



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA

Machine Learning  
2024/2025

AMCO  
ARTIFICIAL INTELLIGENCE, MACHINE  
LEARNING AND CONTROL RESEARCH GROUP

# Lecture #02 Introduction & Basic Statistics

Gian Antonio Susto



# Before starting

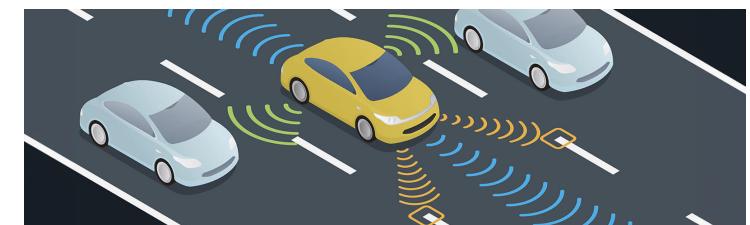
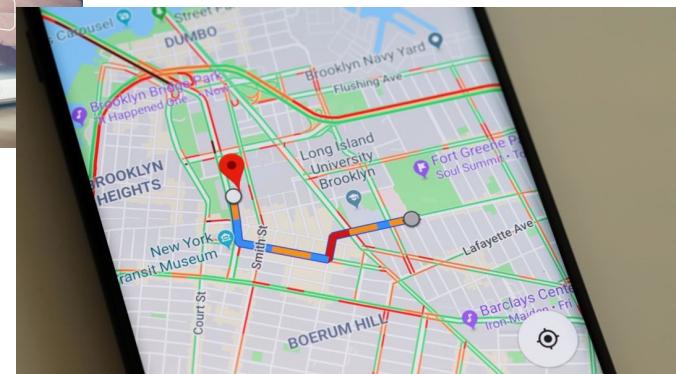
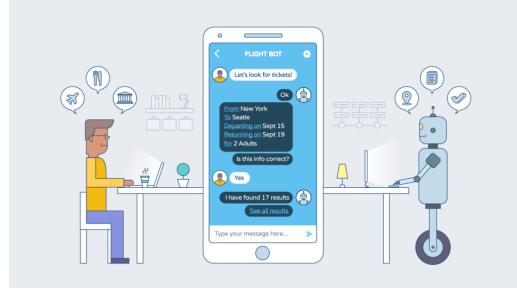
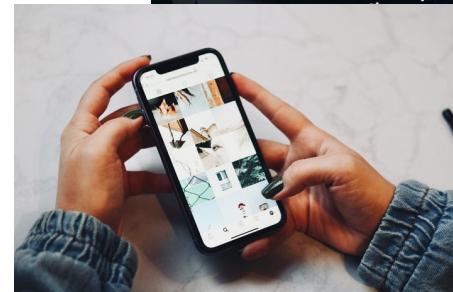
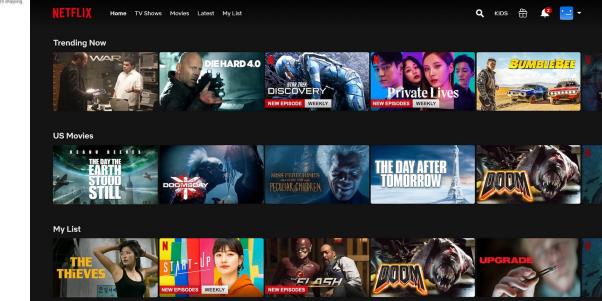
IPSE DIXIT:

‘Lecture and laboratories recordings will be made available shortly after the lecture.’

... from this lecture! As I forgot to record the first one...



# Many technologies, let's put some order



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Different tasks (objectives), different models, different data type... and different stages of development!

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Different **tasks** (**objectives**), different models, different data type... and different stages of development!

A task refers to the specific problem that the model is designed to solve. It defines the objective that the model aims to achieve based on input data

## Supervised Learning

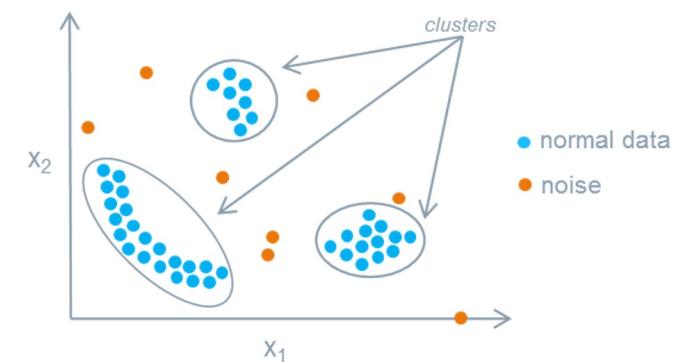


Setup: Observation of the environment

Data:  $(x, y)$

Task: learn a map from inputs  $x$  to outputs  $y$

## Unsupervised Learning



Setup: Observation of the environment

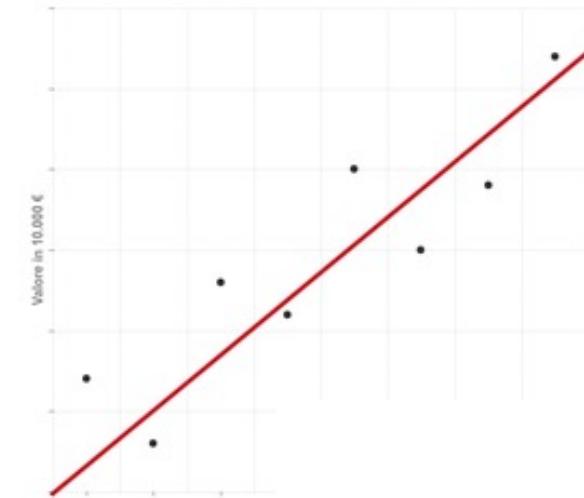
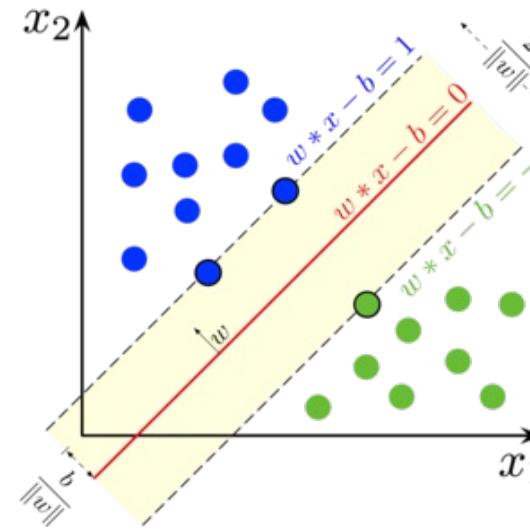
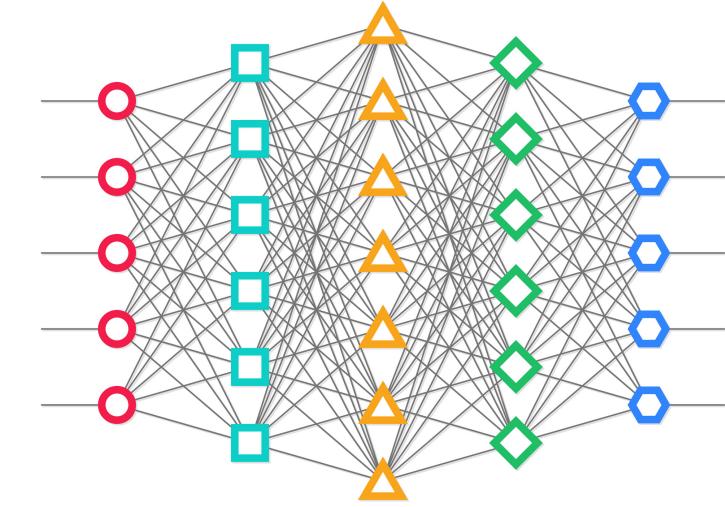
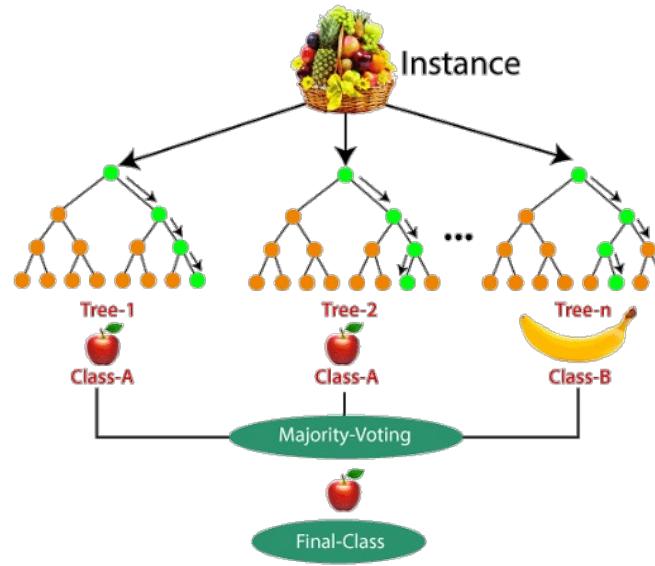
Data:  $x$  (no labels)

Task: learn patterns in input data

# Many technologies, let's put some order

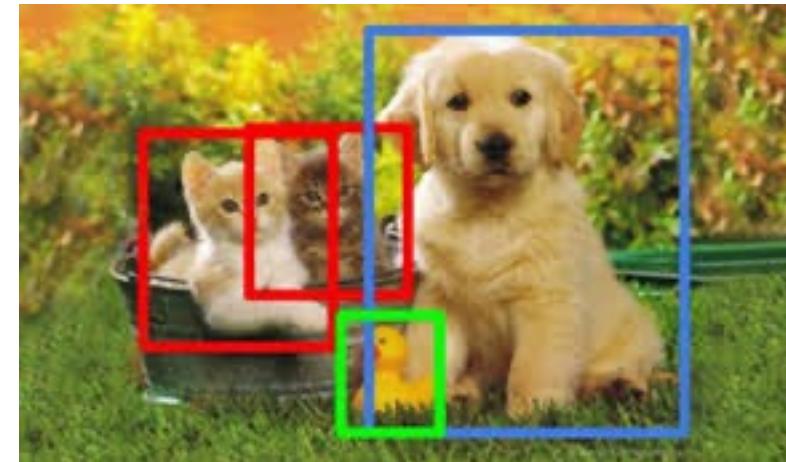
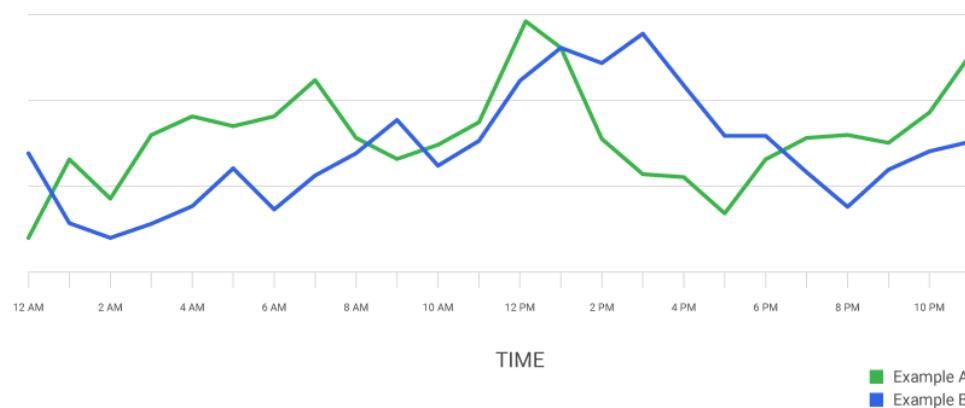
Different tasks (objectives), different **models**, different data type... and different stages of development!

