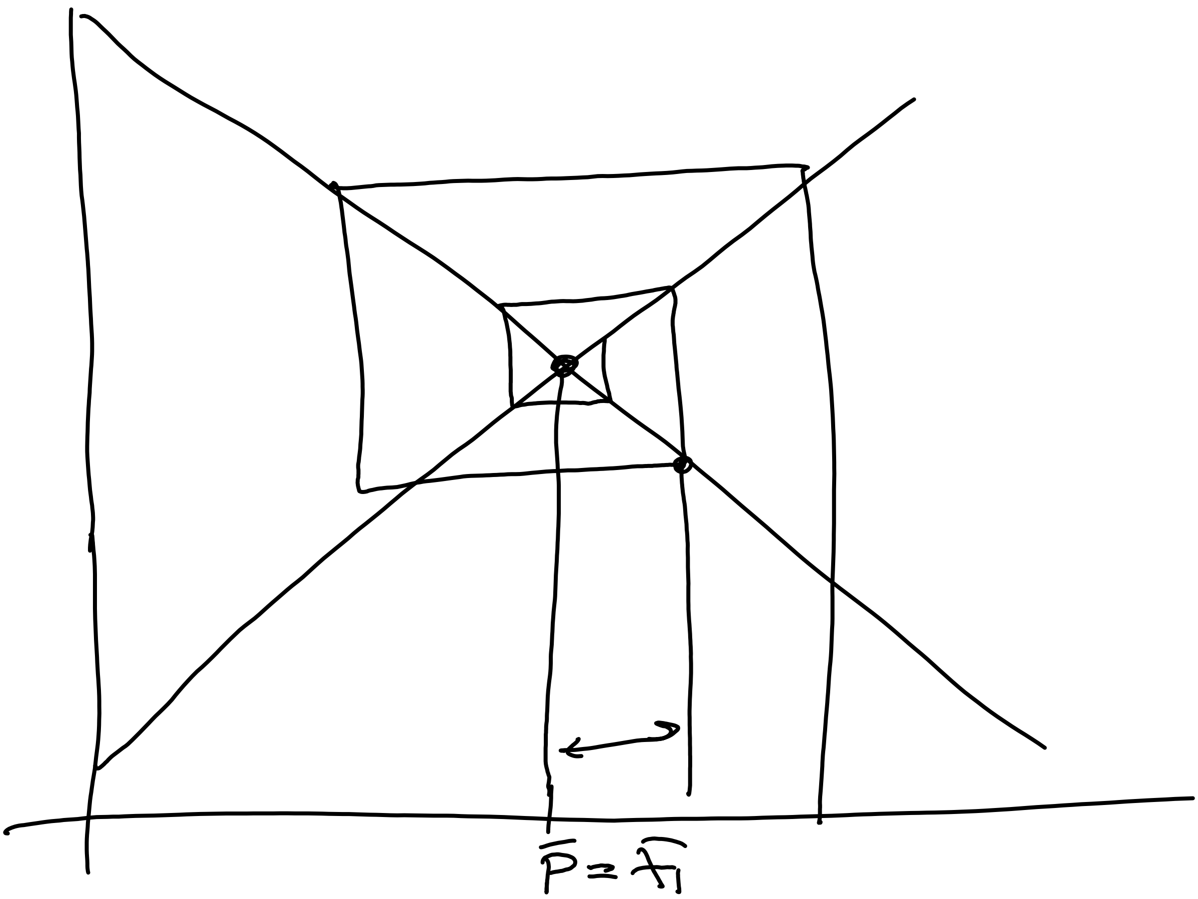
$$x_1(k+1) = p(k+1) \overset{(1)}{=} \frac{1}{a}\left(-q(k+1) + D\right) =$$

$$= -\frac{1}{a}\left(b\,p(k) + Q\right) + \frac{1}{a} D =$$

$$= -\frac{b}{a} x_1(k) + \frac{1}{a} \underbrace{\left(D - Q\right)}_{u_1(k)} = -\frac{b}{a} x_1(k) + \frac{1}{a} u_1(k)$$

