Es.

$$a_n = n^{\alpha} \left[ \frac{1}{n} \operatorname{tg}\left(\frac{1}{n}\right) - \log\left(1+\frac{1}{n^2}\right) \right]$$

1) determinare al variare di $\alpha$

$$\lim_{n \to +\infty} a_n$$

2) determinare al variare di $\alpha$ il carattere della serie

$$\sum_{n=1}^{\infty} |a_n|$$

utilizzo polinomi

$x \to 0$

$$\operatorname{tg} x = x + \frac{1}{3} x^3 + o(x^3)$$

$$\operatorname{tg} \frac{1}{n} = \frac{1}{n} + \frac{1}{3}\left(\frac{1}{n}\right)^3 + o\left(\frac{1}{n}\right)^3 = \frac{1}{n} + \frac{1}{3}\frac{1}{n^3} + o\left(\frac{1}{n^3}\right)$$

$$\frac{1}{n} \cdot \operatorname{tg} \frac{1}{n} = \underbrace{\frac{1}{n^2} + \frac{1}{3}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)}$$

$x \to 0$

$$\lg(1+x) = x - \frac{1}{2} x^2 + o(x^2)$$

$$\lg\left(1 + \frac{1}{n^2}\right) = \frac{1}{n^2} - \frac{1}{2}\left(\frac{1}{n^2}\right)^2 + o\left(\frac{1}{n^2}\right)^2 = +\frac{1}{n^2} - \frac{1}{2}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)$$

$$\underbrace{\frac{1}{n}\operatorname{tg}\frac{1}{n}} - \lg\left(1+\frac{1}{n^2}\right) = \cancel{\frac{1}{n^2}} + \frac{1}{3}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right) \cancel{-\frac{1}{n^2}} + \frac{1}{2}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)$$

$$= \left(\frac{1}{3}+\frac{1}{2}\right)\frac{1}{n^4}+o\left(\frac{1}{n^4}\right) = \frac{5}{6}\,\frac{1}{n^4}+o\left(\frac{1}{n^4}\right)$$

$$= \frac{1}{n^4}\left[\frac{5}{6}+o(1)\right]$$

$$a_n = n^\alpha \cdot \frac{1}{n^4}\left[\frac{5}{6}+o(1)\right]$$

$$\lim_n a_n = \begin{cases} \frac{5}{6} & \text{se } \underline{\underline{\alpha = 4}} \\ +\infty & \text{se } \underline{\underline{\alpha > 4}} \\ 0 & \text{se } \alpha < 4 \end{cases}$$

$$\sum_{n=1}^{\infty} |a_n| \qquad |a_n| = \frac{n^{\alpha}}{n^4}\left(\frac{5}{6}+o(1)\right) =$$

$$= \frac{1}{n^{4-\alpha}}\left(\frac{5}{6}+o(1)\right) \sim \frac{1}{n^{4-\alpha}}$$

criterio confronto asintotico

la serie armonica generalizzata

CONVERGE se $4-\alpha > 1 \rightarrow \alpha < 3$

DIVERGE se $4-\alpha \leq 1 \rightarrow \alpha \geq 3$

se $\alpha < 3$ la serie converge

$\alpha \geq 3$ la serie diverge

Es Stabilire il carattere della serie (a termini positivi)

$$\sum_{n=1}^{+\infty} \frac{5^n + n^2 \lg n + 3^n}{(n-1)!}$$

$$\frac{5^n + n^2 \lg n + 3^n}{(n-1)!} = \frac{5^n \left(1 + \overbrace{\frac{n^2 \lg n}{5^n} + \frac{3^n}{5^n}}^{\to 0}\right)}{(n-1)!}$$

$$a_n \sim \frac{5^n}{(n-1)!}$$

per il criterio confronto asintotico basta controllare se converge la serie $\sum_{n=1}^{\infty} \frac{5^n}{(n-1)!}$

devo studiare la convergenza di

$$\sum_{n=1}^{\infty} \frac{5^n}{(n-1)!}$$

applico criterio del rapporto

$$a_n = \frac{5^n}{(n-1)!} \qquad a_{n+1} = \frac{5^{n+1}}{(n+\cancel{1}-\cancel{1})!} =$$