A model is a mathematical representation of patterns and relationships within data. It is trained using an algorithm to make predictions or decisions based on input data



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Different tasks  
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38	Private	215646	HS-grad	9	Divorced	Handlers-cleaners	Not-in-family	White	Male	0	0	40	United-States	<=50K
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49	Private	160187	9th	5	Married-spouse-absent	Other-service	Not-in-family	Black	Female	0	0	16	Jamaica	<=50K
52	Self-emp-not-inc	209642	HS-grad	9	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	45	United-States	>50K
31	Private	45781	Masters	14	Never-married	Prof-specialty	Not-in-family	White	Female	14084	0	50	United-States	>50K
42	Private	159449	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	5178	0	40	United-States	>50K
37	Private	280464	Some-college	10	Married-civ-spouse	Exec-managerial	Husband	Black	Male	0	0	80	United-States	>50K
30	State-gov	141297	Bachelors	13	Married-civ-spouse	Prof-specialty	Husband	Asian-Pac-Islander	Male	0	0	40	India	>50K
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32	Private	205019	Assoc-acdm	12	Never-married	Sales	Not-in-family	Black	Male	0	0	50	United-States	<=50K
40	Private	121772	Assoc-vc	11	Married-civ-spouse	Craft-repair	Husband	Asian-Pac-Islander	Male	0	0	40	?	>50K
34	Private	245487	7th-8th	4	Married-civ-spouse	Transport-moving	Husband	Amer-Indian-Eskimo	Male	0	0	45	Mexico	<=50K
25	Self-emp-not-inc	176756	HS-grad	9	Never-married	Farming-fishing	Own-child	White	Male	0	0	35	United-States	<=50K
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38	Private	28887	11th	7	Married-civ-spouse	Sales	Husband	White	Male	0	0	50	United-States	<=50K
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# Tabular Data (the 'design matrix') - $X$

$n$   
observations:  
the number  
of times the  
phenomenon  
we need to  
'describe' is  
available in  
our data  
through  
historical  
examples



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What will be better? High values of  $p$  and/or  $n$ ?

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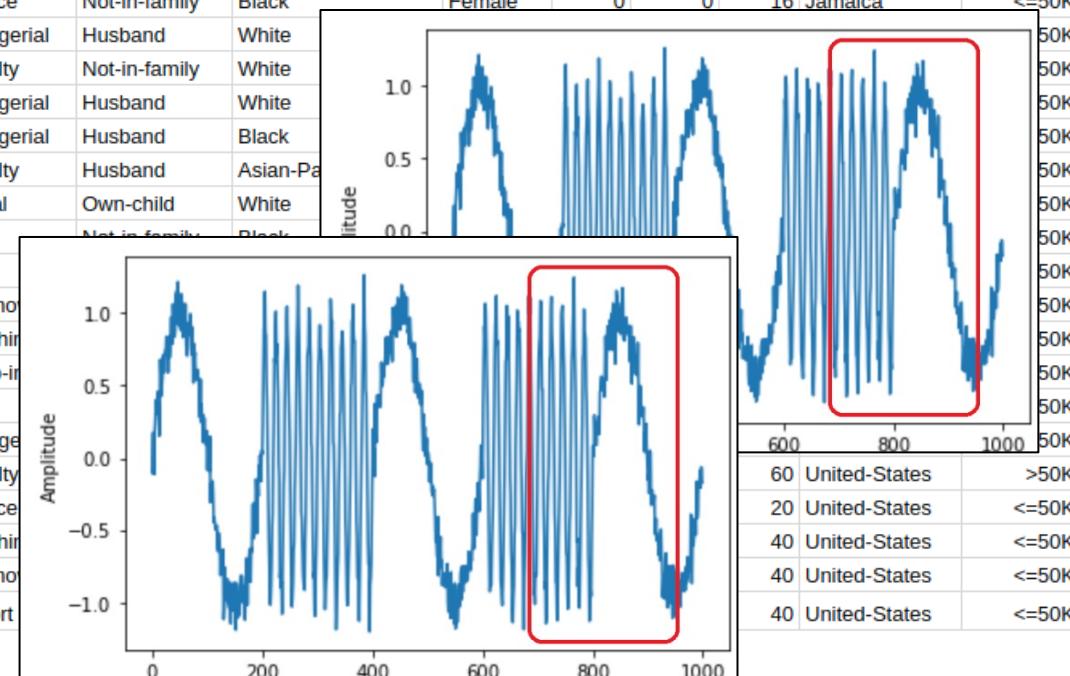
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23	Private					Adm-clerical	Own-child	White						50K
32	Private					Sales								50K
40	Private					Craft-repair								50K
34	Private					Transport-mo								50K
25	Self-emp-not-inc					Farming-fishir								50K
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For example, a system described by (multiple) time-series data: maybe we need to extract quantities from each time window



# Tabular Data (the ‘design matrix’) - $X$

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42	Private	159449	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	5178	0	40	United-States	>50K
37	Private	280464	Some-college	10	Married-civ-spouse	Exec-managerial	Husband	Black	Male	0	0	80	United-States	>50K
30	State-gov	141297	Bachelors	13	Married-civ-spouse	Prof-specialty	Husband	Asian-Pac-Islander	Male	0	0	40	India	>50K
23	Private	122272	Bachelors	13	Never-married	Adm-clerical	Own-child	White	Female	0	0	30	United-States	<=50K
32	Private	205019	Assoc-acdm	12	Never-married	Sales	Not-in-family	Black	Male	0	0	50	United-States	<=50K
40	Private	121772	Assoc-voc	11	Married-civ-spouse	Craft-repair	Husband	Asian-Pac-Islander	Male	0	0	40	?	>50K
34	Private	245487	7th-8th	4	Married-civ-spouse	Transport-moving	Husband	Amer-Indian-Eskimo	Male	0	0	45	Mexico	<=50K
25	Self-emp-not-inc	176756	HS-grad	9	Never-married	Farming-fishing	Own-child	White	Male	0	0	35	United-States	<=50K
32	Private	186824	HS-grad	9	Never-married	Machine-op-inspct	Unmarried	White	Male	0	0	40	United-States	<=50K
38	Private	28887	11th	7	Married-civ-spouse	Sales	Husband	White	Male	0	0	50	United-States	<=50K
43	Self-emp-not-inc	292175	Masters	14	Divorced	Exec-managerial	Unmarried	White	Female	0	0	45	United-States	>50K
40	Private	193524	Doctorate	16	Married-civ-spouse	Prof-specialty	Husband	White	Male	0	0	60	United-States	>50K
54	Private	302146	HS-grad	9	Separated	Other-service	Unmarried	Black	Female	0	0	20	United-States	<=50K
35	Federal-gov	76845	9th	5	Married-civ-spouse	Farming-fishing	Husband	Black	Male	0	0	40	United-States	<=50K
43	Private	117037	11th	7	Married-civ-spouse	Transport-moving	Husband	White	Male	0	2042	40	United-States	<=50K
59	Private	109015	HS-grad	9	Divorced	Tech-support	Unmarried	White	Female	0	0	40	United-States	<=50K

At same point, this matrix (or part of this... more on this later) will be fed to a machine learning model!

To do so, data has:

- to be ‘consistent’;
- all variables should ‘be treated’ equally (unless we have a priori knowledge, all variables can contribute to understand/describe the phenomena);
- we should make life easy for a model and do not provide redundant/useless information

# Tabular Data (the ‘design matrix’) - $X$

39	State-gov	77516	Bachelors	13	Never-married	Adm-clerical	Not-in-family	White	Male	2174	0	40	United-States	<=50K
50	Self-emp-not-inc	83311	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	13	United-States	<=50K
38	Private	215646	HS-grad	9	Divorced	Handlers-cleaners	Not-in-family	White	Male	0	0	40	United-States	<=50K
53	Private	234721	11th	7	Married-civ-spouse	Handlers-cleaners	Husband	Black	Male	0	0	40	United-States	<=50K
28	Private	338409	Bachelors	13	Married-civ-spouse	Prof-specialty	Wife	Black	Female	0	0	40	Cuba	<=50K
37	Private	284582	Masters	14	Married-civ-spouse	Exec-managerial	Wife	White	Female	0	0	40	United-States	<=50K
49	Private	160187	9th	5	Married-spouse-absent	Other-service	Not-in-family	Black	Female	0	0	16	Jamaica	<=50K
52	Self-emp-not-inc	209642	HS-grad	9	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	45	United-States	>50K
31	Private	45781	Masters	14	Never-married	Prof-specialty	Not-in-family	White	Female	14084	0	50	United-States	>50K
42	Private	159449	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	5178	0	40	United-States	>50K
37	Private	280464	Some-college	10	Married-civ-spouse	Exec-managerial	Husband	Black	Male	0	0	80	United-States	>50K
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32	Private	205019	Assoc-acdm	12	Never-married	Sales	Not-in-family	Black	Male	0	0	50	United-States	<=50K
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38	Private	28887	11th	7	Married-civ-spouse	Sales	Husband	White	Male	0	0	50	United-States	<=50K
43	Self-emp-not-inc	292175	Masters	14	Divorced	Exec-managerial	Unmarried	White	Female	0	0	45	United-States	>50K
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35	Federal-gov	76845	9th	5	Married-civ-spouse	Farming-fishing	Husband	Black	Male	0	0	40	United-States	<=50K
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Statistical  
moments/quantities  
can be of help!

