MODELLISTICA

1) Modelli di trasferimento di risorse
2) Modelli di transizione tra stati
3) Modelli di influenza
4) Modelli fisici

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1) Modelli di trasferimento di risorse

Tempo continuo

Modelli di flusso

Tempo discreto

Modelli di decisione

Segnali $x_1(t)$ $x_2(t)$ .. $x_n(t)$

parametri

$X_i(t)$ quantità nel compartimento i $\quad t$ tempo

$$\frac{d}{dt} x_i(t) = f_i(t) \qquad f_i = 0 \quad x_i(t) = x_i(0)$$

$$= \underbrace{f_i^{in}(t)}_{} - \underbrace{f_i^{out}(t)}_{}$$

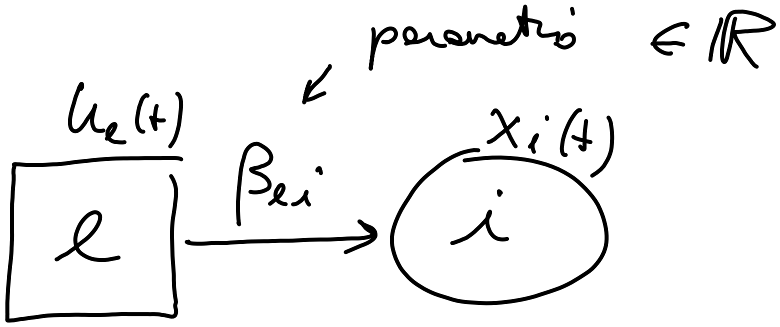
Altri segnali $u_1(t)$ ... $u_p(t)$

Variabili indipendenti (ingressi) che sono inserite dall'esterno

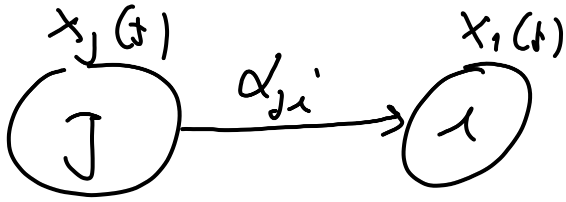
Flussi entranti

1) $f_i^{in}(t) = \dots + \underbrace{\beta_{ei}\, u_e(t)} + \dots$

flussi dall'esterno che dipendono dal fattore esterno indipendente $u_e(t)$

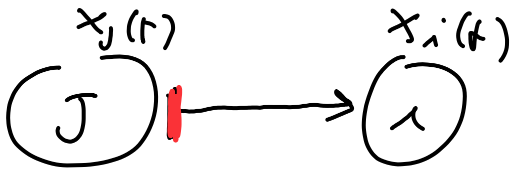


2) $f_i^{(in)}(t) = \dots + \alpha_{ji}\, x_j(t) + \dots$

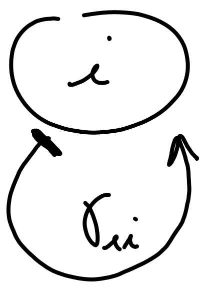


Trasferimento interni tra compartimenti senza generazione / perdite complessive

3) $f_i^{in}(t) = \dots + \gamma_{ji}\, x_j(t) + \dots$



Esempio



$$f_i^{in}(t) = \gamma_{ii}\, x_i(t) + \sum_{j \neq i} \gamma_{ji}\, x_j(t) + \sum_{\substack{j \neq i \\ j=1}}^{n} \alpha_{ji}\, x_j(t) + \sum_e \beta_{ei}\, u_e(t)$$

$\alpha_{ji} = 0$ $\quad \cancel{\alpha_{ji}\, x_j(t)}$

flussi uscenti

1) $f_i^{out}(t) = \dots + \beta_{ih} u_h(t) + \dots$

$$\begin{array}{ccc} & & u_h(t) \\ i & \xrightarrow{\beta_{ih}} & \boxed{h} \end{array}$$

2) $f_i^{out}(t) = \dots + \alpha_{ij} x_i(t) + \dots$

$$\begin{array}{ccc} x_i(t) & & x_j(t) \\ i & \xrightarrow{\alpha_{ij}} & j \end{array}$$