$$= \frac{5^{n+1}}{n!} = \frac{5^n \cdot 5}{n!}$$

$$\lim_n \frac{a_{n+1}}{a_n} = \lim a_{n+1} \cdot \frac{1}{a_n} = \lim_n \frac{\cancel{5^n} \cdot 5}{n!} \cdot \frac{(n-1)!}{\cancel{5^n}} =$$

$$= \lim_{n} \frac{5 \quad (n-1)!}{n!} = \lim_{n} \frac{5 \quad \cancel{(n-1)!}}{n \cdot \cancel{(n-1)!}} = \underline{0 < 1}$$

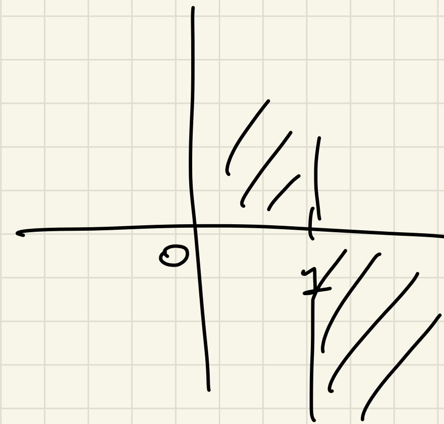
$$n! = n \cdot (n-1)!$$

per il criterio del rapporto, dato che il limite è $< 1$, $\boxed{\text{la serie converge}}$

ES

$$f(x) = \sqrt[4]{x} \cdot \left[\lg\left(\sqrt{x}\right)\right]^5$$

$$D = \{x > 0\} = (0, +\infty)$$



segno $f(x) \geq 0$

$\sqrt[4]{x} > 0 \quad \forall x \in D$

$\left(\lg \sqrt{x}\right)^5 \geq 0 \ ? \implies \lg(\sqrt{x}) \geq 0 = \lg 1$

$\implies \sqrt{x} \geq 1 \implies x \geq 1$

$$\lim_{x \to 0^+} \underbrace{\sqrt[4]{x}}_{0} \cdot \underbrace{\left[\lg(\sqrt{x})\right]^5}_{-\infty} = 0$$

$x = 0$ singolarità eliminabile

$\boxed{\text{aggiungo } x=0 \text{ a } D \qquad f(0) = 0}$

$$\left( \lim_{x\to 0^+} x^h \left(\log(x)\right)^k = 0 \right. \qquad h>0 \quad k>0$$

---

$$\lim_{x\to+\infty} \underbrace{\sqrt[4]{x}}_{\downarrow +\infty} \cdot \underbrace{\left(\log\sqrt{x}\right)^5}_{\downarrow +\infty} = +\infty$$

non ho as. orizzontale

cerco asintoto obliquo

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt[4]{x}\cdot\left(\log\sqrt{x}\right)^5}{x} = \lim_{x\to+\infty} \frac{\overbrace{\left(\log\sqrt{x}\right)^5}^{+\infty}}{\underbrace{x^{3/4}}_{\to+\infty}} = 0$$

$$\frac{x^{1/4}}{x} = \frac{1}{x^{3/4}}$$

NON HO AS. OBLIQUO.

$$f(x) = \sqrt[4]{x}\,(\lg\sqrt{x})^5 = \underbrace{x^{\frac{1}{4}}}\ \underbrace{\left[\lg\left(x^{\frac{1}{2}}\right)\right]^{5}}$$

$$f'(x) = \frac{1}{4}\,x^{\frac{1}{4}-1} \cdot \left[\lg\left(x^{\frac{1}{2}}\right)\right]^{5} +$$

$$+\; x^{\frac{1}{4}} \cdot 5 \cdot \left[\lg\left(x^{\frac{1}{2}}\right)\right]^{4}\ \frac{1}{x^{1/2}}\ \frac{1}{2}\,x^{\frac{1}{2}-1} =$$

