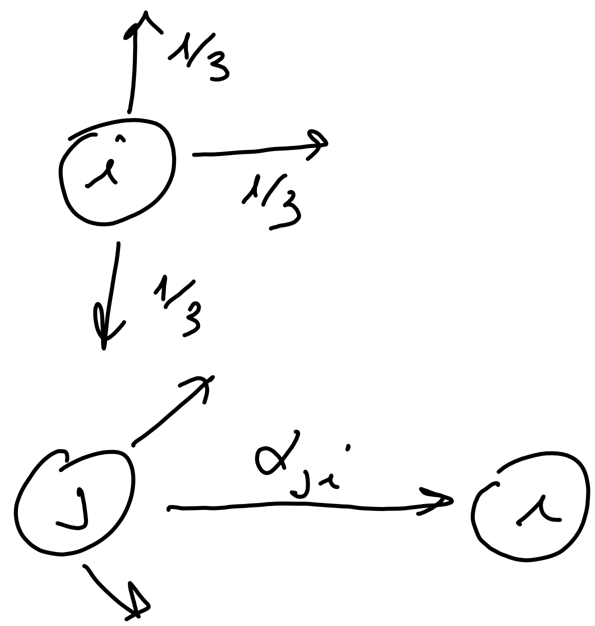
GOOGLE PAGE RANK

1) Ad ogni pagina associamo un nodo

2) Ad ogni link presente nella pagina associamo un arco uscente dal punto al nodo associato alla pagina a cui punta il link

3) Pesi sugli archi uscenti $= 1/$numero di archi uscenti

Esempio



$$\sum_i \alpha_{ji} = 1$$

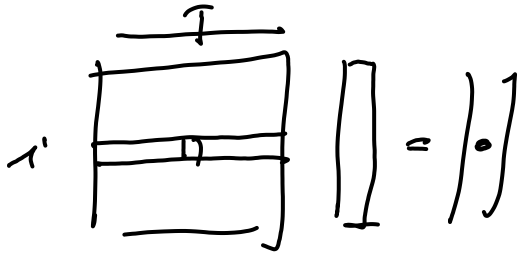
$$x_i(k+1) = \sum_j \alpha_{ji} \, x_j(k)$$

$x_i(k)$ = prob. di essere nello stato i all'istante k

$$x(k+1) = A x(k)$$

$$\boxed{x(k) \longrightarrow x_{eq}}$$

$$\underline{A X_{eq} = X_{eq}} = \begin{bmatrix} x_{eq\,1} \\ \vdots \\ x_{eq\,n} \end{bmatrix}$$



$$x_{eq,i} = \sum_j a_{ij}\, x_{eq,j}$$

$$x_{eq,i} = \sum_j \alpha_{ji}\, \underline{x_{eq,j}} \qquad j \xrightarrow{\alpha_{ji}} i$$

