Es

$$\int_{\frac{1}{e^2}}^{\frac{1}{e}} \frac{1}{x(\lg^2 x - 2\lg x)}\,dx$$

$\lg^2 x - 2\lg x$ è polinomio di grado 2 nella variabile $\lg x$

$(a = \lg b \Leftrightarrow e^a = b)$

$\boxed{y = \lg x} \Longleftrightarrow \boxed{x = e^y}$

$\lg^2 x - 2\lg x \to y^2 - 2y$

$x = e^y \qquad dx = (e^y)'\,dy = e^y\,dy$

$x = \frac{1}{e^2} \to y = \lg\left(\frac{1}{e^2}\right) = \lg(e^{-2}) = -2$

$x = \frac{1}{e} \to y = \lg\left(\frac{1}{e}\right) = \lg(e^{-1}) = -1$

$$\int_{-2}^{-1} \frac{1}{\cancel{e^y}(y^2 - 2y)}\,\cancel{e^y}\,dy$$

Ora quindi basta calcolare

$$\int_{-2}^{-1} \frac{1}{y^2-2y}\, dy$$

$$\int \frac{1}{y^2-2y}\, dy = ?$$

$$y^2-2y=0 \qquad y_{1,2}=\begin{cases} 0 \\ 2 \end{cases}$$

$$y(y-2)=0$$

metodo dei fratti semplici

$$\underbrace{\frac{0\cdot y+1}{y^2-2y}} = \frac{1}{y(y-2)} = \underbrace{\frac{A}{y}+\frac{B}{y-2}} = \frac{Ay-2A+By}{y(y-2)}$$

$$\begin{cases} A+B=0 \\ -2A=1 \end{cases} \qquad \begin{cases} B=\frac{1}{2} \\ A=-\frac{1}{2} \end{cases}$$

$$\frac{1}{y^2-2y} = \frac{-\frac{1}{2}}{y} + \frac{\frac{1}{2}}{y-2} = -\frac{1}{2}\,\frac{1}{y} + \frac{1}{2}\,\frac{1}{y-2}$$

$$\int \frac{1}{y^2-2y}\,dy = -\frac{1}{2}\int \frac{1}{y}\,dy + \frac{1}{2}\int \frac{1}{y-2}\,dy =$$

$$= -\frac{1}{2}\log|y| + \frac{1}{2}\log|y-2| + c$$

$$= \frac{1}{2}\left[-\log|y| + \log|y-2|\right] + c$$

$$= \frac{1}{2}\log\left(\frac{|y-2|}{|y|}\right) + c$$

$$\int_{-2}^{-1} \frac{1}{y^2-2y}\,dy = \frac{1}{2}\lg\left|\frac{-1-2}{-1}\right| - \frac{1}{2}\lg\left|\frac{-2-2}{-2}\right| =$$

$$\int \frac{1}{y^2-2y}\,dy = \underbrace{\frac{1}{2}\lg\left|\frac{y-2}{y}\right|} + C$$

$$= \frac{1}{2}\left(\lg 3 - \lg 2\right) =$$

$$= \frac{1}{2}\lg\left(\frac{3}{2}\right) =$$

$$= \lg\sqrt{\frac{3}{2}}$$

Es

$$\int_{e^{-2}}^{e^{-1}} x\,(\lg^2 x - 2\lg x)\,dx$$

PER PARTI

$$\int f(x)g(x)\,dx = F(x)g(x) - \int F(x)g'(x)\,dx$$

$$\int x\,\underbrace{(\lg^2 x - 2\lg x)}\,dx = \frac{1}{2}x^2\cdot(\lg^2 x - 2\lg x) - \int \frac{1}{\cancel{2}}\cancel{x^2}\left(\frac{\cancel{2}}{\cancel{x}}(\lg x - 1)\right)dx$$

$$f(x) = x \rightarrow F(x) = \frac{1}{2}x^2$$

$$g(x) = \lg^2 x - 2\lg x \rightarrow g'(x) = 2\lg x\,\frac{1}{x} - 2\,\frac{1}{x} =$$

