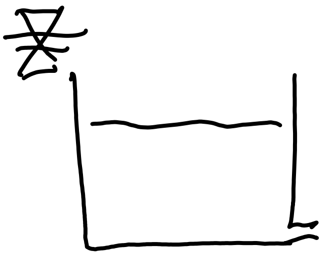
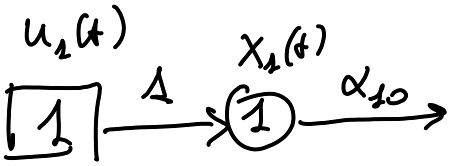
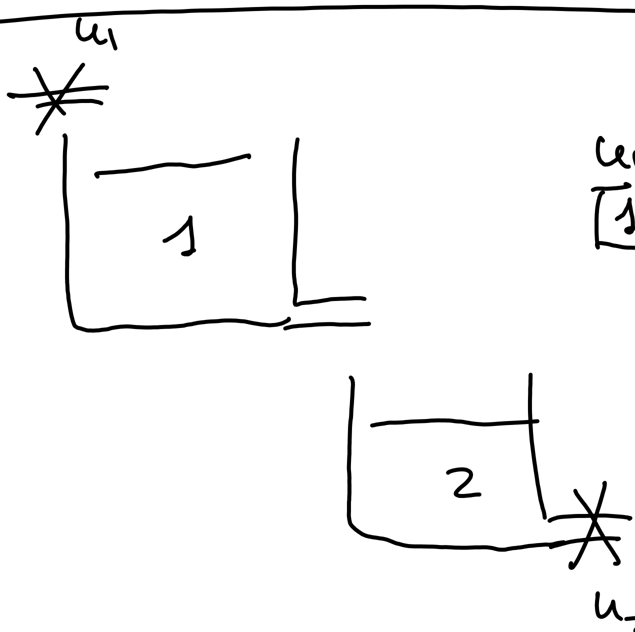
Terza lezione

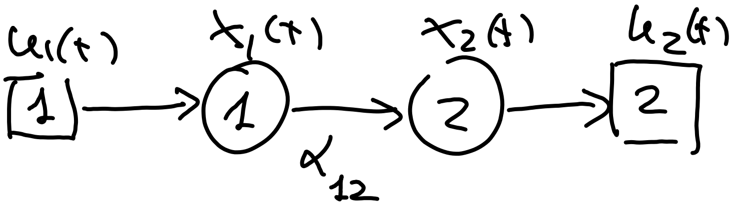
Esempio della rete di serbatoi





$$\dot{x}_1(t) = +u_1(t) - \alpha_{10}\, x_1(t)$$





$u_1(t) = 0$

$$\dot{x}_1(t) = +u_1(t) - \alpha_{12}\, x_1(t) \qquad x_1(t) = x_1(0)e^{-\alpha_{12}t}$$

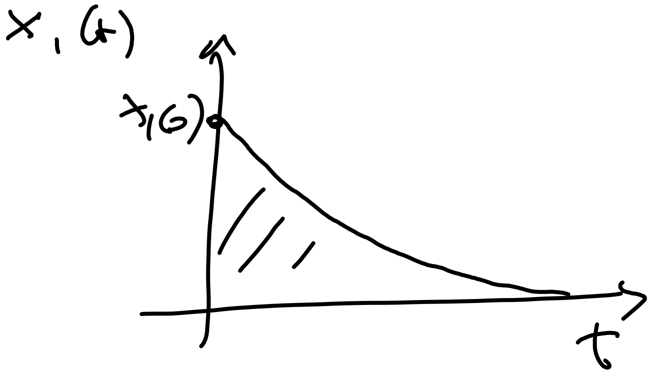
$$\dot{x}_2(t) = +\alpha_{12}\, x_1(t) - u_2(t)$$