Osserviamo

$$f_j^{in}(t) = \dots + \alpha_{ij} x_i(t) + \dots$$

$$i \xrightarrow{\alpha_{i0}}$$

$$f_i^{out}(t) = \dots + \alpha_{i0} x_i(t) + \dots$$

$$f_i^{out}(t) = \sum_{h=1}^{p} \beta_{ih} u_h(t) + \sum_{\substack{j \neq i \\ j=0}}^{n} \alpha_{ij} x_i(t)$$

$$\dot{x}_i(t) = \underbrace{f_i^{in}(t)} - f_i^{out}(t)$$

$$= \gamma_{ii} x_i + \sum_{j \neq i} \alpha_{ji} x_j + \sum_{j \neq i} \gamma_{ji} x_j + \sum_{l} \beta_{li} u_l - \left(\sum_{j \neq i} \alpha_{ij} x_i + \sum_{h} \beta_{hi} u_h \right)$$

$$a_{ii} = \overbrace{\left(\gamma_{ii} - \sum_{j \neq i} \alpha_{ij}\right)} x_i + \sum_{j \neq i} \overbrace{\left(\alpha_{ji} + \gamma_{ji}\right)}^{a_{ij}} x_j$$

$$+ \sum \underbrace{\left(\beta_{li} - \beta_{il}\right)}_{b_{il}} u_l$$

$$\dot{x}_1(t) = a_{11}x_1(t) + \sum_{j\neq i} a_{1j}x_j(t) + \sum b_{1l}u_l(t)$$

Definisco

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \qquad u(t) = \begin{bmatrix} u_1(t) \\ \\ u_p(t) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot & a_{1n} \\ \\ a_{n1} & \cdot & \cdot & a_{nn} \end{bmatrix} \qquad B = \begin{bmatrix} \cdot & \cdot & \cdot \\ & b_{ij} & \\ \cdot & \cdot & \cdot \end{bmatrix}$$

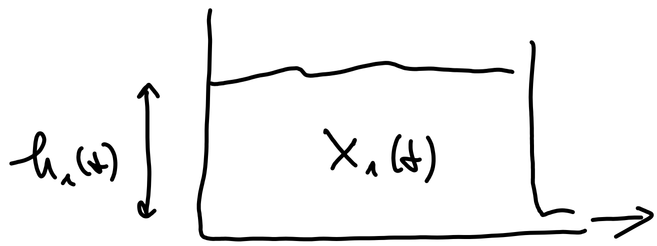
$$\dot{x}(t) \overset{def}{=} \begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}$$

$$\boxed{\dot{x}(t) = Ax(t) + Bu(t)}$$ Sistema lineare

Esempio rete di serbatoi

Risorsa = fluido

Quantità = Volume $\quad x_i(t)$ Volume nel serbatoio i



$x_i(t) = S\, h_i(t)$

$$\begin{cases} \dot{x}_1(t) = -f_1^{out}(t) \\ f_1^{out}(t) = \alpha_{10}\, x_1(t) \end{cases}$$

(*) $$\boxed{\dot{x}_1(t) = -\alpha_{10}\, x_1(t)}$$

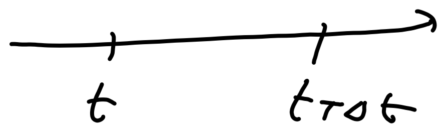
$\dot{x} = Ax + Bu$

$A = [-\alpha_{10}]$

<u>Stima del parametro α_{10}</u>

misuro

$x_1(t) \qquad x_1(t+\Delta t)$



$$\alpha(\Delta t) = \frac{x_1(t) - x_1(t+\Delta t)}{x_1(t)}$$

percentuale di liquido persa in Δt

Confrontiamo con l'evoluzione di $x_1(t)$

Soluzione dell'equazione (*)

$$\underline{x_1(t) = x_1(0)\, e^{-\alpha_{10} t}}$$

Verifichiamo sostituendo

$\frac{d}{dt} e^{at} = a\, e^{at}$

$$\dot{x}_1(t) = x_1(0)(-\alpha_{10})\, e^{-\alpha_{10} t}$$

|| sono uguali

$$-\alpha_{10}\, x_1(t) = (-\alpha_{10})\, x_1(0)\, e^{-\alpha_{10} t}$$

$$\alpha(\Delta t) = \frac{x_1(0)e^{-\alpha_{10}t} - x_1(0)e^{-\alpha_{10}(t+\Delta t)}}{x_1(0)e^{-\alpha_{10}t}}$$

$$= \frac{\cancel{x_1(0)e^{-\alpha_{10}t}} - \cancel{x_1(0)e^{-\alpha_{10}t}}\, e^{-\alpha_{10}\Delta t}}{\cancel{x_1(0)e^{-\alpha_{10}t}}}$$

misto

$$\alpha(\Delta t) = 1 - e^{-\alpha_{10}\Delta t}$$

$$e^{-\alpha_{10}\Delta t} = 1 - \alpha(\Delta t)$$

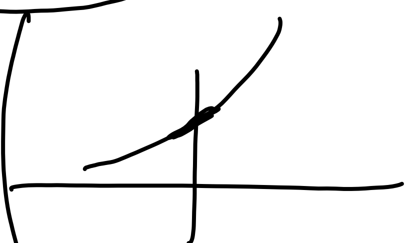
$$-\alpha_{10}\Delta t = \ln(1 - \alpha(\Delta t))$$

$$\alpha_{10} = -\frac{1}{\Delta t}\ln(1-\alpha(\Delta t)) = \frac{1}{\Delta t}\ln\frac{1}{1-\alpha(\Delta t)}$$