At same point, this matrix (or part of this... more on this later) will be fed to a machine learning model!

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# Many technologies, let's put some order

Different tasks  
(objectives), different  
models, different data  
type... and **different  
stages of development!**

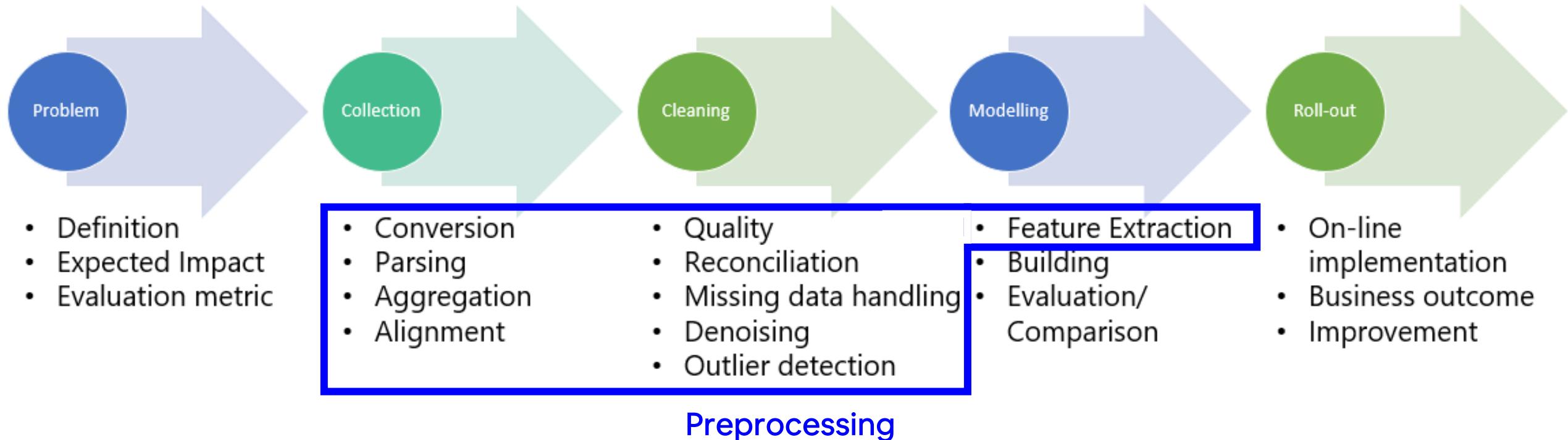


# A Machine Learning pipeline

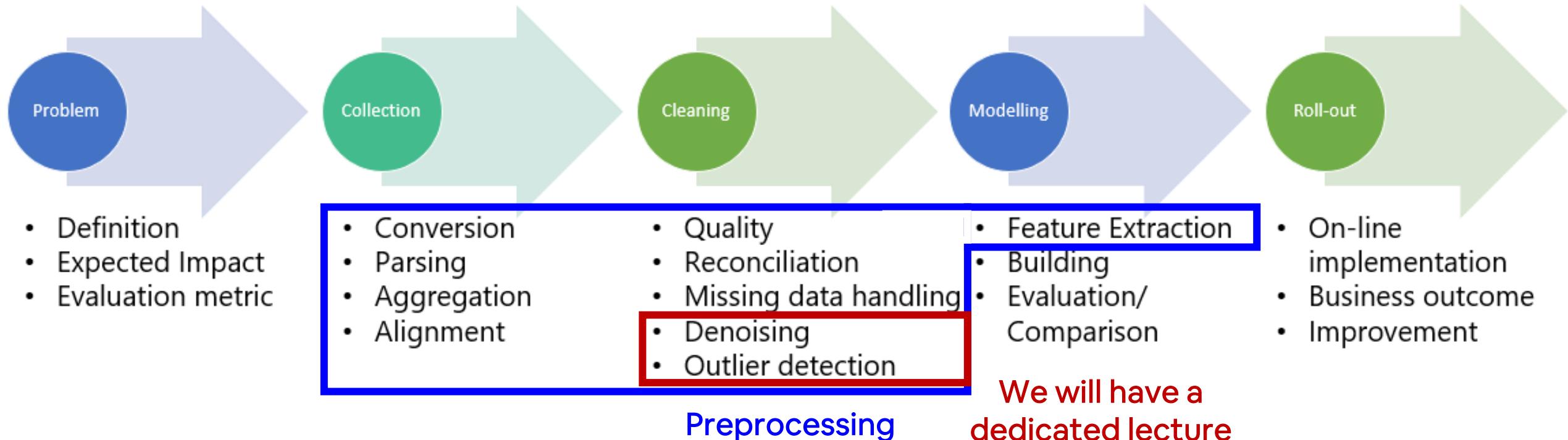


Main focus of  
the course  
(from week 3)