$$u_1(k) = D - Q$$





$$x_1(k+1) = -\frac{b}{a}\, x_1(k) + \frac{1}{a}\, u_1(k) \leftarrow$$

$$u_1(k) = \bar{u}_1$$

$$x_1(k) = \bar{x}_1 \qquad \forall k$$

$$\boxed{\bar{x}_1 = -\frac{b}{a}\,\bar{x}_1 + \frac{1}{a}\,\bar{u}_1} \longrightarrow \bar{x}_1 = \frac{1/a}{1+b/a}\,\bar{u}_1$$

$$\tilde{x}_1(k) \overset{def}{=} x_1(k) - \bar{x}_1 \longrightarrow x_1(k) = \bar{x}_1 + \tilde{x}_1(k)$$

$$\tilde{x}_1(k+1) = x_1(k+1) - \bar{x}_1$$

$$= -\frac{b}{a} x_1(k) + \frac{1}{a}\bar{u}_1 - \bar{x}_1$$

$$= -\frac{b}{a}\left(\tilde{x}_1(k) + \bar{x}_1\right) + \frac{1}{a}\bar{u}_1 - \bar{x}_1$$

$$= -\frac{b}{a}\tilde{x}_1(k) \cancel{- \frac{b}{a}\bar{x}_1 + \frac{1}{a}\bar{u}_1 - \bar{x}_1}$$

$$\boxed{\tilde{x}_1(k+1) = -\frac{b}{a}\tilde{x}_1(k)}$$

$\frac{b}{a} < 1$ convergono verso l'equil.