Trappole

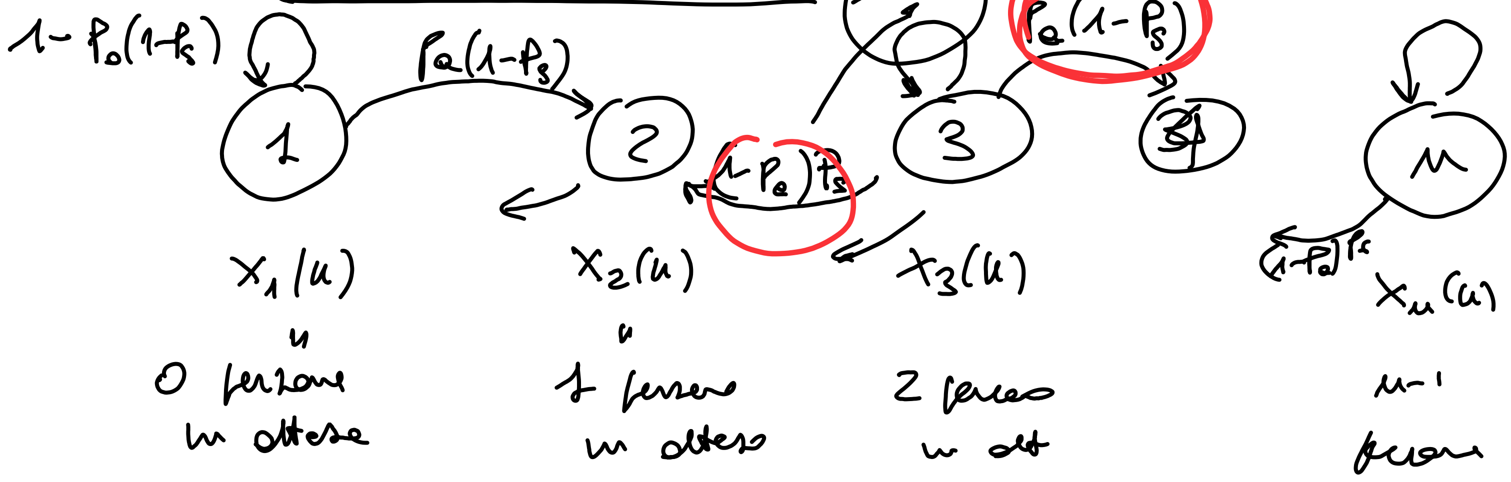
$$\longrightarrow i$$

$$A$$

$$\downarrow$$

$$\tilde{A} = pA + (1-p) \underbrace{\begin{bmatrix} 1/n & 1/n & \cdots & 1/n \\ 1/n & & & \\ \vdots & & & \\ & & & \end{bmatrix}}_{n} \qquad p \simeq 0.99$$

Esempio: Coda

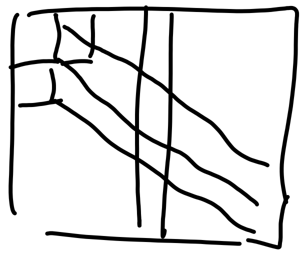


P_a = probabilità che all'istante k arrivi 1 persona

$1-P_a$ = che non arrivi nessuno

P_s = probabilità che all'istante k sia servita una persona

$i = 2, \dots, n-1$

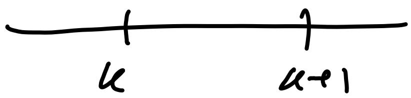
$A =$ 

$$x_i(k+1) = \overbrace{\left[1-(1-P_a)P_s - P_a(1-P_s)\right]}^{a_{ii}} x_i(k) + \overbrace{P_a(1-P_s)}^{a_{i,i-1}} x_{i-1}(k) + \overbrace{(1-P_a)P_s}^{a_{i,i+1}} x_{i+1}(k)$$

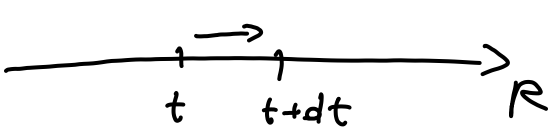
$$x_1(k+1) = \left[1 - P_a(1-P_s)\right] x_1(k) + (1-P_a)P_s\, x_2(k)$$

Tempo continuo

$x_i(k+1) \qquad x_i(k)$



$x_i(t+dt) \qquad x_i(t)$



$$\rightarrow P_a(\overset{k}{t}, \overset{k+1}{t+dt}) = \lambda dt \qquad \lambda, \mu > 0$$

$$\rightarrow P_s(t, t+dt) = \mu dt \qquad dt \quad \mu \text{ ccolo}$$

$$x_i(t+dt) = [1 - \overline{P_s} + \cancel{P_a P_s} - \overline{P_a} + \cancel{P_a P_s}]\, x_i(t) + [P_a - \cancel{P_a P_s}]\, x_{i-1}(t) + [P_s - \cancel{P_a P_s}]\, x_{i+1}(t)$$

$$P_a P_s = \lambda dt\, \mu dt \qquad (dt)^2 \longrightarrow 0$$

$$\frac{x_i(t+dt) - x_i(t)}{dt} = (-\mu - \lambda)\, x_i(t) + \underline{\lambda\, x_{i-1}}(t) + \mu\, x_{i+1}(t)$$

$$\dot{x}_i(t) = (-\lambda - \mu)\, x_i(t) + \mu\, x_{i+1}(t) + \lambda\, x_{i-1}(t)$$