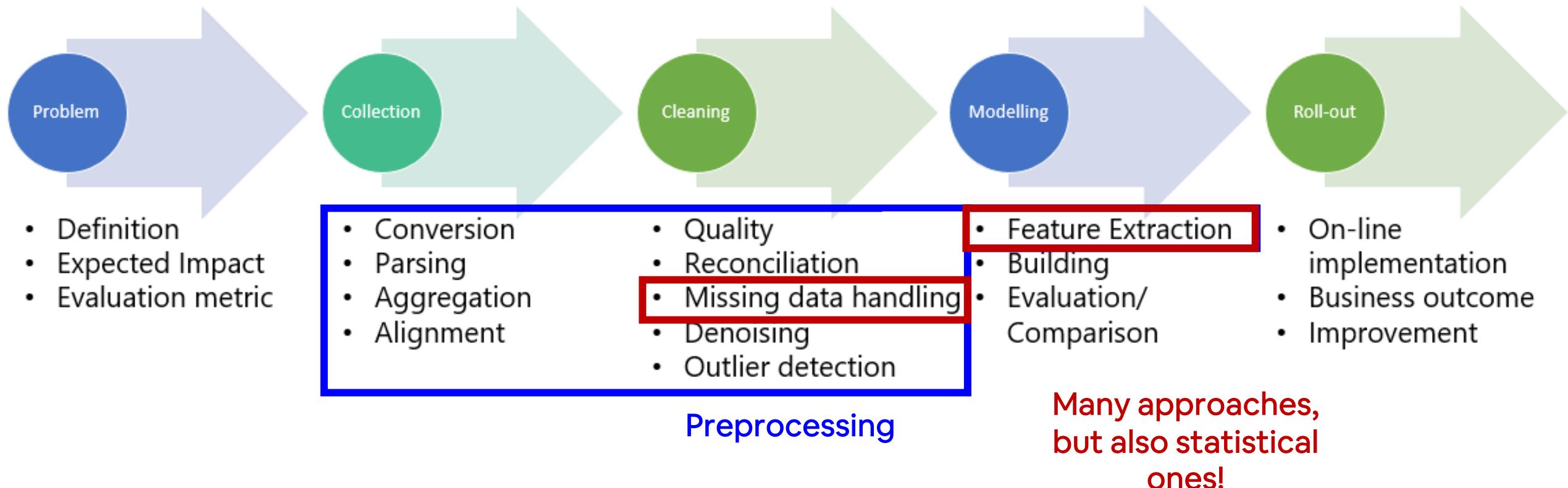
# A Machine Learning pipeline



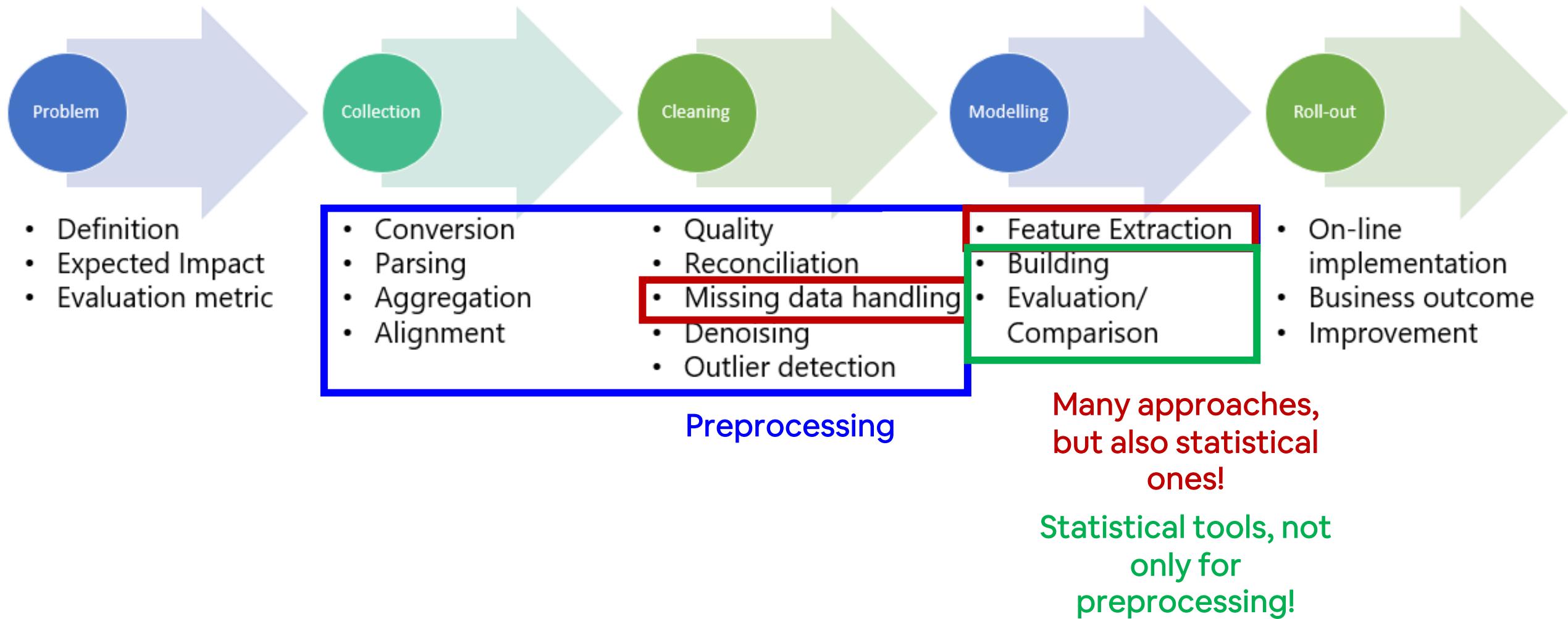
# A Machine Learning pipeline



# A Machine Learning pipeline



# A Machine Learning pipeline



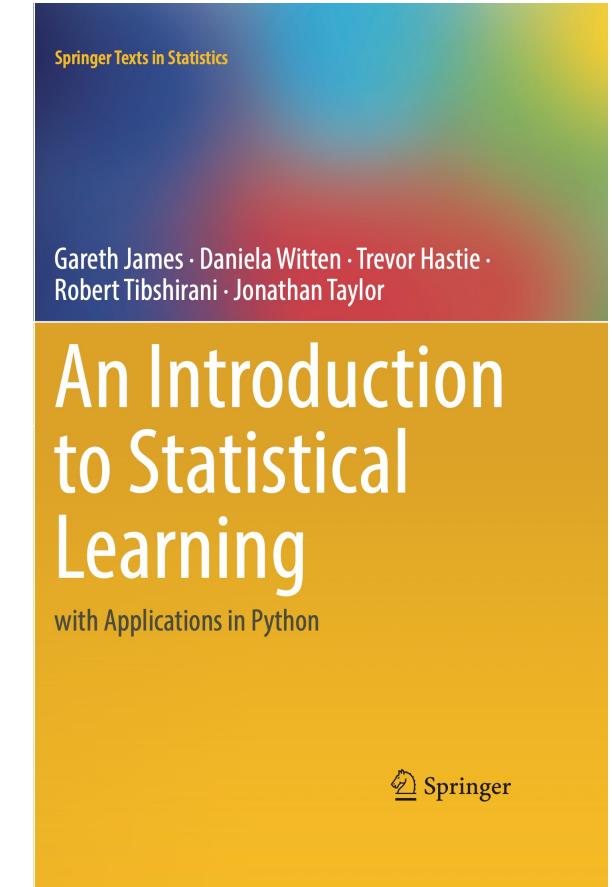
# A Machine Learning pipeline



# Why use Statistics in Machine Learning?

## Why Use Statistics in ML?

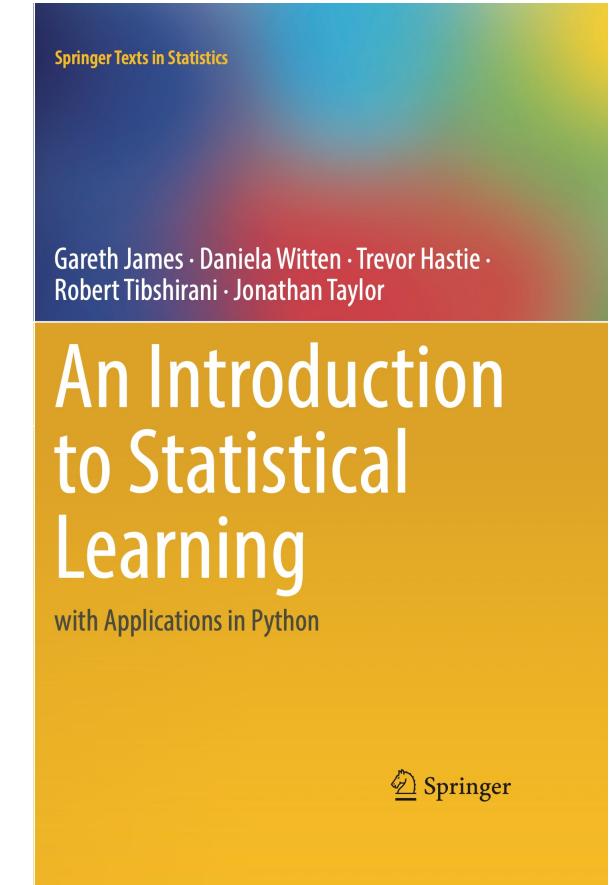
1. [Preprocessing] Data Understanding – Descriptive statistics (mean, variance, distributions) help explore and clean data, identifying patterns and outliers.
2. [Preprocessing] Feature Engineering – Techniques like correlation analysis, PCA, and scaling rely on statistical principles.
3. [Building] Probability & Uncertainty – ML often deals with probabilistic models (e.g., Naïve Bayes) and uncertainty estimation.
4. [Building] Generalization & Inference – Concepts like overfitting, hypothesis testing, and bias-variance tradeoff come from statistics.
5. [Evaluation] Model Evaluation – Metrics like MSE, MAE, accuracy, precision, and recall are rooted in statistical concepts.



# Why use Statistics in Machine Learning?

## Why Use Statistics in ML?

1. [Preprocessing] Data Understanding – **Descriptive statistics (mean, variance, distributions)** help explore and clean data, identifying patterns and outliers.
2. [Preprocessing] **Feature Engineering** – Techniques like **correlation analysis**, PCA, and **scaling** rely on statistical principles.
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# Statistical moments and other statistical quantities

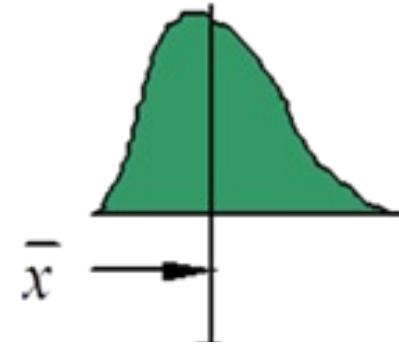
# Statistical moments in Machine Learning

## Moments of a Random Variable X

### 1. First Moment (Mean) – Central Tendency

$$E[X] = \mu$$

The expected value of X, representing the average outcome.



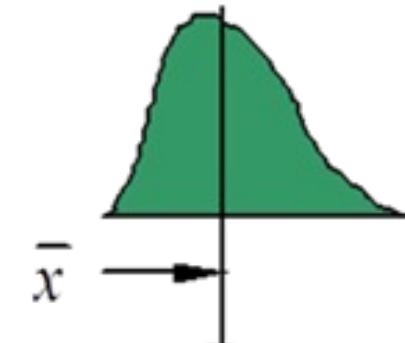
# Statistical moments in Machine Learning

## Moments of a Random Variable X

### 1. First Moment (Mean) – Central Tendency

$$E[X] = \mu$$

The expected value of X, representing the average outcome.



Difference  
between Statistical  
Moments  
(theoretical) and  
Statistical  
Moments

Computed with  
Sampling (with  
available data)

$$\mu_k = E[X^k] = \int_{-\infty}^{\infty} x^k f(x) dx \quad (\text{for continuous distributions})$$

$$\mu_k = \sum_x x^k P(X = x) \quad (\text{for discrete distributions})$$



$$\hat{\mu}_k = \frac{1}{n} \sum_{i=1}^n X_i^k$$

# Statistical moments in Machine Learning

## Moments of a Random Variable X

### 1. First Moment (Mean) – Central Tendency

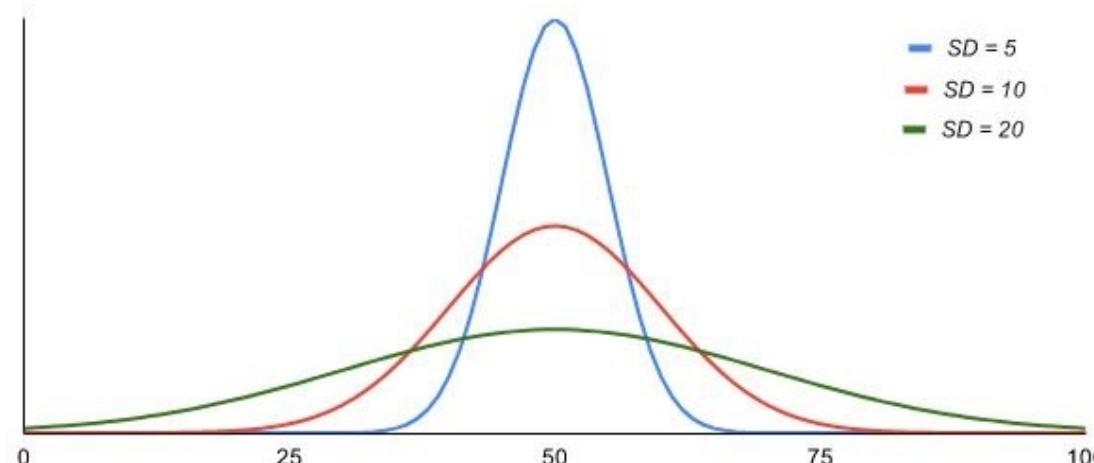
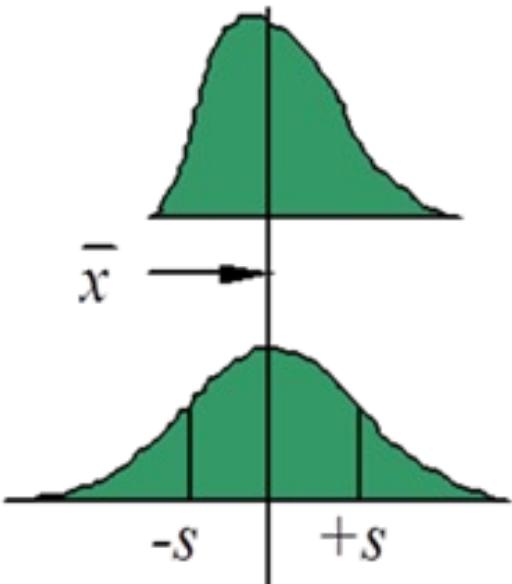
$$E[X] = \mu$$

The expected value of X, representing the average outcome.

### 2. Second Moment (Variance) – Spread of Data

$$Var(X) = E[(X - \mu)^2] = \sigma^2$$

Measures how far values of X deviate from the mean.



