



MCDA – DCM

VULNERABILITY ANALYSIS AND RISK MANAGEMENT FOR WATER-RELATED HAZARDS

Professor: **Canesi R.**

Teaching assistant: **Corti F.** (franco.corti@unipd.it)

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What is MCDA?

- Multiple Criteria Decision Aiding (MCDA) supports the decision-making process by considering multiple perspectives.
- Helps **Decision-Makers (DMs)** reflect, discuss, and argue about choices.
- Involves a co-constructive process:
 - DM shares preferences.
 - **Analyst** builds a decision model using feedback.
- Assign values to preference parameters **based on the chosen method**.

Deck of Cards Method (DCM)

Proposed by Simos (1989; 1990):

- The method was designed for decision makers (DMs), even without prior MCDA knowledge.
- It helps DMs rank and prioritize criteria (not only criteria) in a given context.

Purpose:

- Communicate preferences clearly to the analyst through cards.
- Support in the assignment of numerical weights to criteria.

Applications:

- Successfully applied in diverse real-life contexts.
- Highly accepted by DMs, offering meaningful preference insights.

Deck of Cards Method (DCM)

Brief history

- **Simos (1990a, b)** introduced a technique enabling any Decision Maker (DM), even without prior knowledge of multicriteria decision aiding, to articulate and prioritize the criteria of a family F within a specific context.
- **Figueira and Roy (2002)** developed a modified version of Simos' approach (building on **Maystre, Pictet, & Simos, 1994**) for determining criteria weights in ELECTRE methods and other outranking-based approaches. Known as the SRF1 method, it served as the foundation for the SRF Software. For a list of Deck of Cards Method (DCM) applications, see **Siskos and Tsotsolas (2015)**.
- **Pictet and Bollinger (2008)** introduced an initial approach for constructing interval scales. **Bottero et al. (2018)** enhanced the DCM method by creating more versatile interval scales, defining at least two reference levels relevant to policymakers, decision-makers, users, or experts.
- **Corrente et al. (2021)** proposed a simplified version of the Pairwise Comparison Deck of Cards Method (PaCo-DCM), which was used to construct value functions (interval scales) and criteria weights (ratio scales).

Deck of Cards Method (DCM)

DCM is widely used to elicit preference parameters. E.g., to assess criteria weights in ELECTRE methods ([Figueira & Roy, 2002](#)).

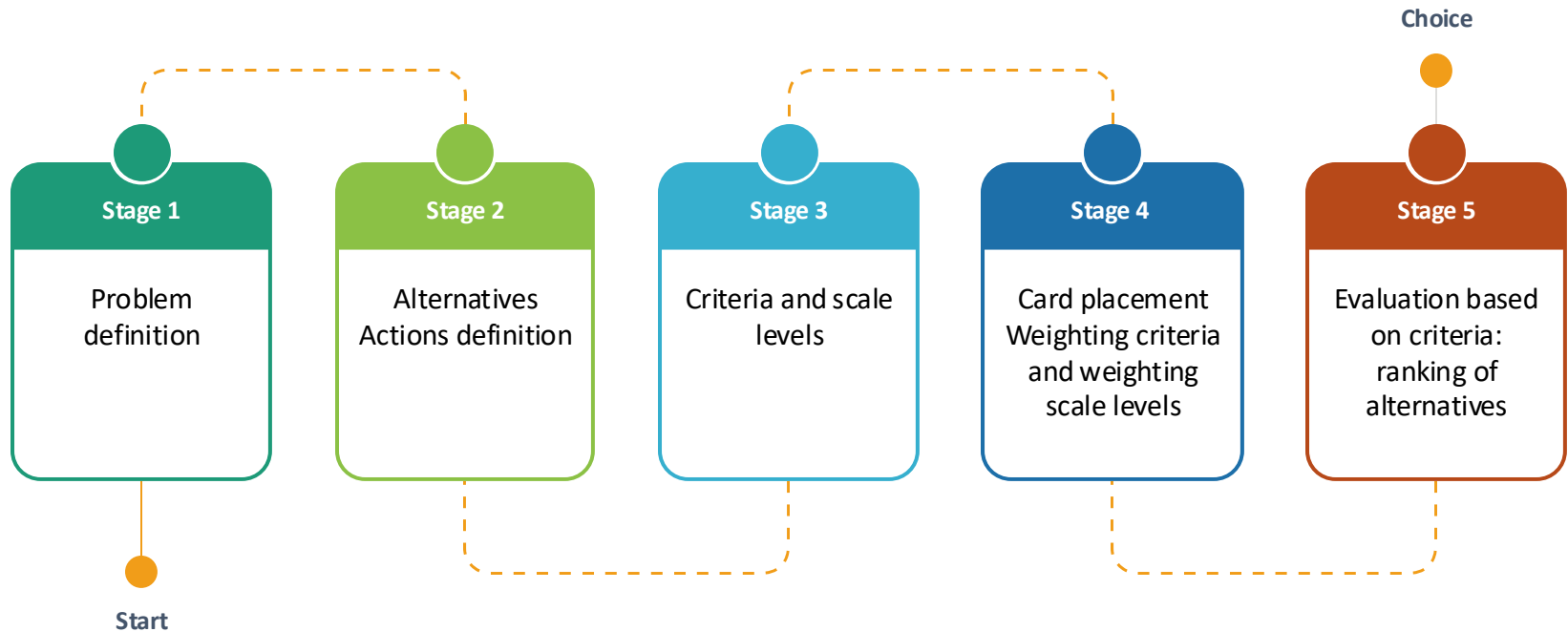
- Steps in the original DCM for outranking methods:
 1. **Rank elements:** From least to most important.
 2. **Express preference strength:** Add blank cards between levels.
 3. **Define weight ratios:** Compare the most and least important criteria.