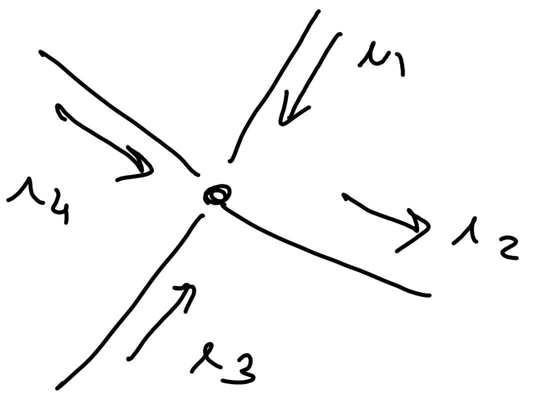
$\frac{b}{a} > 1$ divergono

MODELLI FISICI

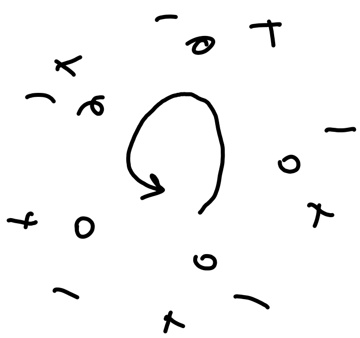
Circuiti

Variabili sono correnti e tensioni

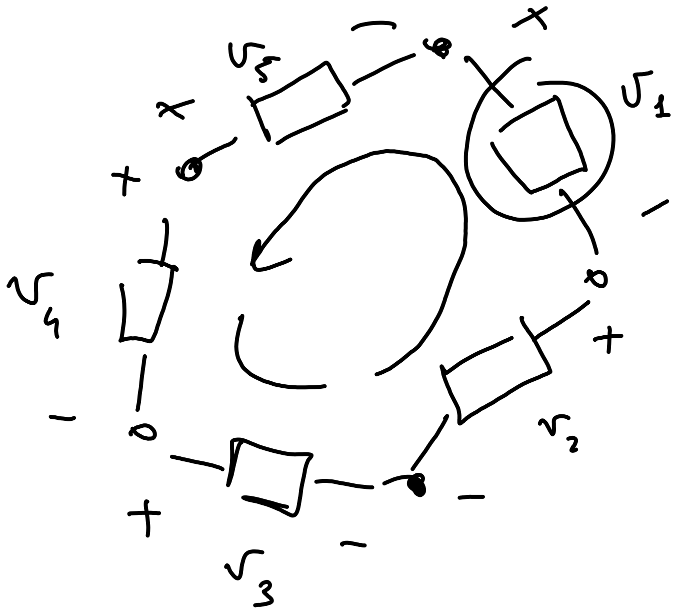
Vincoli sono leggi di Kirchoff

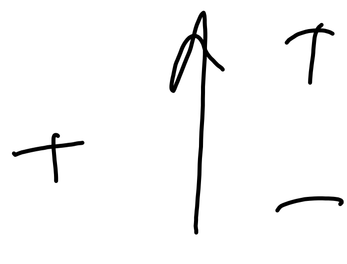


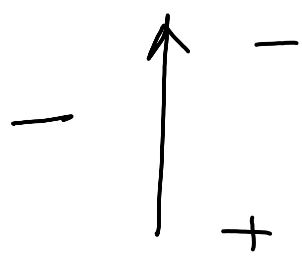
$$i_1 - i_2 + i_3 + i_4 = 0$$



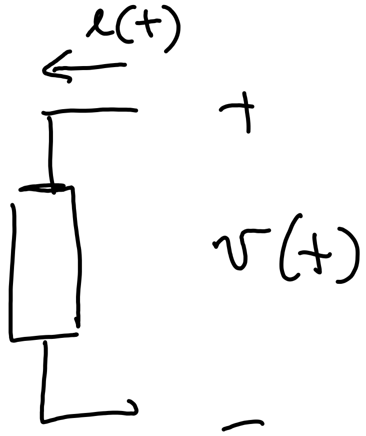
$$v_1 + v_2 + v \cdots = 0$$

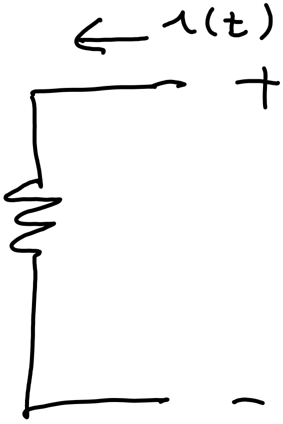






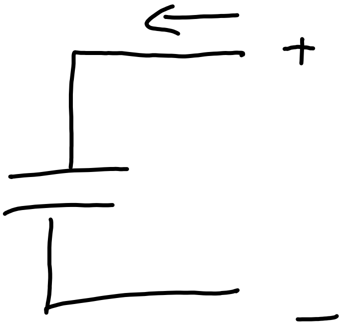
$$v_1 + v_2 - v_3 - v_4 + v_5 = 0$$



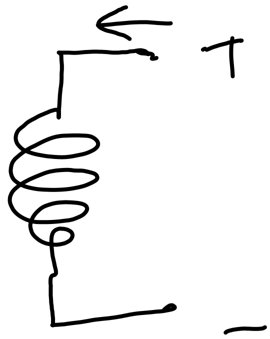


$$v(t) = R\, i(t)$$

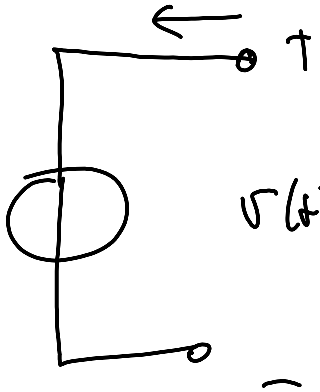
$$R > 0$$



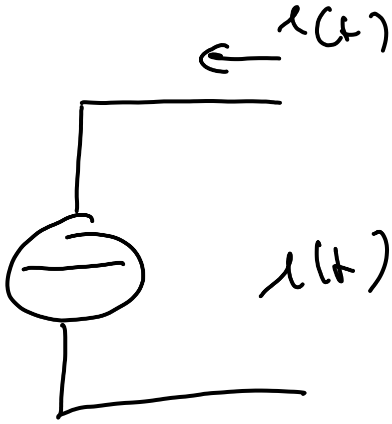
$$i(t) = C \frac{d}{dt} v(t)$$



$$v(t) = L \frac{d}{dt} i(t)$$



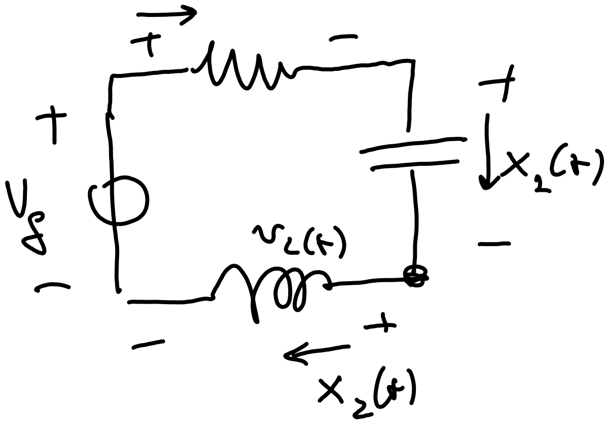
$$v(t) = V_g(t)$$



$$i(t) = I_g(t)$$

Esempio

Stati : tensione del condensatore, corrente degli induttori



$$\dot{x}_1(t) = \frac{1}{C}\, x_2(t)$$

$$\dot{x}_2(t) = \frac{1}{L}\, v_L(t) = \frac{1}{L}\left(-x_1 - Rx_2 + V_g\right)$$