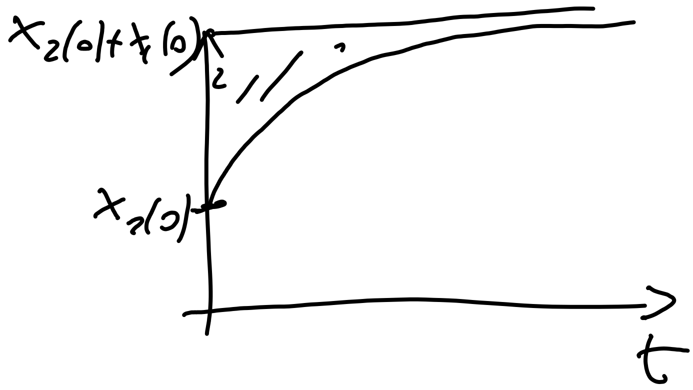
$u_1(t) = 0 \qquad u_2(t) = 0$

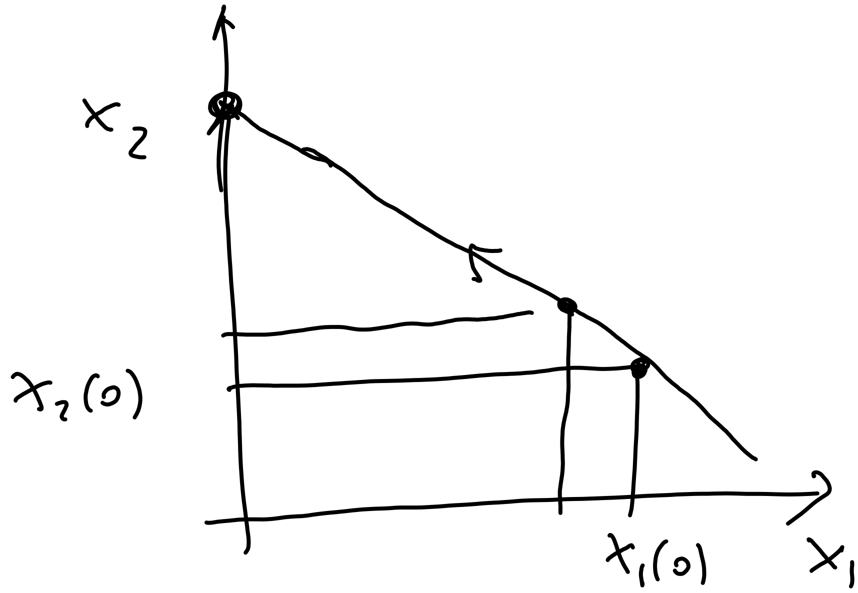
$$y(t) = x_1(t) + x_2(t)$$

$$\dot{y}(t) = \dot{x}_1(t) + \dot{x}_2(t) = \underline{u_1(t) - u_2(t)} = 0$$

$$y(t) = y(0)$$







$$\begin{cases} \dot{x}_1(t) = +u_1(t) - \alpha_{12}\, x_1(t) \\ \dot{x}_2(t) = +\alpha_{12}\, x_1(t) - u_2(t) \end{cases}$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad \dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\alpha_{12} & 0 \\ \alpha_{12} & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{B} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$[1 \quad 1]\, A = [0 \mid 0]$$

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = x_1(t) + x_2(t) = [1 \;\; 1]\, x(t)$$