Recall: the decision process

- **Actor**: the analyst
- **Decision-Maker (DM)**: the client
- (Stakeholders)

- Set of **Actions** that are **Alternatives**
- **Criteria** to evaluate the actions
- Multicriteria **Problem** definition (e.g., *choice, sorting/classification, ranking/rating*)

1. DCM: Problem and flow chart



1. Problem: need to buy a new car



Purpose: Identify the best car

How? Based on what criteria?

2. Alternatives

Alternatives	
Acura TL	Cost \$33,000 Miles per Gallon 25,5 (City/Hwy) Prestige is Above Average Comfort is Excellent
Toyota Camry	Cost \$25,000 Miles per gallon 26 (City/Hwy) Prestige is Average Comfort is Average
Honda Civic	Cost \$18,000 Miles per gallon 34,0 (City/Hwy) Prestige is Below Average Comfort is Below Average

3. Criteria definition

Prestige	Direction: MAX	Ordinal
Price	Direction: MIN	Interval (Continuous)
MPG	Direction: MIN	Interval (Continuous)
Comfort	Direction: MAX	Ordinal

3. Scale levels

Criteria	Rating Scale
Prestige	Excellent Above Average Average Below Average Poor
Price	From 15 to 35 (k€)
MPG	From 20 to 40 (city/hwy)
Comfort	Excellent Above Average Average Below Average Poor