$$= \frac{2}{x}\left[\lg x - 1\right]$$

$$= \frac{1}{2}x^2\left(\lg^2 x - 2\lg x\right) - \int x\cdot(\lg x - 1)\,dx =$$

$$= \frac{1}{2}x^2\left(\lg^2 x - 2\lg x\right) - \int x\left(\lg x - 1\right)dx$$

$$f(x) = x \longrightarrow F(x) = \frac{1}{2}x^2$$

$$g(x) = \lg x - 1 \longrightarrow g'(x) = \frac{1}{x} - 0 = \frac{1}{x}$$

$$= \frac{1}{2}x^2\left(\lg^2 x - 2\lg x\right) - \left[\frac{1}{2}x^2\left(\lg x - 1\right) - \int \frac{1}{2}x^{\cancel{2}} \cdot \frac{1}{\cancel{x}}\,dx\right] =$$

$$= \frac{1}{2}x^2\left(\lg^2 x - 2\lg x\right) - \frac{1}{2}x^2\left(\lg x - 1\right) + \frac{1}{2}\int x\,dx =$$

$$= \frac{1}{2}x^2\left(\lg^2 x - 2\lg x\right) - \frac{1}{2}x^2\left(\lg x - 1\right) + \frac{1}{2} \cdot \frac{1}{2}x^2 + C$$

$$= \frac{1}{2}x^2\left[\lg^2 x - 2\lg x - \lg x + 1 + \frac{1}{2}\right] + C = \frac{1}{2}x^2\left[\lg^2 x - 3\lg x + \frac{3}{2}\right] + c$$

$$\int x\left(\lg^2 x - 2\lg x\right)dx = \underline{\frac{1}{2}x^2\left[\lg^2 x - 3\lg x + \frac{3}{2}\right]} + c$$

$$\int_{e^{-2}}^{e^{-1}} x\left(\lg^2 x - 2\lg x\right)dx = \frac{1}{2}\left(e^{-1}\right)^2\left[\lg^2\left(e^{-1}\right) - 3\lg\left(e^{-1}\right) + \frac{3}{2}\right] +$$

$$- \frac{1}{2}\left(e^{-2}\right)^2\left[\lg^2\left(e^{-2}\right) - 3\lg\left(e^{-2}\right) + \frac{3}{2}\right] =$$

$$= \frac{1}{2}e^{-2}\left[(-1)^2 - 3(-1) + \frac{3}{2}\right] - \frac{1}{2}e^{-4}\left[(-2)^2 - 3(-2) + \frac{3}{2}\right] =$$

$$= \frac{1}{2}e^{-2}\left[1 + 3 + \frac{3}{2}\right] - \frac{1}{2}e^{-4}\left[4 + 6 + \frac{3}{2}\right] -$$

es

$$\int_1^e \frac{\lg^3 x - 2\lg x}{x^2}\, dx$$

$y = \lg x \qquad \underbrace{x = e^y} \qquad dx = e^y\, dy \qquad x^2 = (e^y)^2 = e^{2y}$

$x = 1 \to y = \lg 1 = 0 \qquad x = e \to y = \lg e = 1$

$$\int_0^1 \frac{(y^3 - 2y)}{e^{2y}}\, e^y\, dy = \int_0^1 \frac{(y^3-2y)}{e^y}\, dy =$$

$$= \int_0^1 (y^3-2y)\, e^{-y}\, dy$$

$$\int_0^1 (y^3 - 2y)\, e^{-y}\, dy$$

$$\int (y^3-2y)\, e^{-y}\, dy = -e^{-y} \cdot (y^3-2y) - \int (-e^{-y})(3y^2-2)\, dy$$