Distributions with the same mean, but different variance

# Statistical moments in Machine Learning

## Moments of a Random Variable X

### 1. First Moment (Mean) – Central Tendency

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The expected value of X, representing the average outcome.

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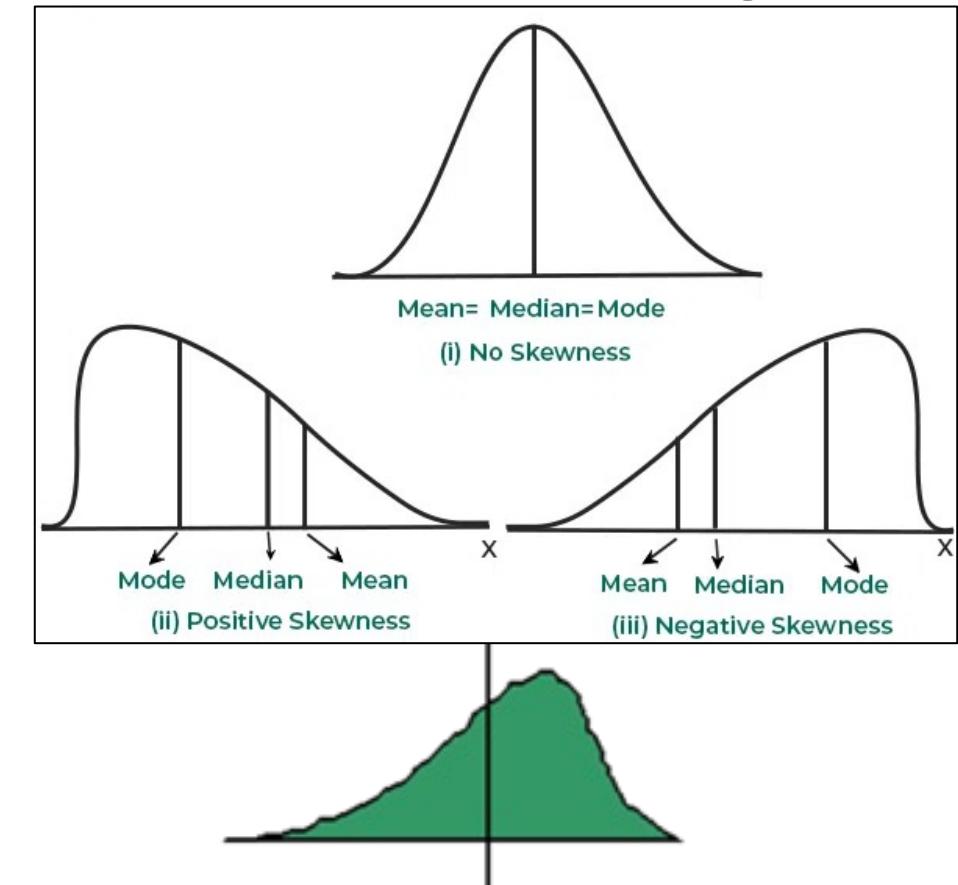
$$Var(X) = E[(X - \mu)^2] = \sigma^2$$

Measures how far values of X deviate from the mean.

### 3. Third Moment (Skewness) – Asymmetry of Distribution

$$Skew(X) = E[(X - \mu)^3] / \sigma^3$$

Indicates whether the distribution leans right (negative skew) or left (positive skew).



# Statistical moments in Machine Learning

## Normal Kurtosis ( $K = 3$ ) - Mesokurtic

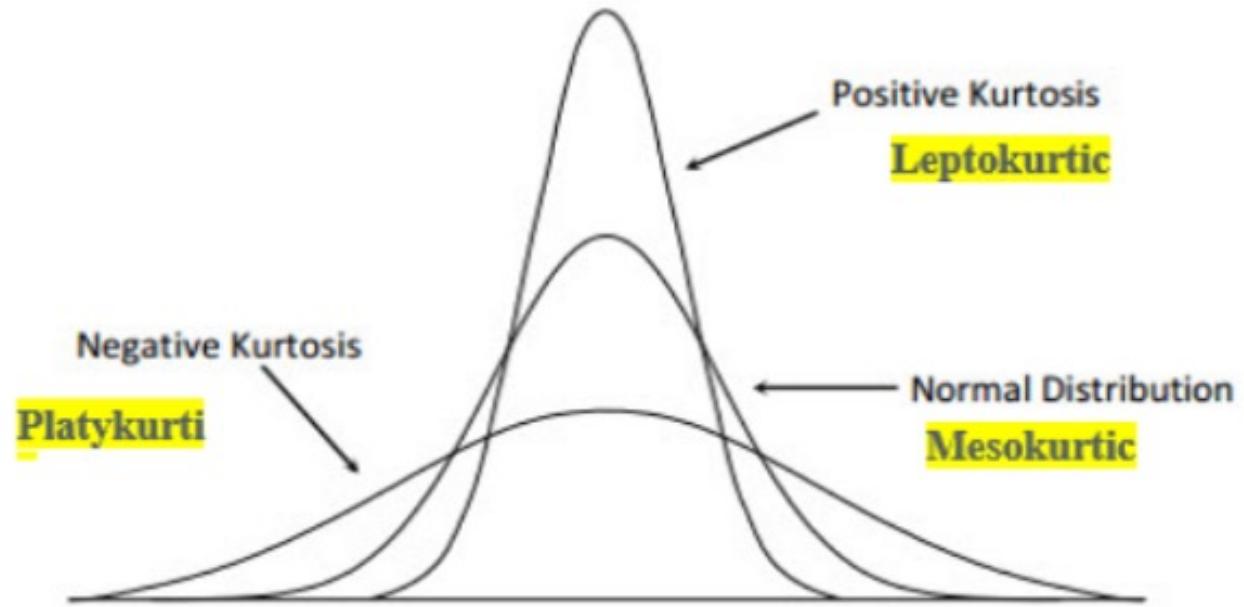
- The distribution has the same shape as the normal distribution.

## High Kurtosis ( $K > 3$ ) - Leptokurtic

- Heavier tails than the normal distribution, meaning more extreme values (outliers).
- The distribution is more "peaked" in the center and has longer tails.

## Low Kurtosis ( $K < 3, K_{\text{excess}} < 0$ ) - Platykurtic

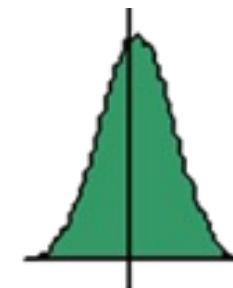
- Lighter tails than the normal distribution, meaning fewer extreme values.
- The distribution is "flatter" in the center with shorter tails.



## 4. Fourth Moment (Kurtosis) – Tailedness of Distribution

$$Kurt(X) = E[(X - \mu)^4] / \sigma^4$$

Measures how heavy or light the tails of the distribution are compared to a normal distribution.



# Statistical moments in Machine Learning

## Moments of a Random Variable X

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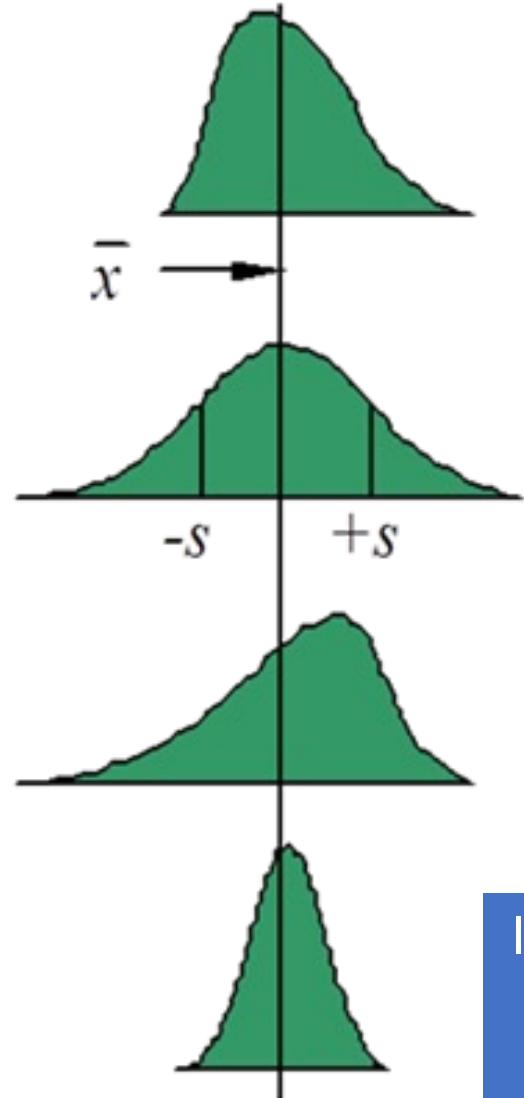
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Measures how heavy or light the tails of the distribution are compared to a normal distribution.