Δt piccolo $\rightarrow$ $-\alpha_{10}\Delta t$ piccolo

$$e^{a} \cong 1 + a$$

$$a = -\alpha_{10}\Delta t$$



$$\alpha(\Delta t) \simeq 1 - (1 - \alpha_{10}\Delta t)$$

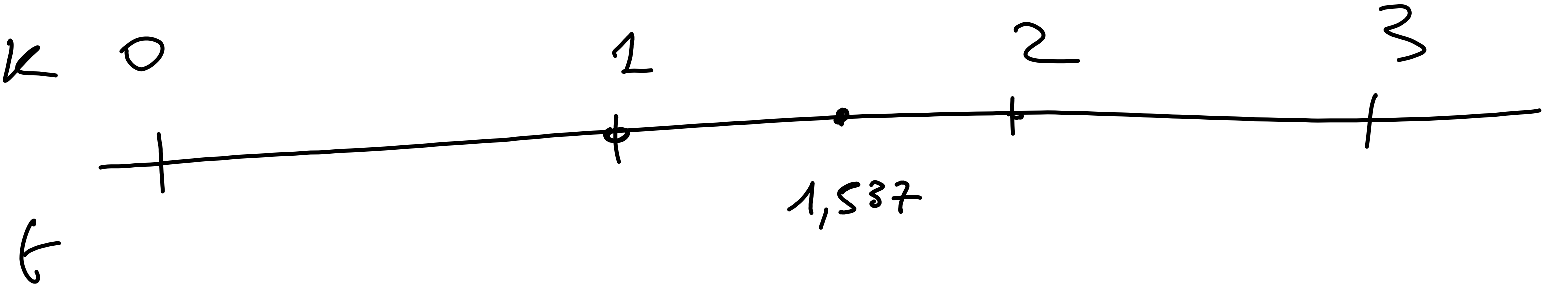
$$\alpha(\Delta t) \simeq \alpha_{10}\Delta t$$

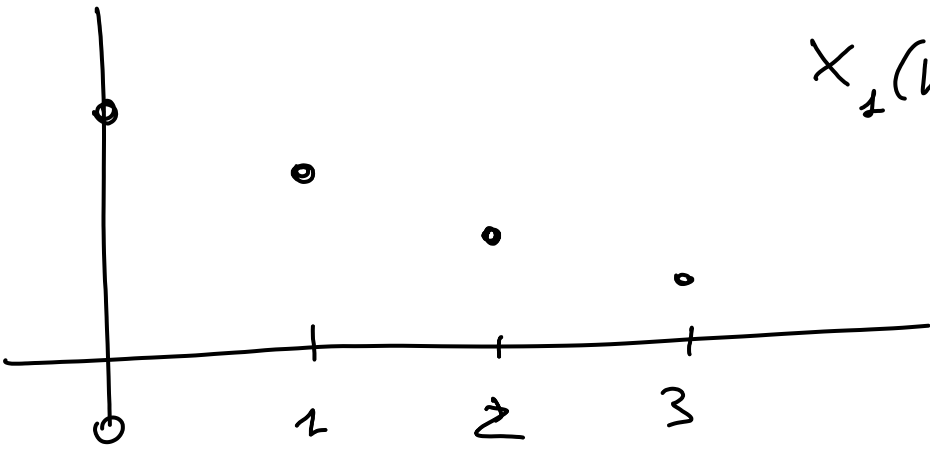
$$\alpha_{10} \simeq \frac{\alpha(\Delta t)}{\Delta t}$$

Emigrazione

Italia

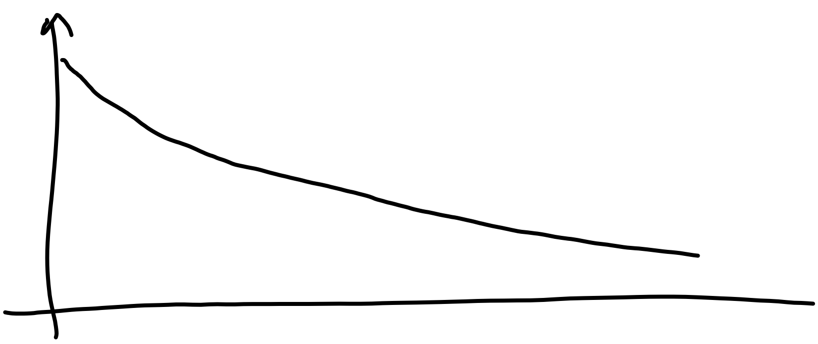
$$X_1(k+1) = X_1(k) - \frac{1}{1000} X_1(k)$$





$$X_1(k) = \left(1 - \frac{1}{1000}\right)^k X_1(0)$$

$$\boxed{\dot{X}_1(t) = -\alpha_{10} X_1(t)} \rightarrow X_1(t) = e^{-\alpha_{10} t} X_1(0)$$