$f(y) = e^{-y} \rightarrow F(y) = -e^{-y}$

$$\int e^{\alpha y}\, dy = \frac{1}{\alpha} e^{\alpha y} + C$$

$g(y) = y^3 - 2y \rightarrow g'(y) = 3y^2 - 2$

$$= -e^{-y}(y^3-2y) + \int e^{-y}(3y^2-2)\, dy =$$

$f(y) = e^{-y} \rightarrow F(y) = -e^{-y}$

$g(y) = 3y^2 - 2 \rightarrow g'(y) = 6y$

$$= -e^{-y}(y^3-2y) + \left[-e^{-y}(3y^2-2) - \int (-e^{-y}) \cdot 6y\, dy \right] =$$

$$= -e^{-y}(y^3-2y) - e^{-y}(3y^2-2) + 6\int e^{-y}\, y\, dy =$$

$f(y) = e^{-y} \Rightarrow F = -e^{-y}$

$g(y) = y \rightarrow g'(y) = 1$

$$= -e^{-y}(y^3-2y) - e^{-y}(3y^2-2) \cancel{+}\, 6\left[-e^{-y}\cdot y - \int(\cancel{-}e^{-y})\cdot 1\, dy\right] =$$

$$= -e^{-y}(y^3-2y) - e^{-y}(3y^2-2) - 6e^{-y}y + 6\int e^{-y}\, dy =$$

$$= -e^{-y}(y^3-2y) - e^{-y}(3y^2-2) - 6e^{-y}y - 6e^{-y} + C$$

$$= -e^{-y}\left[y^3 - 2y + 3y^2 - 2 + 6y + 6\right] + C =$$

$$= \boxed{-e^{-y}\left[y^3 + 3y^2 + 4y + 4\right] + C}$$

$$\int_0^1 e^{-y}(y^3-2y)\,dy = -e^{-1}[1+3+4+4] - \left(\overset{1}{\overline{-e^{-0}}}\right)[0+0+4]$$

$$\int e^{-y}(y^3-2y)\,dy = -e^{-y}[\underline{y^3}+\underline{3y^2}+\underline{4y}+4]+C$$

$$= -\frac{12}{e}+4$$

Es $\int e^x \cos(3x)\,dx$?

$$\int e^x \cos(3x)\,dx = e^x \cdot \cos(3x) - \int e^x(-3\sin 3x)\,dx$$

$f(x) = e^x \rightarrow F(x) = e^x$

$g(x) = \cos 3x \rightarrow g'(x) = -3\sin(3x)$

$$= e^x \cos(3x) + 3\int e^x \sin 3x\,dx =$$

$\int$

$f(x) = e^x \rightarrow F = e^x$

$g(x) = \sin 3x \rightarrow g'(x) = 3\cos 3x$

$$= e^x \cos(3x) + 3\left[e^x \cdot \sin(3x) - \int e^x\, 3\cos 3x\,dx\right]$$

$$= e^x \cos(3x) + 3e^x \sin(3x) - 9\int e^x \cos 3x\,dx \quad + C$$

$$\int e^x \cos(3x)\,dx = e^x\cos(3x) + 3e^x \sin(3x) - 9\int e^x \cos 3x\,dx + C$$

$$\int e^x \cos(3x)\,dx + 9\int e^x \cos(3x)\,dx = e^x\cos(3x) + 3e^x \sin 3x + C$$

$$\frac{\cancel{10}}{\cancel{10}}\int e^x \cos(3x)\,dx = \frac{e^x\cos(3x) + 3e^x \sin(3x)}{10} + C$$

$$\boxed{\int e^x \cos(3x)\,dx = \frac{1}{10}e^x\cos(3x) + \frac{3}{10}e^x\sin(3x) + C}$$

Es

$$\int_0^{\pi} \frac{\sin x\,(\cos x + 1)}{\sin^2 x + 3\cos^2 x + 1}\,dx = \int_0^{\pi} \frac{\sin x\,(\cos x + 1)}{\underbrace{2}\cos^2 x + \underbrace{2}}\,dx$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x + 3\cos^2 x + 1 = 1 - \cos^2 x + 3\cos^2 x + 1 = 2\cos^2 x + 2$$

$$= \sin^2 x + 3(1 - \sin^2 x) + 1 =$$

$$= \sin^2 x + 3 - 3\sin^2 x + 1 = 4 - 2\sin^2 x$$

$$y = \cos x$$

$$\int_0^{\pi} \frac{\sin x\,(\cos x+1)}{2\cos^2 x+2}\,dx = \int_0^{\pi} \frac{(\cos x+1)}{2\cos^2 x+2}\,\underbrace{\sin x \cdot dx}$$