$$= \frac{1}{4}\,x^{-\frac{3}{4}}\left[\lg\left(x^{\frac{1}{2}}\right)\right]^{5} + \frac{5}{2}\,\underbrace{x^{\frac{1}{4}-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-1}}\left[\lg\left(x^{\frac{1}{2}}\right)\right]^{4} =$$

$$= \frac{1}{4} x^{-3/4} (\lg\sqrt{x})^5 + \frac{5}{2} x^{-3/4} (\lg\sqrt{x})^4 =$$

$$= \frac{1}{2} x^{-3/4} (\lg\sqrt{x})^4 \left[ \frac{1}{2} (\lg\sqrt{x}) + 5 \right]$$

derivata definita per $x > 0$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \frac{1}{2} \underbrace{x^{-3/4}}_{+\infty} \underbrace{[\lg\sqrt{x}]^4}_{+\infty} \cdot \underbrace{\left[\frac{1}{2}(\lg\sqrt{x}) + 5\right]}_{-\infty}$$

$= -\infty$ ATTACCO VERTICALE in $x = 0$

segno di $f'$

$$x > 0$$

$$f'(x) = \frac{1}{2} \underbrace{x^{-3/4}}_{> 0} \underbrace{(\lg \sqrt{x})^4}_{\geq 0} \left[\frac{1}{2} \lg(\sqrt{x}) + 5\right]$$

$$f'(x) \geq 0$$

$$\Updownarrow$$

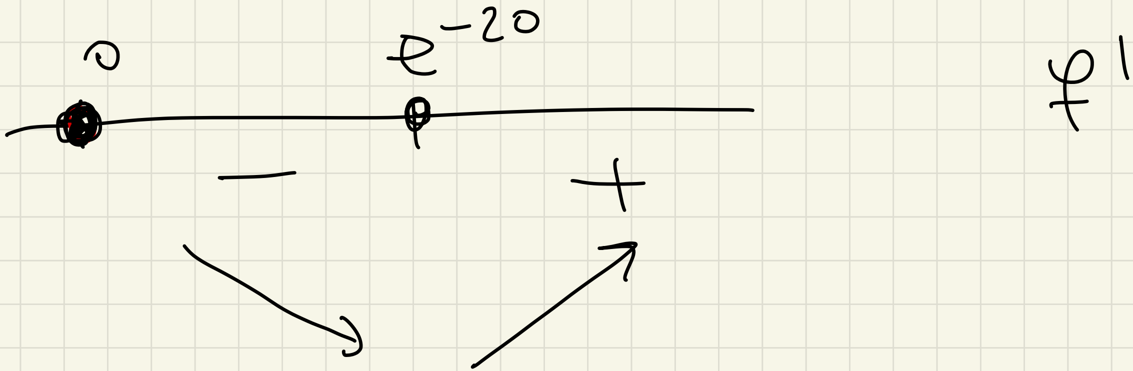
$$\frac{1}{2} \lg \sqrt{x} + 5 \geq 0 \; ?$$