$\Delta t = 1$

$\alpha(\Delta t)$

$X_1(k)$ $X_1(k+1)$

$$\frac{X_1(k) - X_1(k+1)}{X_1(k)} = \frac{1}{1000}$$

$$\alpha_{10} = \frac{1}{\Delta t} \ln \frac{1}{1 - \frac{1}{1000}}$$

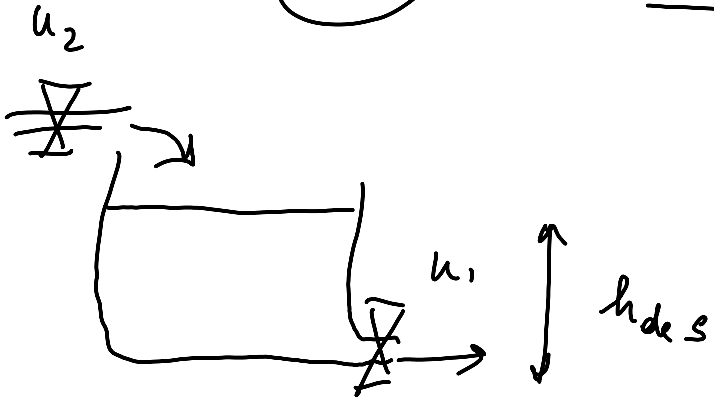
$$\alpha_{10} \simeq \frac{\alpha(\Delta t)}{\Delta t}$$

$$\alpha_{10} \simeq \frac{1/1000}{1}$$

Esempio

2) $\dot{x}_1(t) = -u_1(t)$

$$\underset{x_1(t)}{\textcircled{1}} \xrightarrow{1} \underset{u_1(t)}{\boxed{1}}$$



3)

$\dot{x}_1(t) = u_2(t) - u_1(t)$

$A = [0]$ $\quad \begin{vmatrix} u_1 \\ u_2 \end{vmatrix}$

$B = [-1 \mid 1]$

$$\boxed{2} \xrightarrow{1} \textcircled{2} \xrightarrow{1} \boxed{1}$$

$\dot{x}_1(t) = u(t)$ $\qquad u(t) \overset{def}{=} u_2(t) - u_1(t)$

$\dot{x}_1(t) = u(t)$

$h(t)$ misurabile

Legge di controllo

$$u(t) = k(h_{des} - h(t))$$

$$\dot{x}_1(t) = k(h_{des} - h(t)) \qquad \boxed{h(t)\,S = x_1(t)}$$

$$\dot{x}_1(t) = k\left(h_{des} - \frac{x_1(t)}{S}\right) = \frac{k}{S}(S h_{des} - x_1(t))$$

Fissiamo $\tilde{x}_1(t) \overset{def}{=} x_1(t) - S h_{des}$

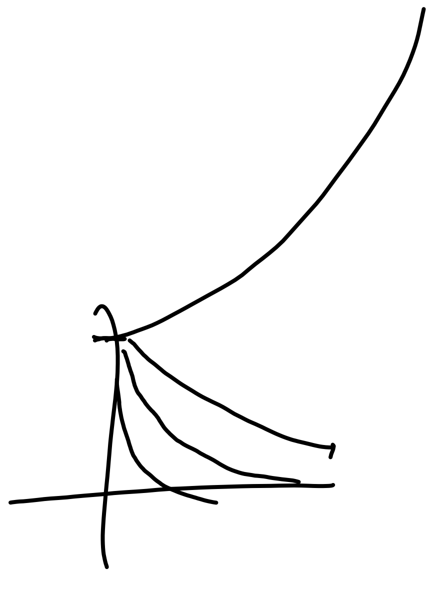
$$\dot{\tilde{x}}_1(t) = \dot{x}_1(t) - \cancel{S \dot{h}_{des}} = \frac{k}{S}(-\tilde{x}_1(t))$$

$$\boxed{\dot{\tilde{x}}_1(t) = -\frac{k}{S}\,\tilde{x}_1(t)}$$

$- \alpha_{10}$

Soluzione

$$\tilde{x}_1(t) = \tilde{x}_1(0)\, e^{-\frac{k}{S}t}$$



Se $\underline{k > 0}$

Realizzazione

Se $u(t) > 0$ flusso entrante $u_2(t) = u(t)$

$u_1(t) = 0$

Se $u(t) < 0$ flusso uscente $u_1(t) = -u(t)$

$u_2(t) = 0$