$$\dot{y}(t) = [1 \;\; 1]\, \dot{x}(t)$$

$$= [1 \;\; 1]\,(A\,x(t) + B\,u(t))$$

$$= [0 \;\; 0]\, x(t) + [1 \;\; 1]\, B\, u(t)$$

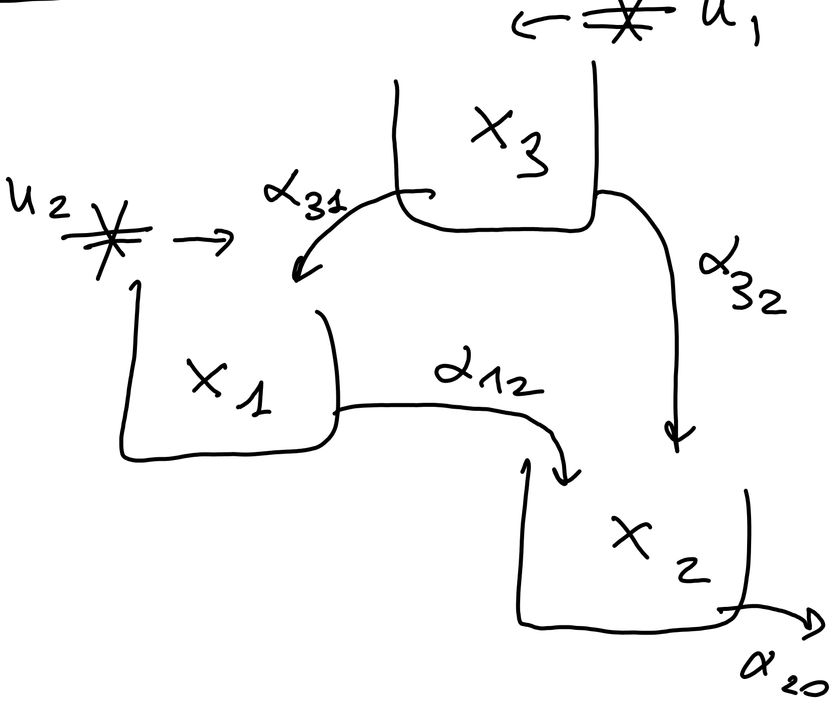
$$= [1 \;\; 1]\, B\, u(t)$$

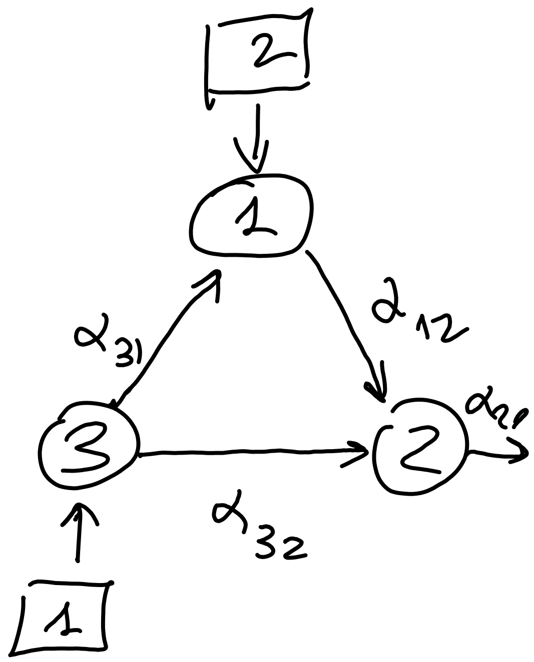
$u(t) = 0$ $\qquad$ $\dot{y}(t) = 0$

$\boxed{[1 \;\; 1]\, A = 0}$ $\qquad$ $\lambda = 0$

$A\underline{v} = \lambda \underline{v}$ $\qquad$ $w^T A = \lambda w^T$

autovettore $\qquad$ autovalore





$$
\begin{cases}
\dot{x}_1(t) = u_2(t) - \alpha_{12}\, x_1(t) + \alpha_{31}\, x_3(t) \\
\dot{x}_2(t) = \alpha_{12}\, x_1(t) + \alpha_{32}\, x_3(t) - \alpha_{20}\, x_2(t) \\
\dot{x}_3(t) = u_1(t) - \alpha_{31}\, x_3(t) - \alpha_{32}\, x_3(t)
\end{cases}
$$

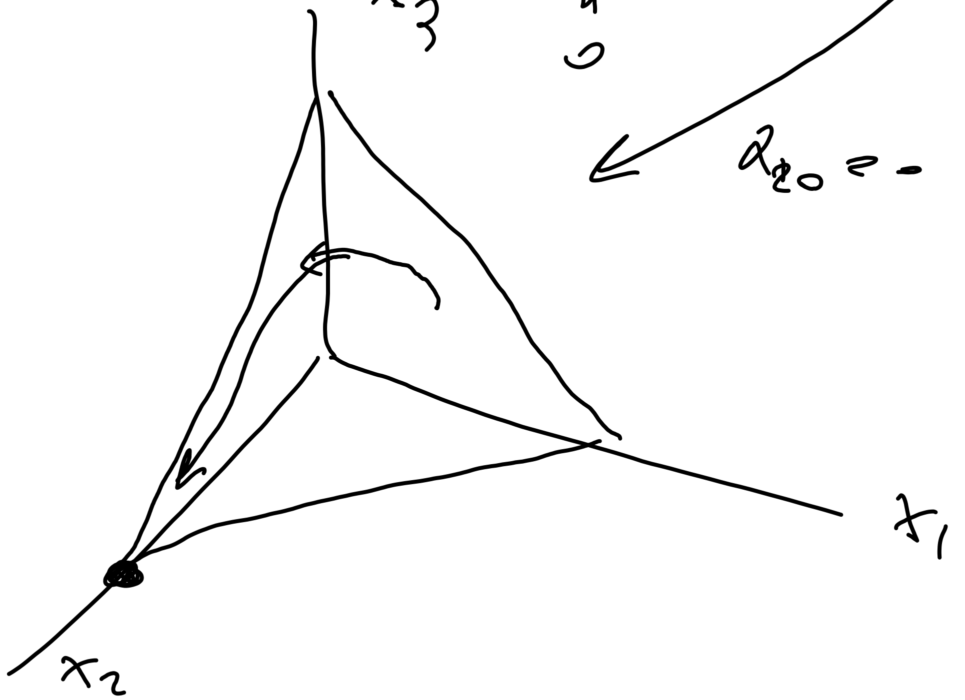
$$
\begin{array}{cc}
 & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\
A = & \left[\begin{array}{c|c|c} -\alpha_{12} & 0 & \alpha_{31} \\ \hline \alpha_{12} & -\alpha_{20} & \alpha_{32} \\ \hline 0 & 0 & -\alpha_{31}-\alpha_{32} \end{array}\right]
\end{array}
$$