$$\frac{1}{2} \lg \sqrt{x} \geq -5$$

$$\boxed{\lg \sqrt{x} \geq -10 = \lg(e^{-10})}$$

$$(\sqrt{x})^2 \geq (e^{-10})^2$$

$$\boxed{x \geq e^{-20}}$$




$f$ è decrescente in $(0, e^{-20})$

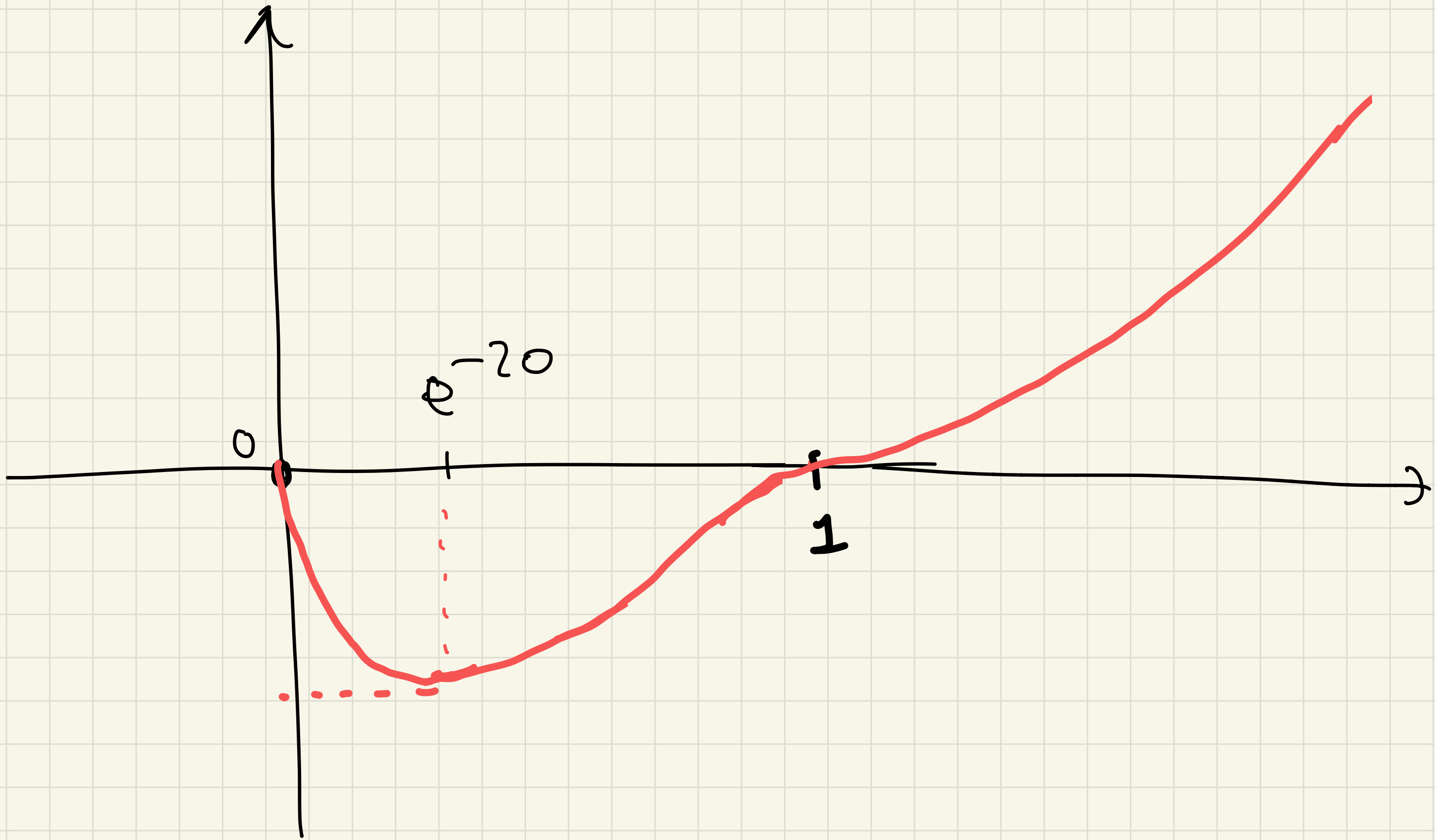
crescente in $(e^{-20}, +\infty)$

$x = e^{-20}$ pto di MINIMO LOCALE

e anche ASSOLUTO

$x = 0$ pto di MASSIMO LOCALE

NON ASSOLUTO


0
$e^{-20}$
1

Es

Calcolare se esiste

$$\int_1^2 \frac{1}{e^{2x}\left(e^{-4x}+2e^{-2x}-3\right)}\,dx =$$

$$\int_1^2 \frac{1}{\underbrace{e^{2x}}\left(e^{-4x}+2e^{-2x}-3\right)}\,dx$$

$$e^{-4x} = \left(e^{-2x}\right)^2$$

$$\int_1^2 \frac{1}{e^{2x}\left(e^{-4x}+2e^{-2x}-3\right)}\,dx =$$

$$= \int_1^2 \frac{e^{-2x}}{e^{-4x}+2e^{-2x}-3}\,dx$$

$$y = e^{-2x}$$

$$-2x = \lg y$$

$$x = -\frac{1}{2}\lg y$$

$$dx = -\frac{1}{2}\frac{1}{y}\,dy$$

$$= \int_{e^{-2}}^{e^{-4}} \frac{\cancel{y}}{y^2+2y-3}\left(-\frac{1}{2}\frac{1}{\cancel{y}}\right)dy =$$