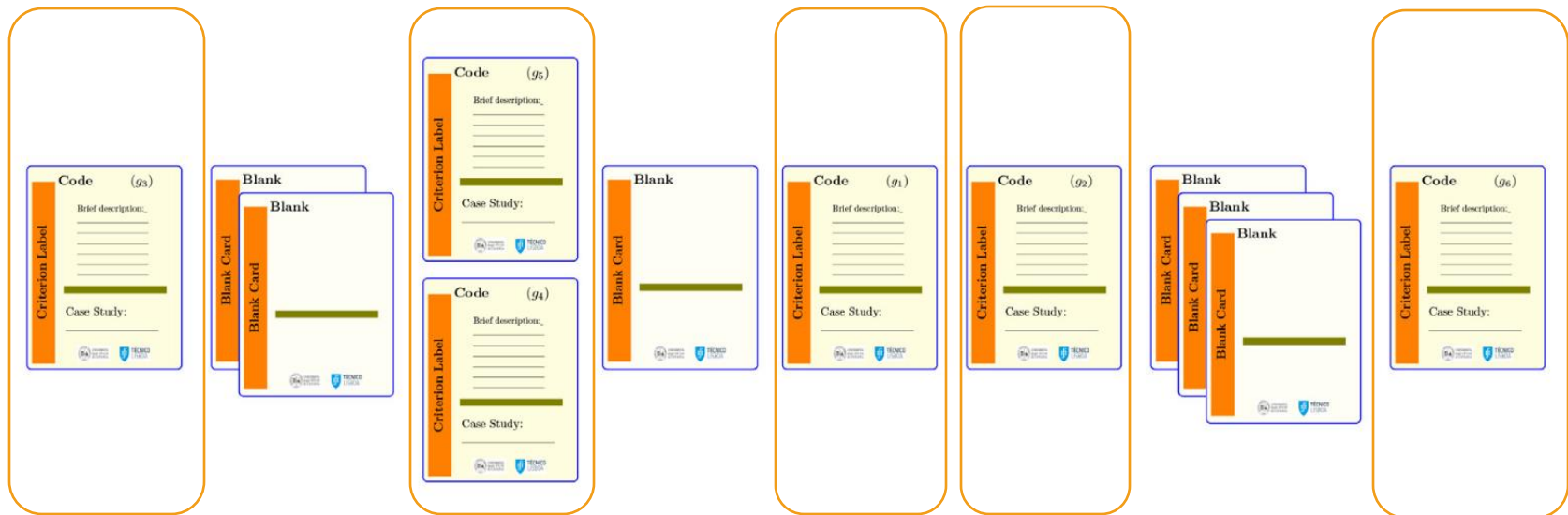
For each criterion, two reference levels have been considered.

4. How the DCM works

Setup:

- Provide cards labeled with criteria names (plus complementary info if needed).
 - Number of cards = Number of criteria (n).
1. Rank cards from least preferred to most preferred.
 2. Evaluate differences in preference:
 - Place white cards between ranked cards to show the weight (or strength) of preference:
 - No (zero) white cards = Minimal difference (unit = α).
 - 1 white card = Difference of 2α
 - 2 white cards = Difference of 3α

4. DCM: criteria ranking



Example of ranking of criteria with blank cards by [Corrente et al. \(2021\)](#)

4. DCM: criteria ranking

How to overcome uncertainties in this ranking?

1. Dummy actions have been strategically constructed, ensuring that the dummy scenario scores highest on a certain criterion and lowest on the remaining ones.
2. The dummy scenarios have been ranked by the swing from best to worst value.

Example:

Prestige $g_1 = (\text{Excellent}, 35, 40, \text{Poor}) = (\mathbf{100}, 0, 0, 0)$

is preferred to

Price $g_2 = (\text{Poor}, \mathbf{15}, 40, \text{Poor}) = (0, \mathbf{100}, 0, 0)$

Criteria ranking:

$g_1 \prec g_2 \prec g_3 \prec g_4$

4. Zeta value: substitution rate

After ranking the dummy scenarios, the DCM has been used to model their greater or lesser closeness regarding weights.

	Dummy	C1	C2	C3	C4
Prestige	C1		2		
Price	C2			1	
MPG	C3				1
Comfort	C4				

Then, the DM was asked to establish a relation between the first and last in the ranking, and in this example, we consider that *Prestige* is 10 times more important than *Comfort*.

This is called the z -ratio intended as the substitution rate between the criterion with the highest and lowest weights. In this example, we state this value is **10**

4. Alpha and weight calculation

Given the z-ratio = **10/1 = 10**, it is possible to compute the Alpha-value, and by comparing the values obtained, it is possible to calculate the weight assigned.