$u_2(t) = 0$
$u_1(t) = 0$

$\alpha_{20} = 0$

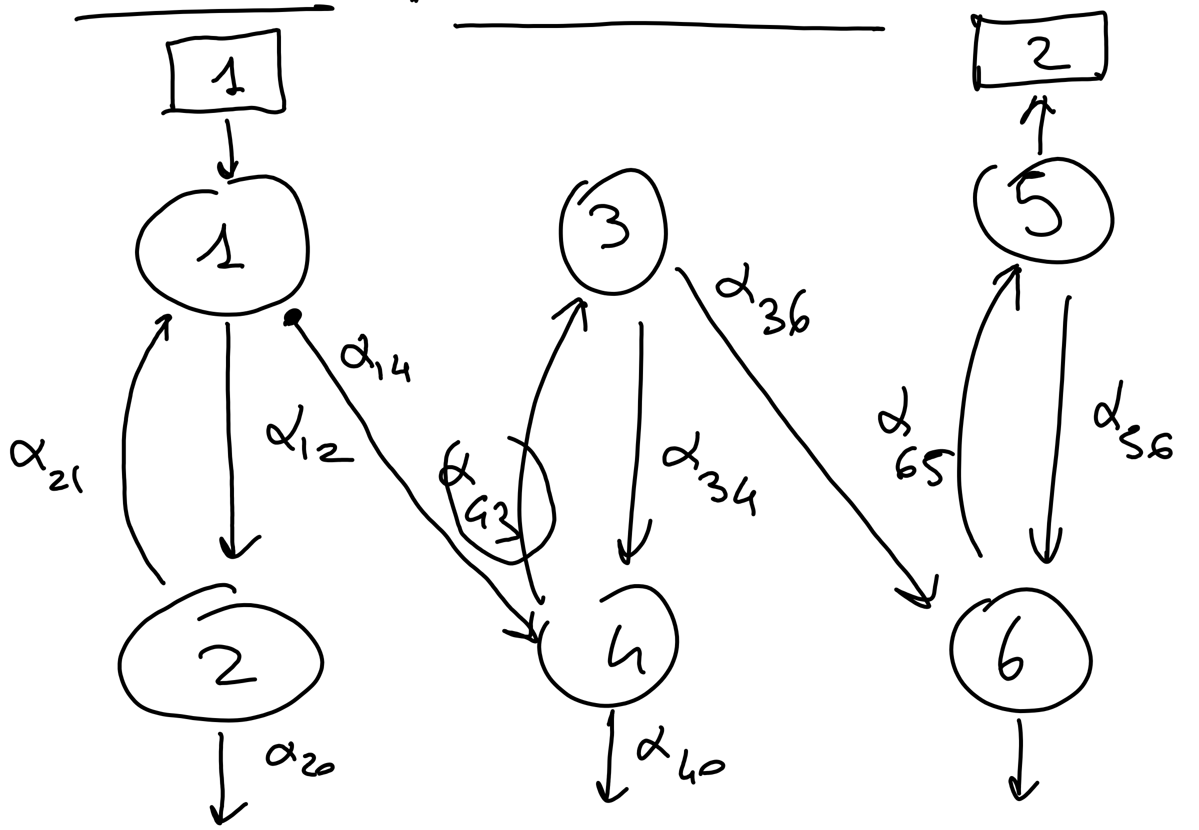
$$[1 \;\; 1 \;\; 1]\, A = [\,0 \mid -\alpha_{20} \mid 0\,]$$