$$= -\frac{1}{2}\int_{e^{-2}}^{e^{-4}} \frac{1}{y^2+2y-3}\,dy =$$

$$= \frac{1}{2}\int_{e^{-4}}^{e^{-2}} \frac{1}{y^2+2y-3}\,dy$$

fratti semplici $\quad y^2+2y-3=0 \to y=\begin{cases}1\\-3\end{cases}$

$$\frac{0\cdot y+1}{y^2+2y-3} = \frac{A}{y-1} + \frac{B}{y+3} = \frac{Ay+3A+By-B}{(y-1)(y+3)}$$

$$\begin{cases} A+B=0 \\ 3A-B=1 \end{cases} \qquad \begin{cases} A=-B \\ 3A+A=1 \end{cases} \qquad \begin{cases} B=-\frac{1}{4} \\ A=\frac{1}{4} \end{cases}$$

$$\frac{1}{2}\int \frac{1}{y^2+2y-3}\,dy = \frac{1}{2}\left[\frac{1}{4}\int \frac{1}{y-1}\,dy - \frac{1}{4}\int \frac{1}{y+3}\,dy\right]$$

$$= \frac{1}{2}\cdot\left[\frac{1}{4}\log|y-1| - \frac{1}{4}\log|y+3| + c\right] =$$

$$= \frac{1}{2}\cdot\frac{1}{4}\log\left(\frac{|y-1|}{|y+3|}\right) + c = \frac{1}{8}\log\left|\frac{y-1}{y+3}\right| + c$$

$$\frac{1}{2}\int_{e^{-4}}^{e^{-2}} \frac{1}{y^2+2y-3}\,dy = \frac{1}{8}\lg\left(\left|\frac{e^{-2}-1}{e^{-2}+3}\right|\right) - \frac{1}{8}\lg\left|\frac{e^{-4}-1}{e^{-4}+3}\right|$$

$$\int_{\cdot\cdot}^{\cdot\cdot} \frac{1}{x\left[(\lg x)^2+3\lg x-2\right]}\,dx \qquad y=\lg x$$

$$\int_0^{\pi/2} x^2 \cos(3x)\, dx \qquad\qquad \int_0^1 x^2 e^{3x}\, dx$$

$$\int x^2 \cos 3x \, dx = \frac{1}{3}\sin 3x \cdot x^2 - \int \frac{1}{3} \sin 3x \cdot 2x\, dx$$

$f(x) = \cos 3x \rightarrow F(x) = \frac{1}{3}\sin(3x)$

$g(x) = x^2 \longrightarrow g'(x) = 2x$

$$= \frac{1}{3}\sin 3x \cdot x^2 - \frac{2}{3}\int \sin 3x \cdot x\, dx$$

$$f(x) = \sin 3x \quad \rightarrow \quad F(x) = -\frac{1}{3}\cos(3x)$$

$$g(x) = x \quad \rightarrow \quad g'(x) = 1$$

$$= \frac{1}{3}\sin(3x)\,x^2 - \frac{2}{3}\left[-\frac{1}{3}\cos 3x \cdot x - \int\left(-\frac{1}{3}\cos 3x\right)\cdot 1\,dx\right]$$

$$= \frac{1}{3}\sin 3x \cdot x^2 + \frac{2}{9}\cos(3x)\cdot x - \frac{2}{9}\int \cos 3x\,dx$$

$$= \frac{1}{3}\sin 3x \cdot x^2 + \frac{2}{9}\cos 3x \cdot x - \frac{2}{9}\cdot\frac{1}{3}\sin 3x + C$$