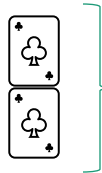
$$a_c = \frac{10 - 1}{(2+1)+(1+1)+(1+1)} = 1,29$$

	Dummy	C1	C2	C3	C4
Prestige	C1	1	2		
Price	C2		1	1	
MPG	C3			1	1
Comfort	C4				1

Z value	10,00		
α value	1,29		Weights
	10,00		42,9%
	7,43		31,9%
	4,86		20,9%
	1,00		4,3%
	23,29		100%

4. Ordinal Criterion

Example within one criterion:



Prestige	L ₁	L ₂	L ₃	L ₄	L ₅
Level 1		2			
Level 2			1		
Level 3				0	
Level 4					2
Level 5					

$v_1(l_{1,1})$	0,00	Poor
$v_1(l_{1,2})$	33,33	Below Average
$v_1(l_{1,3})$	55,56	Average
$v_1(l_{1,4})$	66,67	Above Average
$v_1(l_{1,5})$	100,00	Excellent

The value of a unit α has also been computed for each criterion after the card placement by dividing the values of the two reference levels (we use 0 and 100) by the number of units between them.

$$\alpha_1 = \frac{100 - 0}{(2+1)+(1+1)+(0+1)+(2+1)} = 11,11$$

$$v_1(l_{1,3}) = v_1(l_{1,1}) + (2 + 1 + 1 + 1) * \alpha = 0 + (5) \times 11,11 = 55,56$$

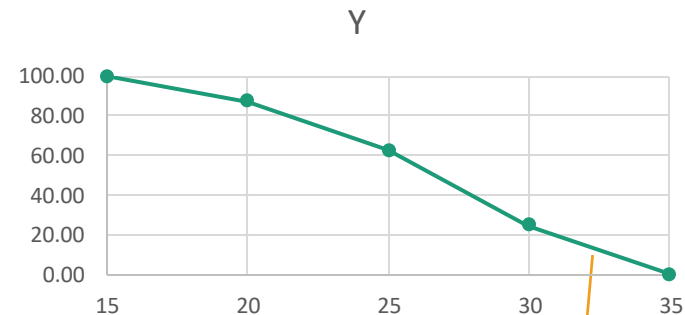
4. Interval Criterion

Example within one criterion:

Price	L1	L2	L3	L4	L5
Level 1	■	→ 1			
Level 2		■	2		
Level 3			■	1	
Level 4				■	0
Level 5					■



		X	Y
$v_2(l_{2,1})$	0,00	35	0,00
$v_2(l_{2,2})$	25,00	30	25,00
$v_2(l_{2,3})$	62,50	25	62,50
$v_2(l_{2,4})$	87,50	20	87,50
$v_2(l_{2,5})$	100,00	15	100,00



Given that the criterion is on a continuous scale, the value between the levels is obtained through the linear interpolation between two levels.

$$a_1 = \frac{100 - 0}{(2+1)+(1+1)+(0+1)+(2+1)} = 11,11$$

$$v_2(33) = v_2(l_{2,2}) - \frac{(v_2(l_{2,2}) - v_2(l_{2,1}))}{(l_{2,2} - l_{2,1})} (l_{2,2} - 33) = 25 - \frac{(25 - 0)}{30 - 35} (30 - 33) \approx 10$$

5. Model results

$$v_1(l_{1,3}) = v_1(l_{1,1}) + (2 + 1 + 1 + 1) * \alpha = 0 + (5) \times 11,11 = 55,56$$

Information Table	Prestige	Price	MPG	Comfort
Acura TL	Above Average	33	25,5	Excellent
Toyota Camry	Average	25	26	Average
Honda Civic	Below Average	18	34	Below Average

Calculations Table	Prestige	Price	MPG	Comfort
Acura TL	66,67	10,00	27,50	100,00
Toyota Camry	55,56	50,00	30,00	55,56
Honda Civic	33,33	85,00	70,00	33,33

5. Ranking

Calculations Table	Prestige	Price	MPG	Comfort
Acura TL	66,67	10,00	27,50	100,00
Toyota Camry	55,56	50,00	30,00	55,56
Honda Civic	33,33	85,00	70,00	33,33

Z value	10,00		
α value	1,29		Weights
	10,00		42,9%
	7,43		31,9%
	4,86		20,9%
	1,00		4,3%
	23,29		100%

Weighted value of the action
 $= 55,56 * 42,9\% = 23,86$

Performance Table	Prestige	Price	MPG	Comfort	Ranking
Acura TL	28,63	3,19	5,74	4,29	41,85
Toyota Camry	23,86	15,95	6,26	2,39	48,45
Honda Civic	14,31	27,12	14,60	1,43	57,46

References

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