$$L(t) = x_1(t)\,0 + x_2(t)\,1 + x_3(t)\,2 + \cdots$$

MODELLI DI INFLUENZA

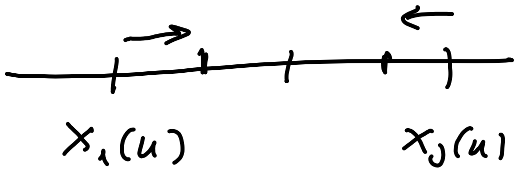
Modello per la dinamica delle opinioni

$x_i(k)$ descrive l'orientamento dell'individuo i rispetto a un tema

Esempio per due persone l'interazione è

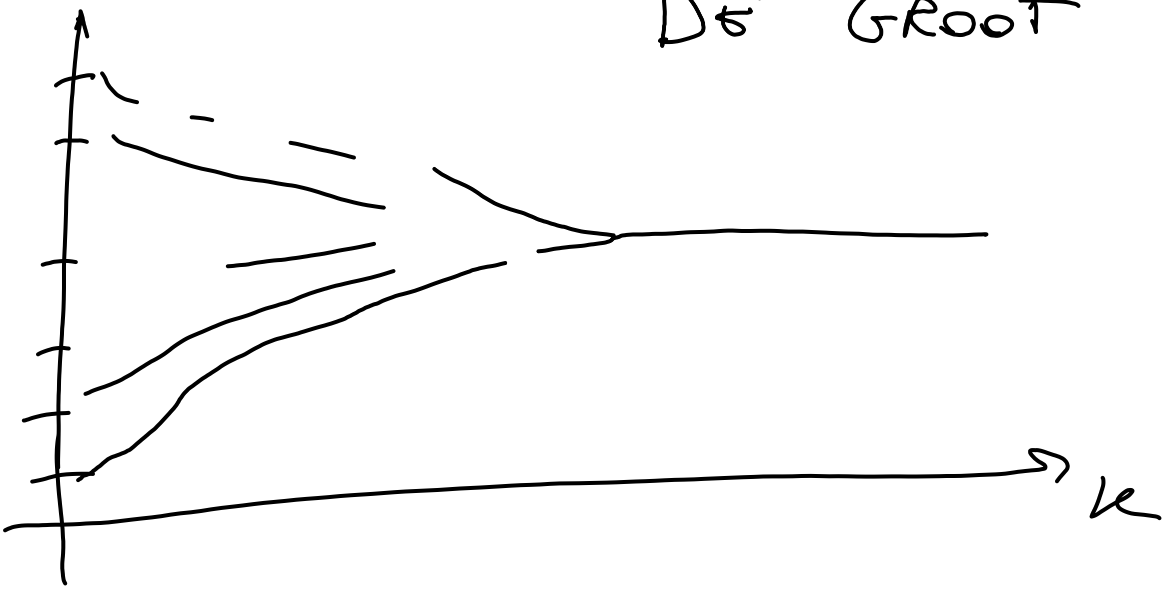
$$x_i(k+1) = \overbrace{(1-p)}^{\alpha_{ii}} x_i(k) + \overbrace{p}^{\alpha_{ji}} x_j(k)$$