$= 0$



$\alpha_{20} = 0$

Esempio: Parco auto



1 Auto con km $\in [0, 10000 \text{ km}]$

2 Auto " " "
in riparazione

3 Auto con km $\in [10.000, 20.000]$

4

5 Auto con km $\in [20000 + \infty)$

6

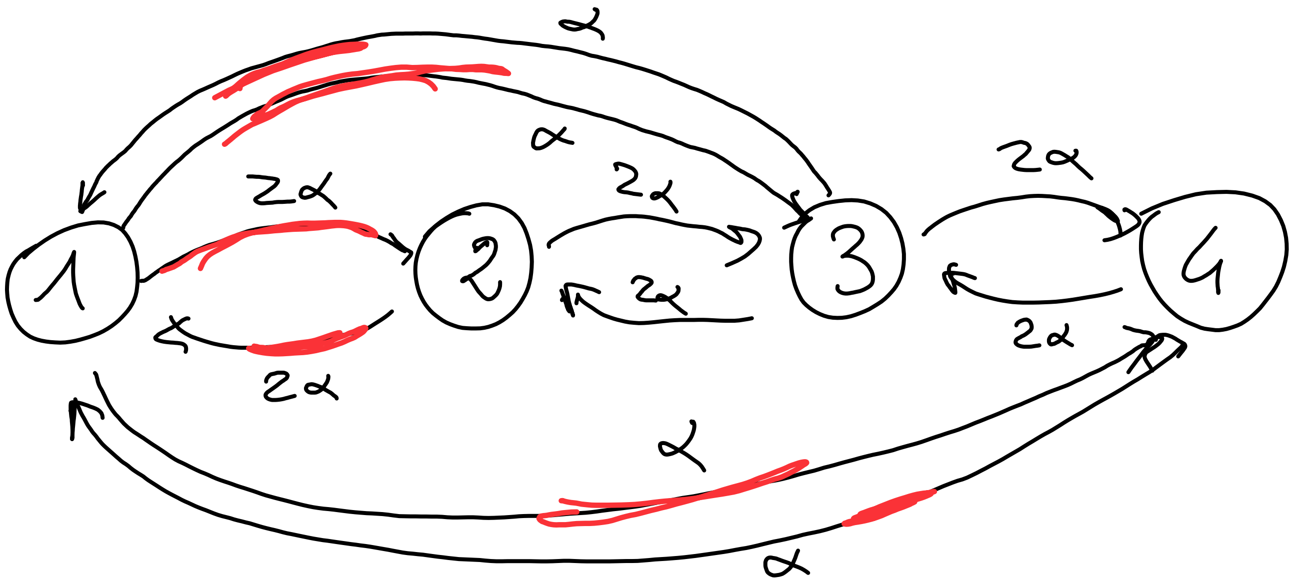
$-\alpha_{14}$

$$A = \begin{array}{c|cccccc} & x_1 & & & & & \\ \hline \dot{x}_1 \to & -\alpha_{12} & \alpha_{21} & 0 & 0 & 0 & 0 \\ \dot{x}_2 \to & \alpha_{12} & & 0 & 0 & 0 & 0 \\ \dot{x}_3 \to & & & & \alpha_{43} & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array}$$

$-\alpha_{21} - \alpha_{20}$ $\qquad$ $-\alpha_{34} - \alpha_{36}$

$$B = \begin{array}{|c|c|} 1 & 0 \\ \hline & \\ & \\ & \end{array}$$

Esempio Rete stradale



$$\dot{x}_1(t) = -4\alpha x_1(t) + 2\alpha x_2(t) + \alpha x_3(t) + \alpha x_4(t)$$

$$\dot{x}_2(t) = -4\alpha x_2(t) + 2\alpha x_1(t) + 2\alpha x_3(t)$$

$$\dot{x}_3(t) = -5\alpha x_3(t) + 2\alpha x_2(t) + 2\alpha x_4(t) + \alpha x_1(t)$$

$$\dot{x}_4(t) = -3\alpha x_4(t) + 2\alpha x_3(t) + \alpha x_1(t)$$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} \dot{x}_1 \\ \\ \\ \\ \end{array} & \underline{A} = \alpha \begin{bmatrix} -4 & 2 & 1 & 1 \\ 2 & -4 & 2 & 0 \\ 1 & 2 & -5 & 2 \\ 1 & 0 & 2 & -3 \end{bmatrix} \end{array}$$