Increasingly  
adding  
more  
information

# A numerical example

A  $n = 10$  dataset

$$X = [10, 15, 20, 25, 30, 35, 40, 45, 50, 55]$$

Let's compute the mean:

$$\mu = \frac{\sum X_i}{n} = \frac{10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + 55}{10} = \frac{325}{10} = 32.5$$

# A numerical example

Let's compute the variance:

$$\sigma^2 = \frac{\sum(X_i - \mu)^2}{n}$$

# A numerical example

Let's compute the variance:

$$\sigma^2 = \frac{\sum(X_i - \mu)^2}{n}$$

$$(10 - 32.5)^2 = 552.25$$

$$(15 - 32.5)^2 = 306.25$$

$$(20 - 32.5)^2 = 156.25$$

$$(25 - 32.5)^2 = 56.25$$

$$(30 - 32.5)^2 = 6.25$$

$$(35 - 32.5)^2 = 6.25$$

$$(40 - 32.5)^2 = 56.25$$

$$(45 - 32.5)^2 = 156.25$$

$$(50 - 32.5)^2 = 306.25$$

$$(55 - 32.5)^2 = 552.25$$

# A numerical example

Let's compute the variance:

$$\sigma^2 = \frac{\sum(X_i - \mu)^2}{n}$$

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$$(35 - 32.5)^2 = 6.25$$

$$(40 - 32.5)^2 = 56.25$$

$$(45 - 32.5)^2 = 156.25$$

$$(50 - 32.5)^2 = 306.25$$

$$(55 - 32.5)^2 = 552.25$$

$$552.25 + 306.25 + 156.25 + 56.25 + 6.25 + 6.25 + 56.25 + 156.25 + 306.25 + 552.25 = 2062.5$$

$$\sigma^2 = \frac{2062.5}{10} = 206.25$$

# A numerical example

Let's compute the skewness:

$$\text{Skew}(X) = \frac{\sum(X_i - \mu)^3}{n \cdot \sigma^3}$$

$$(10 - 32.5)^3 = -12903.125$$

$$(15 - 32.5)^3 = -5359.375$$

$$(20 - 32.5)^3 = -1953.125$$

$$(25 - 32.5)^3 = -421.875$$

$$(30 - 32.5)^3 = -15.625$$

$$(35 - 32.5)^3 = 15.625$$

$$(40 - 32.5)^3 = 421.875$$

$$(45 - 32.5)^3 = 1953.125$$

$$(50 - 32.5)^3 = 5359.375$$

$$(55 - 32.5)^3 = 12903.125$$

$$-12903.125 + (-5359.375) + (-1953.125) + (-421.875) + (-15.625) + 15.625 + 421.875 + 1953.125 + 5359.375 + 12903.125 = 0$$

$$\text{Skew}(X) = \frac{0}{10 \cdot (206.25)^{3/2}} = 0$$

# A numerical example

Let's compute the skewness:

$$\text{Skew}(X) = \frac{\sum(X_i - \mu)^3}{n \cdot \sigma^3}$$

We have a symmetric distribution!

$$(10 - 32.5)^3 = -12903.125$$

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$$-12903.125 + (-5359.375) + (-1953.125) + (-421.875) + (-15.625) + 15.625 + 421.875 + 1953.125 + 5359.375 + 12903.125 = 0$$

$$\text{Skew}(X) = \frac{0}{10 \cdot (206.25)^{3/2}} = 0$$

# A numerical example

Let's compute the kurtosis:

$$\text{Kurt}(X) = \frac{\sum(X_i - \mu)^4}{n \cdot \sigma^4}$$

The kurtosis is less than 3, so the distribution is **platykurtic** (fewer "tails" compared to the normal distribution).

$$(10 - 32.5)^4 = 3013025.5625$$

$$(15 - 32.5)^4 = 938906.25$$

$$(20 - 32.5)^4 = 244140.625$$

$$(25 - 32.5)^4 = 31640.625$$

$$(30 - 32.5)^4 = 39.0625$$

$$(35 - 32.5)^4 = 39.0625$$

$$(40 - 32.5)^4 = 31640.625$$

$$(45 - 32.5)^4 = 244140.625$$

$$(50 - 32.5)^4 = 938906.25$$

$$(55 - 32.5)^4 = 3013025.5625$$

$$3013025.56 + 938906.25 + 244140.62 + 31640.62 + 39.06 + 39.06 + 31640.62 + 244140.62 + 938906.25 + 3013025.56 = 8282825$$

$$\text{Kurt}(X) = \frac{8282825}{10 \cdot (206.25)^2} = \frac{8282825}{42514.0625} = 1.8$$

# Quartiles

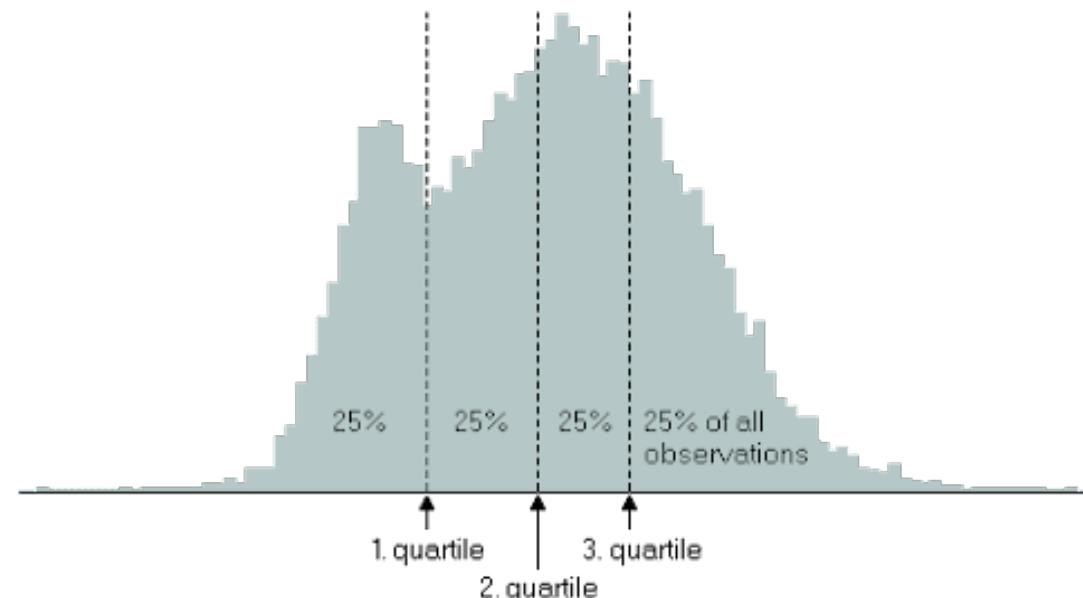
Quartiles divide a dataset into **four equal parts**, helping to understand the distribution and spread of the data.

Quartile Definitions:

- $Q_1$  (First Quartile, 25%) – The value below which 25% of the data falls.
- $Q_2$  (Second Quartile, 50%) – The **median**, the value that splits the data into two equal halves.
- $Q_3$  (Third Quartile, 75%) – The value below which 75% of the data falls.
- Interquartile Range (IQR) – The range between  $Q_1$  and  $Q_3$ , measuring the spread of the middle 50% of the data:

$$IQR = Q_3 - Q_1$$

First Quarter	First Quartile	Median	Second Quartile	Third Quartile	Third Quartile	Fourth Quarter
24, 25, 26,	26½	36	27, 30, 32,	40, 44, 50,	52, 55, 57	
	$Q_1$	$Q_2$		$Q_3$		



# Quartiles

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Quartile Definitions:

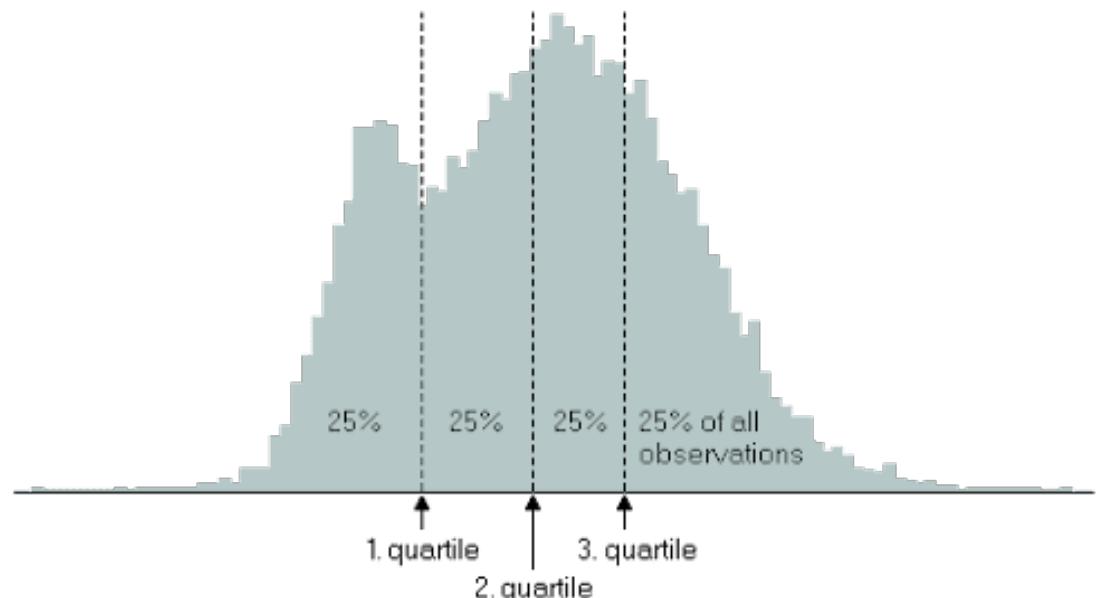
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- $Q_3$  (Third Quartile, 75%) – The value below which 75% of the data falls.
- Interquartile Range (IQR) – The range between  $Q_1$  and  $Q_3$ , measuring the spread of the middle 50% of the data:

$$IQR = Q_3 - Q_1$$

The median is used many times instead of the mean, as it is a **robust quantity w.r.t. 'outliers'** (strange data)

First Quarter	First Quartile	Median	Second Quartile	Third Quartile	Fourth Quarter
24, 25, 26,	26½	36	27, 30, 32,	40, 44, 50,	52, 55, 57
	$Q_1$	$Q_2$	$Q_3$		

MathBits.com



# A numerical example (odd numbers)

Same dataset previously seen ( $n = 11$ ).

$$X = [5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55]$$

$$Q2 =$$

$$Q3 =$$

$$Q1 =$$

$$IQR =$$

# A numerical example (odd numbers)

Same dataset previously seen ( $n = 11$ ).

$$X = [5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55]$$

$$Q2 = 30$$

$$Q3 =$$

$$Q1 =$$

$$IQR =$$

# A numerical example (odd numbers)

Same dataset previously seen ( $n = 11$ ).

$$X = [5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55]$$

$$Q2 = 30$$

$$Q3 = 45$$

$$Q1 = 15$$

$$IQR = Q3 - Q1 = 45 - 15 = 30$$

# A numerical example (even numbers)

Same dataset previously seen ( $n = 10$ ).

$$X = [10, 15, 20, 25, 30, 35, 40, 45, 50, 55]$$

$$Q1 =$$

$$Q2 =$$

$$Q3 =$$

$$IQR =$$

# A numerical example (even numbers)

Same dataset previously seen ( $n = 10$ ).

$$X = [10, 15, 20, 25, 30, 35, 40, 45, 50, 55]$$

$$Q1 = \frac{20 + 25}{2} = 22.5$$

$$Q2 = \frac{30 + 35}{2} = 32.5$$

$$Q3 = \frac{45 + 50}{2} = 47.5$$

$$IQR = Q3 - Q1 = 47.5 - 22.5 = 25$$

# Common convention (for the theoretic part of the exam): no interpolation!

Procedure:

- Order the array
- Compute the exact percentage
- Always take the closest number which percentage is AT LEAST 25%/50%/75% for Q1/Q2/Q3 respectively

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[10, 15, 20, 25, 30, 35, 40, 45, 50, 55]

[10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100%]

# Common convention (for the theoretic part of the exam): no interpolation!

Procedure:

- Order the array
- Compute the exact percentage
- Always take the closest number which percentage is AT LEAST 25%/50%/75% for  $Q_1$ / $Q_2$ / $Q_3$  respectively

[10, 15, 20, 25, 30, 35, 40, 45, 50, 55]  
[10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100%]

# Common convention (for the theoretic part of the exam): no interpolation!

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- Compute the exact percentage
- Always take the closest number which percentage is AT LEAST 25%/50%/75% for Q1/Q2/Q3 respectively

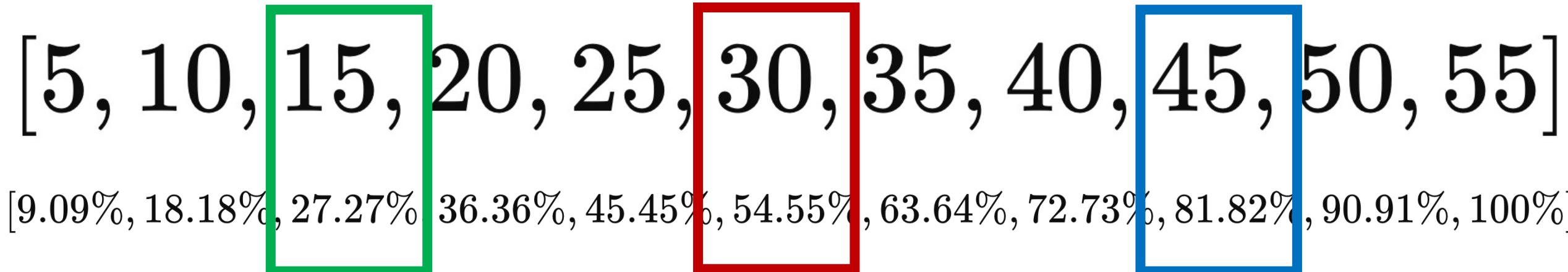
[5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55]

[9.09%, 18.18%, 27.27%, 36.36%, 45.45%, 54.55%, 63.64%, 72.73%, 81.82%, 90.91%, 100%]

# Common convention (for the theoretic part of the exam): no interpolation!

Procedure:

- Order the array
- Compute the exact percentage
- Always take the closest number which percentage is AT LEAST 25%/50%/75% for Q1/Q2/Q3 respectively



# Mode

The value that appears the most on a dataset. It is an important quantity when dealing with categorical data.

Example:

$$X = [1, 2, 2, 3, 4, 7, 9, 10, 10, 10, 12]$$

$$\text{Mean} = \frac{\sum X_i}{n} = \frac{1 + 2 + 2 + 3 + 4 + 7 + 9 + 10 + 10 + 10 + 12}{11} = \frac{70}{11} \approx 6.36$$

$$\text{Median} = 7$$

$$\text{Mode} = 10$$

# Statistical moments (and quantities) in Machine Learning

Statistical moments and quartiles provide a structured way to summarize and **understand data** distributions, which is essential for building, evaluating, and interpreting machine learning models.

Moments help in other pre-processing steps, such as feature engineering or missing data handling.

Not only that: they can be useful in easing the ‘training’ procedure\*

*\*What ‘training’ means it will be clearer when we talk about modelling, but it means tuning/finding the ‘right’ parameters in a given model*

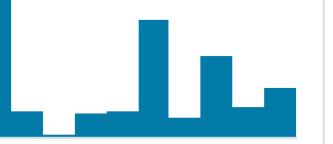
# ‘Understand’ a dataset: Iris dataset

- ‘Iris Classification’ dataset, Ronald Fisher (1936)
- Available on UCI ML Repository/Kaggle/... everywhere!
- L = 3 classes problem: classify **Setosa**, **Versicolour** and **Virginica** iris from data containing sepal and petal width and length – n = 150 samples, p = 4 variables



# Iris dataset

<https://www.kaggle.com/datasets/uciml/iris>

	# SepalLengthCm Length of the sepal (in cm)	# SepalWidthCm Width of the sepal (in cm)	# PetalLengthCm Length of the petal (in cm)	# PetalWidthCm Width of the petal (in cm)	Species Species name
	 4.3 7.9	 2 4.4	 1 6.9	 0.1 2.5	<b>3</b> unique values
	5.1	3.5	1.4	0.2	Iris-setosa
	4.9	3.0	1.4	0.2	Iris-setosa
	4.7	3.2	1.3	0.2	Iris-setosa
	4.6	3.1	1.5	0.2	Iris-setosa
	5.0	3.6	1.4	0.2	Iris-setosa
	5.4	3.9	1.7	0.4	Iris-setosa
	4.6	3.4	1.4	0.3	Iris-setosa
	5.0	3.4	1.5	0.2	Iris-setosa
	4.4	2.9	1.4	0.2	Iris-setosa
	4.9	3.1	1.5	0.1	Iris-setosa
	-	-	-	-	-

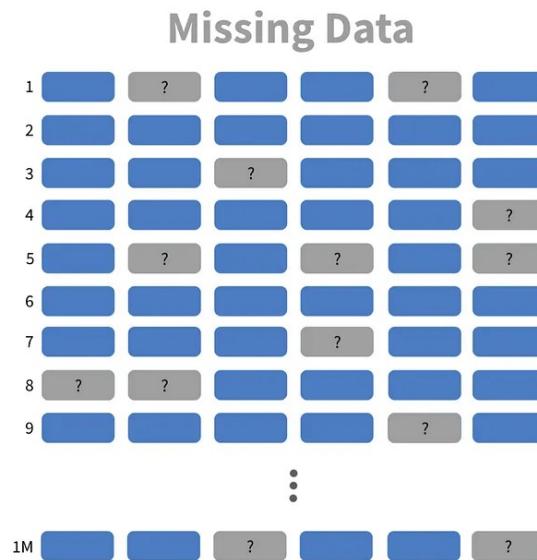
# Iris dataset

<https://www.kaggle.com/datasets/uciml/iris>

	# SepalLengthCm	# SepalWidthCm	# PetalLengthCm	# PetalWidthCm	Species
	Length of the sepal (in cm)	Width of the sepal (in cm)	Length of the petal (in cm)	Width of the petal (in cm)	Species name
	Statistic	Value			
0	Mean (First Moment)	3.758000			
1	Variance (Second Moment)	3.095503			
2	Skewness (Third Moment)	-0.274884			
3	Kurtosis (Fourth Moment)	-1.402103			
4	Q1 (First Quartile, 25%)	1.600000			
	4.7	3.2			
	4.6	3.1			
	5.0	3.6			
	5.4	3.9			
	4.6	3.4			
	5.0	3.4			
	4.4	2.9			
	4.9	3.1			
	-	-			

# ‘Correct’ a dataset

- Missing data are common in practical problems!
- Several models do not work with missing information
- Typically, we prefer not to throw away sample, instead we prefer to ‘impute’ data



# ‘Correct’ a dataset

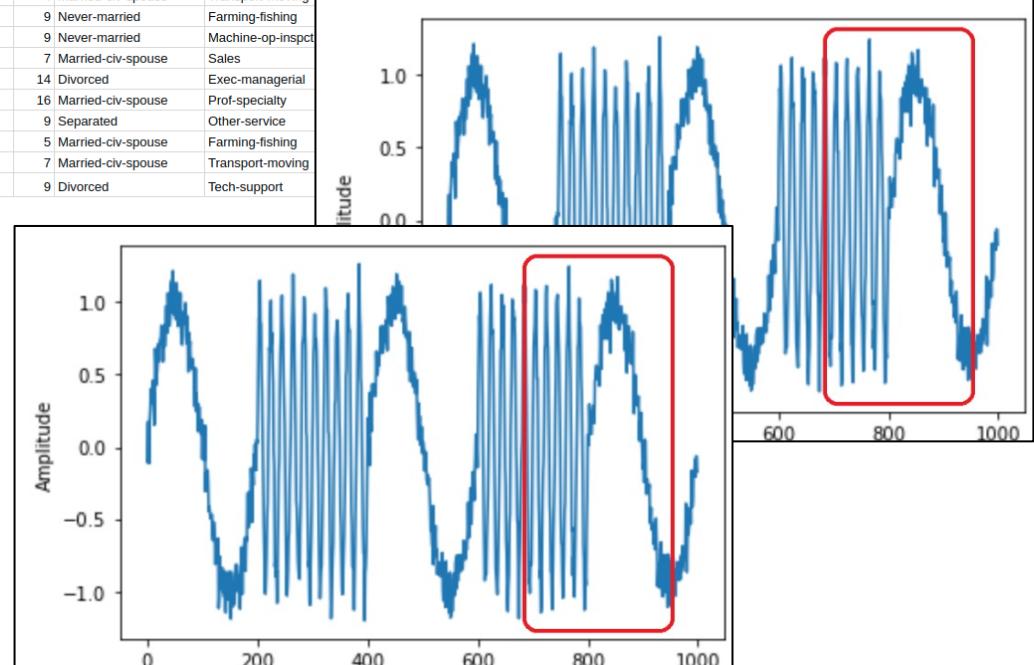
- Missing data are common in practical problems!
- Several models do not work with missing information
- Typically, we prefer not to throw away sample, instead we prefer to ‘impute’ data
- Mean and median of a variable are typical choices



# Enhance a dataset: feature engineering

- As said, data do not always present themselves in an easy tabular form
- We may use statistical moments/quantities (but also rule-based) for **feature engineering**
- Feature engineering is the process of creating, selecting and transforming ‘features’ (variables)