$$v_L + x_1 + v_R = V_g \qquad V_g = u_1(t)$$

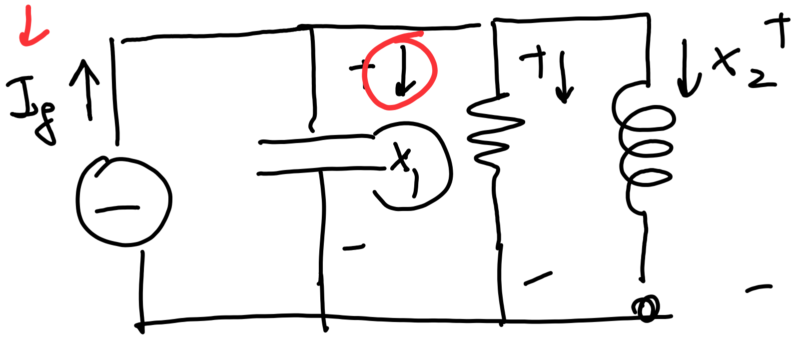
$$v_R = R\, x_2$$

$$\dot{x} = Ax + Bu$$

$$\begin{cases} \dot{x}_1(t) = \frac{1}{C}\, x_2(t) \\ \dot{x}_2(t) = -\frac{1}{L}\, x_1(t) - \frac{R}{L}\, x_2(t) + \frac{1}{L}\, u(t) \end{cases}$$

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$$

Esempio



$$\dot{x}_2 = \frac{1}{L} x_1$$

$$v_R = R i_R$$

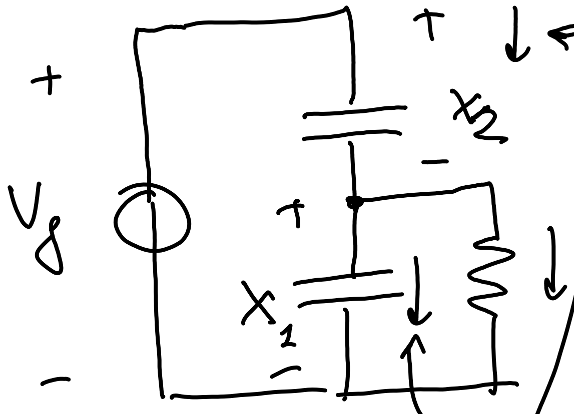
$$\dot{x}_1 = \frac{1}{C} i_c = \frac{1}{C}\left(I_g - x_2 - \frac{x_1}{R}\right)$$

$$I_g = u_1(t)$$

$$\begin{cases} \dot{x}_1 = -\frac{1}{CR} x_1 - \frac{1}{C} x_2 + \frac{1}{C} u_1 \\ \dot{x}_2 = \frac{1}{L} x_1 \end{cases}$$

$$A = \begin{bmatrix} -\frac{1}{CR} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1/C \\ 0 \end{bmatrix}$$

Esempio



$$V_g = x_1 + x_2$$

$$\boxed{C_2 \dot{x}_2 = C_1 \dot{x}_1 + \frac{x_1}{R}}$$

$$v_R = R\, i_R$$

$$\dot{V}_g = \dot{x}_1 + \dot{x}_2 \longrightarrow \dot{x}_2 = -\dot{x}_1 + \dot{V}_g$$

$$C_2\left(-\dot{x}_1 + \dot{V}_g\right) = C_1 \dot{x}_1 + \frac{x_1}{R}$$

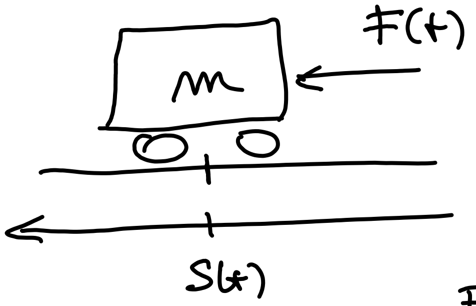
$$C_1 \dot{x}_1 + C_2 \dot{x}_1 = -\frac{x_1}{R} + C_2 \dot{V}_g$$

$$\dot{x}_1 = -\frac{1}{(C_1+C_2)R} x_1 + \frac{C_2}{C_1+C_2} \underbrace{\dot{V}_g}_{u_1(t)}$$

$$A = \left[-\frac{1}{(C_1+C_2)R} \right] \qquad B = \left[\frac{C_2}{C_1+C_2} \right]$$