$$A = A^T$$

$$\mathbb{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \qquad \mathbb{1}^T A = 0$$

$$A^T \mathbb{1} = 0$$

$$A \mathbb{1} = 0$$

Equilibrio = soluzione costante del sistema dinamico

$x(t)$ equilibrio se $x(t) = x(0)$ $\forall t$

$$\begin{cases} \dot{x} = Ax + Bu \\ x(t) \overset{= \bar{x} \quad \forall t}{\text{costante}} \end{cases}$$

$u(t) =$ costante

$u(t) = \bar{u}$ $\forall t$

$$0 = A \underset{\bar{x}}{\underset{\shortparallel}{x(t)}} + B \underset{\bar{u}}{\underset{\shortparallel}{u(t)}}$$

$$\boxed{A\bar{x} + B\bar{u} = 0}$$

$$\bar{x} = -A^{-1} B \bar{u}$$

Esempio

$$A \mathbb{1} = 0$$

Condizioni iniziali soluzione

$$x(0) = \beta \mathbb{1} = \begin{bmatrix} \beta \\ \vdots \\ \beta \end{bmatrix}$$

$$x(t) = \beta \mathbb{1}$$

$\dot{x} = 0$ $\qquad$ $\dot{x} \stackrel{?}{=} Ax$

$$A\beta\mathbb{1} = \beta A\mathbb{1} = 0$$

$\mathbb{1}^T A = 0$ ← conservazione

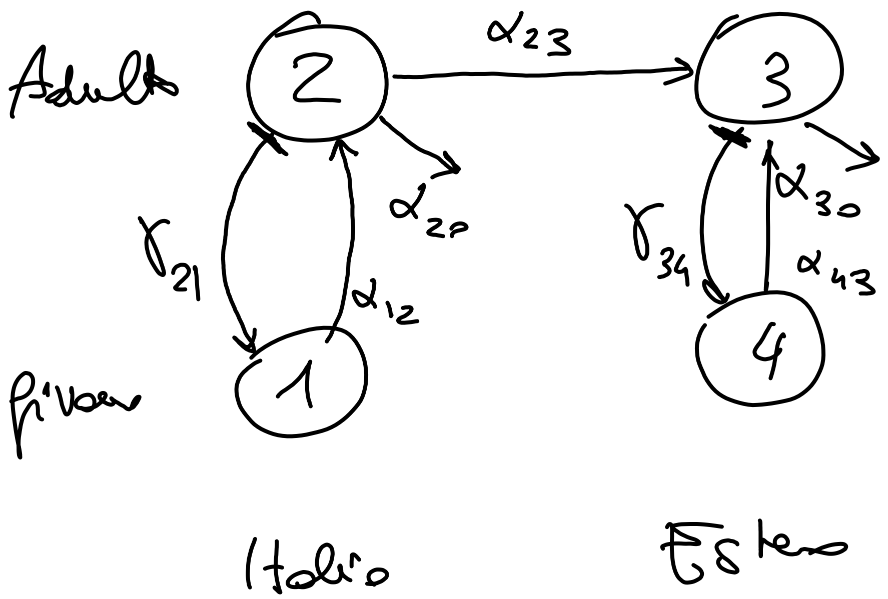
$A = A^T$ ← simmetria dei flussi

$\dot{x} = Ax$ $\qquad$ $x(t) = \bar{x}$

$A\bar{x} = 0$ equazione dell'equilibrio

$\mathbb{1}^T A = 0$ $\qquad$ $A\mathbb{1} \neq 0$

Esempio E un giovane



(2) Popolazione in Italia adulto

(1) Popolazione in Italia giovane

(3)
(4) estero

$$\dot{x}_1 = -\alpha_{12} x_1 + \gamma_{21} x_2$$

$$\dot{x}_2 = \alpha_{12} x_1 - \alpha_{20} x_2 - \underline{\alpha_{23} x_2}$$

$$\dot{x}_3 = +\underline{\alpha_{23} x_2} - \alpha_{30} x_3 + \alpha_{43} x_4$$

$$\dot{x}_4 = -\alpha_{43} x_4 + \gamma_{34} x_3$$