39	State-gov	77516	Bachelors	13	Never-married	Adm-clerical	Not-in-family	White	Male	2174	0	40	United-States	<=50K
50	Self-emp-not-inc	83311	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	13	United-States	<=50K
38	Private	215646	HS-grad	9	Divorced	Handlers-cleaners	Not-in-family	White	Male	0	0	40	United-States	<=50K
53	Private	234721	11th	7	Married-civ-spouse	Handlers-cleaners	Husband	Black	Male	0	0	40	United-States	<=50K
28	Private	338409	Bachelors	13	Married-civ-spouse	Prof-specialty	Wife	Black	Female	0	0	40	Cuba	<=50K
37	Private	284582	Masters	14	Married-civ-spouse	Exec-managerial	Wife	White	Female	0	0	40	United-States	<=50K
49	Private	160187	9th	5	Married-spouse-absent	Other-service	Not-in-family	Black	Female	0	0	16	Jamaica	<=50K
52	Self-emp-not-inc	209642	HS-grad	9	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	45	United-States	>50K
31	Private	45781	Masters	14	Never-married	Prof-specialty	Not-in-family	White	Female	14084	0	50	United-States	>50K
42	Private	159449	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	5178	0	40	United-States	>50K
37	Private	280464	Some-college	10	Married-civ-spouse	Exec-managerial	Husband	Black	Male	0	0	80	United-States	>50K
30	State-gov	141297	Bachelors	13	Married-civ-spouse	Prof-specialty	Husband	Asian-Pac-Islander	Male	0	0	40	India	>50K
23	Private	122272	Bachelors	13	Never-married	Adm-clerical	Own-child	White	Female	0	0	30	United-States	<=50K
32	Private	205019	Assoc-acdm	12	Never-married	Sales	Not-in-family	Black	Male	0	0	50	United-States	<=50K
40	Private	121772	Assoc-voc	11	Married-civ-spouse	Craft-repair	Husband	Asian-Pac-Islander	Male	0	0	40	?	>50K
34	Private	245487	7th-8th	4	Married-civ-spouse	Transport-moving	Husband	Amer-Indian-Eskimo	Male	0	0	45	Mexico	<=50K
25	Self-emp-not-inc	176756	HS-grad	9	Never-married	Farming-fishing								
32	Private	186824	HS-grad	9	Never-married	Machine-op-insct								
38	Private	28887	11th	7	Married-civ-spouse	Sales								
43	Self-emp-not-inc	292175	Masters	14	Divorced	Exec-managerial								
40	Private	193524	Doctorate	16	Married-civ-spouse	Prof-specialty								
54	Private	302146	HS-grad	9	Separated	Other-service								
35	Federal-gov	76845	9th	5	Married-civ-spouse	Farming-fishing								
43	Private	117037	11th	7	Married-civ-spouse	Transport-moving								
59	Private	109015	HS-grad	9	Divorced	Tech-support								



# Reduce a dataset!

- If not informative for the task, variables should be removed for efficiency and for better ‘engineering’ of a productive solutions
- This is typically not known a priori, and it should be done after/during modelling

	Feature_1	Feature_2	Constant_Var
1	54.9671415301123 3	23.636499344142 013	100
2	48.617356988288 15	23.668090197068 675	100
3	56.476885381006 92	26.084844859190 753	100
4	65.230298564080 26	30.495128632644 757	100
5	47.658466252766 644	28.638900372842 315	100
6	47.658630430508 2	25.824582803960 837	100
7	65.7921281550739 1	32.237057894447 59	100
8	57.6743472915290 9	22.789877213040 835	100
9	45.305256140650 48	25.842892970704 362	100
10	55.425600435859 65	27.327236865873 836	100

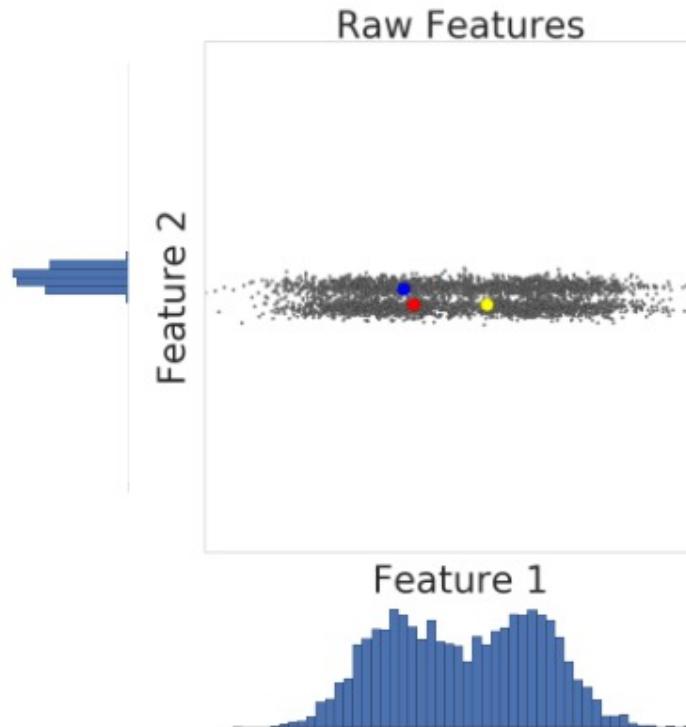
# Reduce a dataset!

- If not informative for the task, variables should be removed for efficiency and for better ‘engineering’ of a productive solutions
- This is typically not known a priori, and it should be done after/during modelling
- However, if a variable is constant (variance = 0), we should get rid of it!

	Feature_1	Feature_2	Constant_Var
1	54.9671415301123 3	23.636499344142 013	100
2	48.617356988288 15	23.668090197068 675	100
3	56.476885381006 92	26.084844859190 753	100
4	65.230298564080 26	30.495128632644 757	100
5	47.658466252766 644	28.638900372842 315	100
6	47.658630430508 2	25.824582803960 837	100
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9	45.305256140650 48	25.842892970704 362	100
10	55.425600435859 65	27.327236865873 836	100

# Optimize a dataset for modelling: data normalization

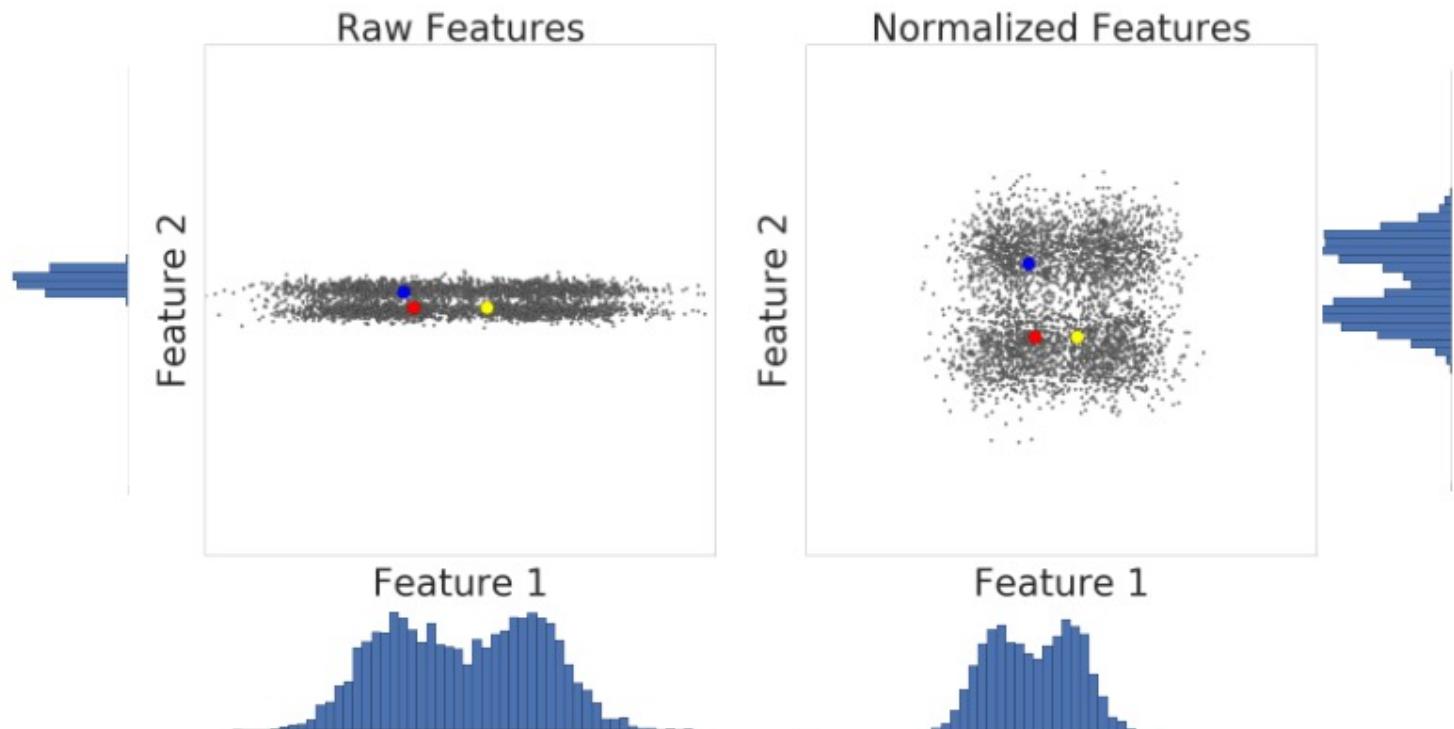
- A-priori, variables can be equally important in a ML task
- However, variable have different range values and one can 'dominate' the others
- Normalization can be of great help and it can speed up processing



# Optimize a dataset for modelling: data normalization

- Z-score normalization (standardization) transforms each data (variables) to have a mean of 0 and a standard deviation of 1

$$Z = \frac{X - \mu}{\sigma}$$



# Optimize a dataset for modelling: data normalization

X	Y
10	200
12	220
14	250
16	260
18	280

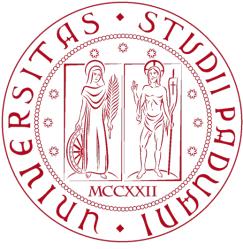


	X	Y
0	-1.264911	-1.315071
1	-0.632456	-0.688847
2	0.000000	0.250490
3	0.632456	0.563602
4	1.264911	1.189826

# Optimize a dataset for modelling: data normalization

Pay attention: data normalization is a task that can make you save a lot of time during building of a model, but it is typically a forgotten step





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# Machine Learning

## 2024/2025

**AMCO**  
ARTIFICIAL INTELLIGENCE, MACHINE  
LEARNING AND CONTROL RESEARCH GROUP

# Thank you!

Gian Antonio Susto