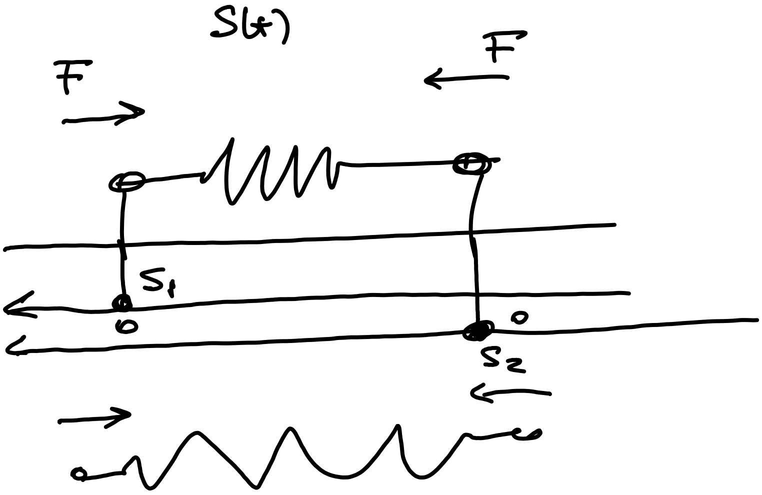
Modelli Meccanici

$\sum$ Forze $= 0$



accelerazione $\ddot{S}(t)$

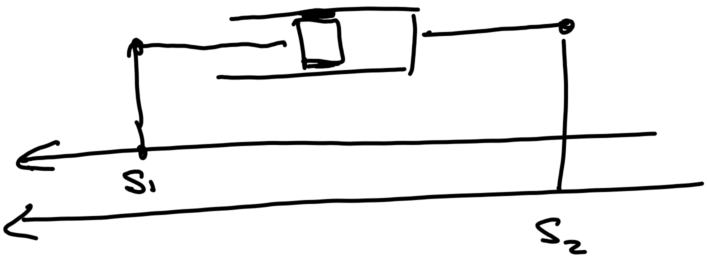
$$F_i(t) = -m\ddot{S}(t)$$



$F_e(t) = k$ allungamento

$$= -k(s_2 - s_1)$$

Smorzatore

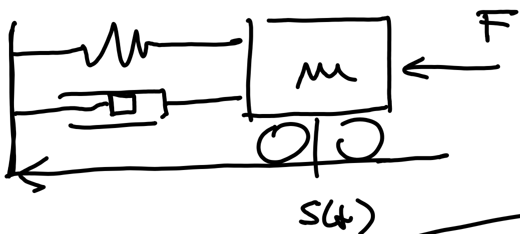


$$F_a(t) = -q\frac{d}{dt}(s_2 - s_1)$$

$$= -q(\dot{s}_2 - \dot{s}_1)$$

Esistono forze esterne

Esempio



$S(t) = 0$ corrispondente alla molla a riposo

$$-m\ddot{S} - kS(t) - q\dot{S}(t) + F(t) = 0$$

le variabili di stato posizione e velocità della massa in movimento

$$x_1(t) \overset{def}{=} s(t)$$

$$x_2(t) \overset{def}{=} \dot{s}(t)$$

$$F(t) = u_1(t)$$

$$\dot{x}_1(t) = \dot{s}(t) = \underline{\underline{x_2(t)}}$$

$$\dot{x}_2(t) = \ddot{s}(t) = \frac{1}{m}\left(-k s(t) - q \dot{s}(t) + F(t)\right)$$

$$= -\frac{k}{m} \underline{x_1(t)} - \frac{q}{m} \underline{x_2}(t) + \frac{1}{m} u_1(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{q}{m} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$