$$p \in [0, 1/2]$$

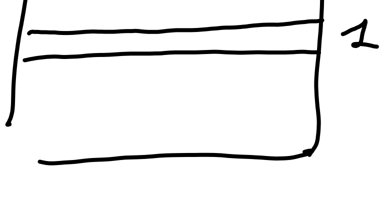


$$x_i(k+1) = \sum_{j=1}^{n} \alpha_{ji} \, x_j(k)$$

DE GROOT



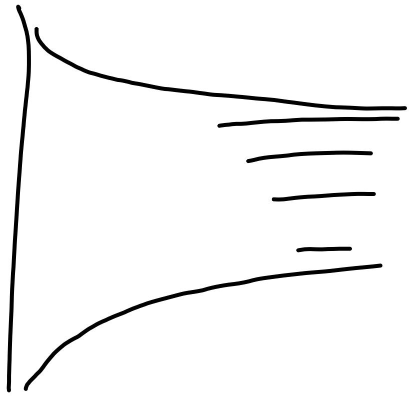
$x(k+1) = A\,x(k)$ $A =$



FRIEDKIN JOHNSEN

$$x(k+1) = \lambda A x(k) + (1-\lambda) x(0)$$

$\lambda \in [0,1]$



$\lambda = 0 \quad x(k+1) = x(0)$

$\lambda = 1 \quad$ DE GROOT

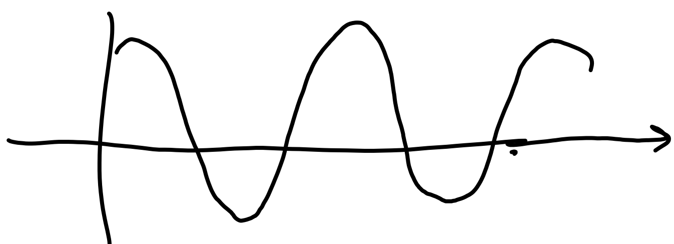
AFFARI DI CUORE

$x_1(t)$ grado di amore di Romeo verso Giulietta

$x_2(t)$ grado di amore di Giulietta verso Romeo

$$\begin{cases} \dot{x}_1(t) = -a x_2(t) \\ \dot{x}_2(t) = b x_1(t) \end{cases}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & -a \\ b & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

