Tema 2

Es 1 $\quad f(x) = \arctan\left(\dfrac{x}{x-2}\right)$

$D = \{x \neq 2\} = (-\infty, 2) \cup (2, +\infty)$

f non è simmetrica né periodica

$\arctan\left(\dfrac{x}{x-2}\right) \geq 0 \iff \dfrac{x}{x-2} \geq 0$

		0		2	
x	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
	$+$		$-$		$+$

$x > 2$

$x \leq 0$

$f(x) \geq 0$ se $x > 2$ oppure $x \leq 0$

$\lim_{x \to 2^+} \arctan\left(\dfrac{x}{\underbrace{x-2}_{\to 0^+}}\right) = \arctan(+\infty) = \dfrac{\pi}{2}$

$\lim_{x \to 2^-} \arctan\left(\dfrac{x}{\underbrace{x-2}_{\to 0^-}}\right) = \arctan(-\infty) = -\dfrac{\pi}{2}$

$x = 2$ discontinuità di SALTO

$\lim_{x \to +\infty} \arctan\left(\dfrac{x}{x-2}\right) = \arctan 1 = \dfrac{\pi}{4} = \lim_{x \to -\infty} \arctan\left(\dfrac{x}{x-2}\right)$

$y = \dfrac{\pi}{4}$ è ASINT. ORIZZ. a $+\infty, -\infty$.

3) $f'(x) = \dfrac{1}{1+\left(\dfrac{x}{x-2}\right)^2} \cdot \dfrac{1\cdot(x-2) - x\cdot 1}{(x-2)^2} = \dfrac{1}{\dfrac{(x-2)^2 + x^2}{\cancel{(x-2)^2}}} \cdot \dfrac{-2}{\cancel{(x-2)^2}} = -\dfrac{2}{(x-2)^2 + x^2}$

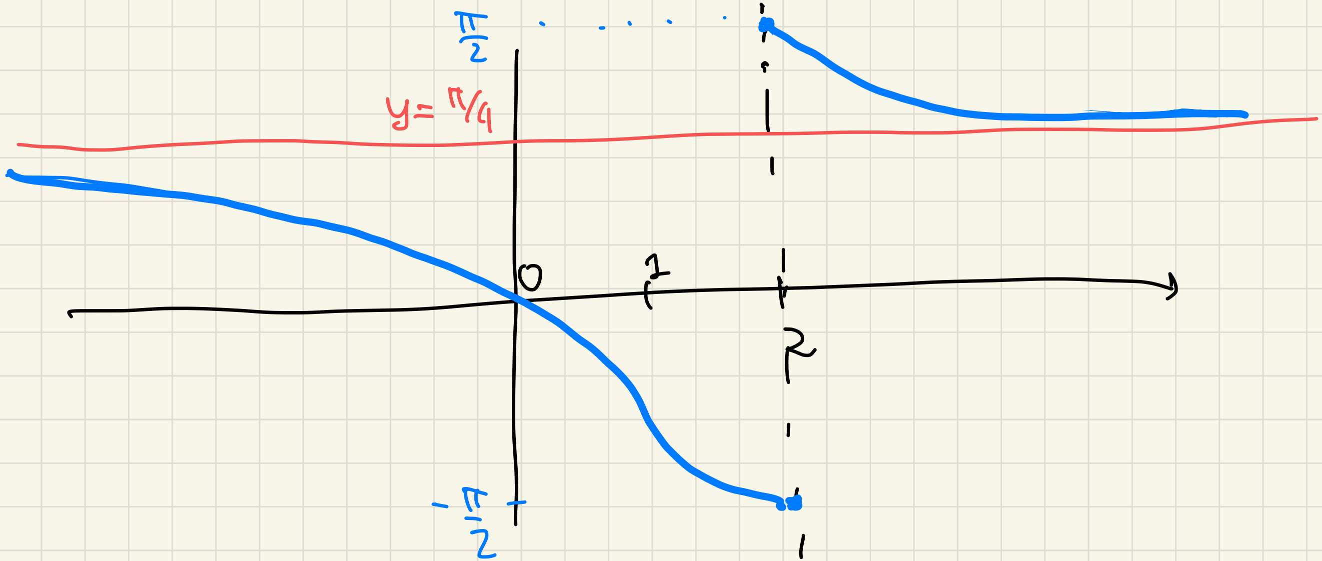
$f'(x) < 0 \quad \forall x \in D$ f è strettam. decrescente in $(-\infty, 2)$ e in $(2, +\infty)$

$\lim_{x \to 2^+} f'(x) = -\dfrac{2}{4} = -\dfrac{1}{2} = \lim_{x \to 2^-} f'(x)$.