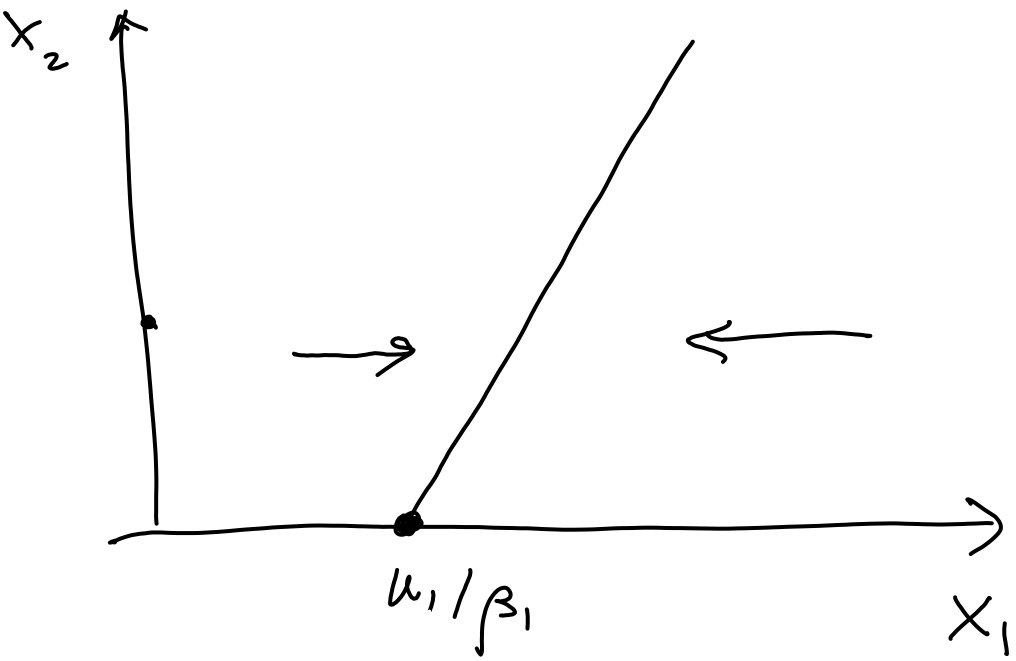


CORSA AGLI ARMAMENTI

$x_1(t)$ armamenti della nazione i ^{1,2} all'istante t

$$\dot{x}_1(t) = u_1(t) + \alpha_{21} x_2(t) - \beta_1 x_1(t)$$

$$\dot{x}_2(t) = u_2(t) + \alpha_{12} x_1(t) - \beta_2 x_2(t)$$



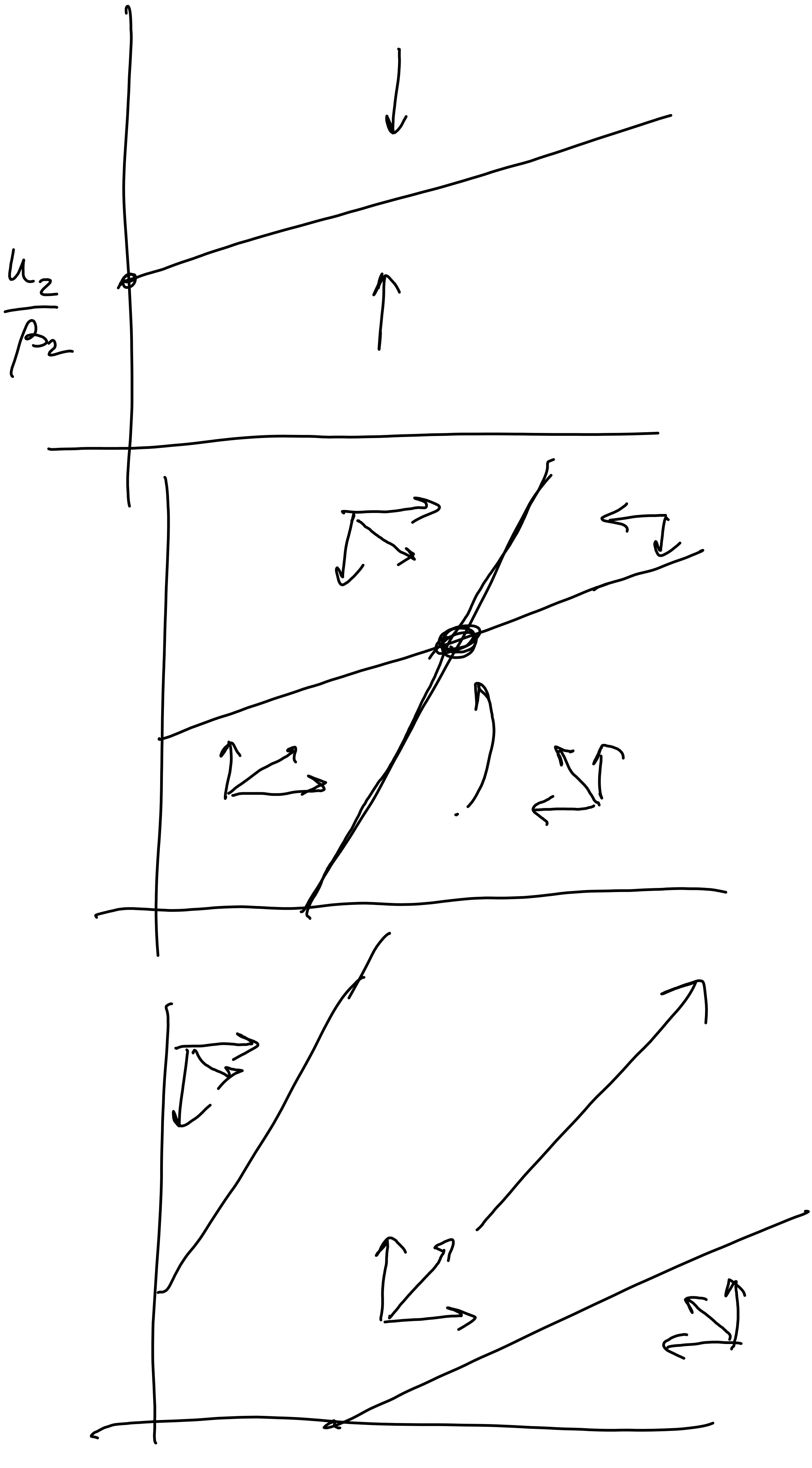
$$\dot{x}_1(t) > 0$$

$$u_1 + \alpha_{21} x_2 - \beta_1 x_1 > 0$$

$$x_1 < \frac{u_1}{\beta_1} + \frac{\alpha_{21}}{\beta_1} x_2$$

$u_1 =$ cost

$$x_1 = \frac{u_1}{\beta_1} + \cancel{\frac{\alpha_{21}}{\beta_1} x_2}$$


$\frac{u_2}{\beta_2}$