fac. $f''(x) = \dfrac{+2 \cdot [2(x-2) + 2x]}{[(x-2)^2 + x^2]^2} = \dfrac{2[2x - 4 + 2x]}{((x-2)^2 + x^2)^2} = \dfrac{8(x-1)}{[(x-2)^2 + x^2]^2}$

$f''(x) > 0$ se $x > 1$ $(x \neq 2)$ $\rightarrow$ f convessa in $(1, 2) \cup (2, +\infty)$

$f''(x) < 0$ se $x < 1$ $\rightarrow$ f concava in $(-\infty, 1)$



Es 2

$$\log\left(1+\frac{1}{n^2}\right) = (\text{Taylor}) = \frac{1}{n^2} - \frac{1}{2}\left(\frac{1}{n^2}\right)^2 + o\left(\frac{1}{n^2}\right)$$

$$a_n = 3n^{a+1}\left[\cancel{\frac{1}{n^2}} - \cancel{\frac{1}{n^2}} + \frac{1}{2}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)\right] = 3\cdot\frac{n^{a+1}}{n^4}\left[\frac{1}{2} + o(1)\right]$$

1) $$\lim_n a_n = \begin{cases} \frac{3}{2} & \text{se } a+1=4 \quad a=3 \\ +\infty & \text{se } a+1>4 \quad a>3 \\ 0 & \text{se } a+1<4 \quad a<3 \end{cases}$$

2) $a_n \sim \frac{1}{n^{4-a-1}} = \frac{1}{n^{3-a}}$ per criterio del confronto asintotico la serie $\sum_n a_n$ converge se $3-a>1$ $(a<2)$

e diverge se $3-a \leq 1$ $(a \geq 2)$

Es 3) Procediamo per parti (2 volte) per trovare primitive

$$\int (x^2+1)\sin x \, dx = (x^2+1)\cdot(-\cos x) - \int 2x\cdot(-\cos x)\, dx =$$

$$= -(x^2+1)\cos x + 2\int x\cos x \, dx = -(x^2+1)\cos x + 2\left[x\sin x - \int 1\cdot \sin x \, dx\right]$$

$$= -(x^2+1)\cos x + 2x\sin x - 2\int \sin x \, dx =$$

$$= -(x^2+1)\cos x + 2x\sin x + 2\cos x + c$$

per il teorema fondam. del calcolo integrale

$$\int_0^{\pi} (x^2+1)\sin x\, dx = \left[-(x^2+1)\cos x + 2x\sin x + 2\cos x\right]_0^{\pi} =$$

$$= -(\pi^2+1)\cos\pi + 2\pi\sin\pi + 2\cos\pi - \left[-(0^2+1)\cos 0 + \cancel{2\cdot 0\sin 0} + 2\cos 0\right]$$

$$= -(\pi^2+1)\cdot(-1) + 0 + 2\cdot(-1) + 1 - 2 =$$

$$= +\pi^2+1-2+1-2 = \pi^2-2.$$

ES 4 $\quad f$ è continua e derivabile in $\mathbb{R}^2$

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = 3x^2-2y=0 \\ \dfrac{\partial f}{\partial y}(x,y) = -2x+2y=0 \end{cases} \rightarrow \begin{cases} 3x^2-2x=0 \\ x=y \end{cases} \begin{cases} x=0 \\ x=\dfrac{2}{3} \\ y=0 \\ y=\dfrac{2}{3} \end{cases} \rightarrow (0,0) \text{ e } \left(\frac{2}{3},\frac{2}{3}\right) \text{ sono}$$

i PUNTI CRITICI

$$\frac{\partial^2 f}{\partial x^2}(x,y) = 6x \qquad \frac{\partial^2 f}{\partial x \partial y}(x\,y) = -2 \qquad \frac{\partial^2 f}{\partial y^2}(x,y) = 2$$

$$Hf(0,0) = \begin{pmatrix} 0 & -2 \\ -2 & 2 \end{pmatrix}$$

$\det Hf(0,0) = -4 < 0$

$(0,0)$ pto di sella

$$Hf\left(\frac{2}{3},\frac{2}{3}\right) = \begin{pmatrix} 4 & -2 \\ -2 & 2 \end{pmatrix}$$

$\det Hf\left(\frac{2}{3}\,\frac{2}{3}\right) = 8-4 = 4 > 0$

$\operatorname{tr} Hf\left(\frac{2}{3},\frac{2}{3}\right) = 6 > 0$

$\left(\frac{2}{3},\frac{2}{3}\right)$ é pto di minimo locale.