$$y = \cos x \qquad x=0 \rightarrow y=\cos 0 = 1 \qquad x = \arccos y$$

$$x=\pi \rightarrow y = \cos\pi = -1$$

$$\downarrow$$

$$dy = (\cos x)'\,dx = -\sin x\,dx \qquad (-1)\,dy = \sin x\,dx$$

$$dy = -\sin x\,dx = (-1)\cdot\sin x\,dx$$

$$(-1)\,dy = \cancel{(-1)}\,\cancel{(-1)}\,\sin x\,dx$$

$$\int_1^{-1} \frac{(y+1)}{2y^2+2}\,(-1)\,dy = -\int_1^{-1} \frac{(y+1)}{2y^2+2}\,dy = \int_{-1}^{1} \frac{y+1}{2(y^2+1)}\,dy$$

$$\int_{-1}^{1} \frac{y+1}{2(y^2+1)}\,dy = \cancel{\int_{-1}^{1} \frac{y}{2(y^2+1)}\,dy} + \underbrace{\int_{-1}^{1} \frac{1}{2(y^2+1)}\,dy}$$

$$\frac{y+1}{2(y^2+1)} = \frac{y}{2(y^2+1)} + \frac{1}{2(y^2+1)}$$

$$\int_{-1}^{1} \frac{y}{2(y^2+1)}\,dy = 0$$ perché $\frac{y}{2(y^2+1)}$ è DISPARI

se non me ne accorgo calcolo la primitiva

$$\int \frac{y}{2(y^2+1)}\,dy = \frac{1}{2}\int \frac{y}{y^2+1}\,dy = \frac{1}{2}\int \frac{\frac{1}{2}\,dz}{z} =$$

$$z = y^2+1$$

$$dz = 2y\,dy$$

$$\frac{1}{2}\,dz = y\,dy$$

$$= \frac{1}{4}\int \frac{1}{z}\,dz = \frac{1}{4}\log|z| + C =$$

$$= \frac{1}{4}\log(y^2+1) + c$$

$$\int_{-1}^{1} \frac{y}{2(y^2+1)}\,dy = \frac{1}{4}\log(1+1) - \frac{1}{4}\log((-1)^2+1) = 0$$

a, b

$$\int \frac{\alpha x}{ax^2+b}\,dx = \frac{\alpha}{2a}\lg|ax^2+b| + c$$

$\alpha = 1 \quad a = 2 = b$

$$\int_{-1}^{1} \frac{1}{2(y^2+1)}\,dy = \frac{1}{2}\int_{-1}^{1} \frac{1}{y^2+1}\,dy = \frac{1}{2}\operatorname{arctg} 1 - \frac{1}{2}\operatorname{arctg}(-1)$$

$$= \frac{1}{2}\operatorname{arctg} 1 + \frac{1}{2}\operatorname{arctg}(+1) = \operatorname{arctg} 1 = \frac{\pi}{4}$$

ES $\int_4^9 \frac{1}{\sqrt{x}-1}\,dx$

$\frac{1}{\sqrt{x}-1}$ $x \geq 0$ $x \neq 1$

$y=\sqrt{x} \to \underbrace{x=y^2}$

$\sqrt{x}-1 \to y-1$

$dx=(y^2)'\,dy=2y\,dy$

$\cancel{\int \frac{1}{\sqrt{x}}\,dx=\int x^{-\frac{1}{2}}\,dx=x^{-\frac{1}{2}+1}+c=\frac{1}{-\frac{1}{2}+1}}$

$x=4 \to y=\sqrt{4}=2$
$x=9 \to y=\sqrt{9}=3$

$\int_2^3 \frac{1}{y-1}\,2y\,dy = 2\int_2^3 \frac{y}{y-1}\,dy$

$$2\int_2^3 \frac{y}{y-1}\,dy = 2\Big[3+\lg|3-1| - (2+\lg|2-1|)\Big]$$

$$= 2\,[3+\lg 2 - 2 - \cancel{\lg 1}] = 2\cdot[1+\lg 2]$$

$$\frac{y}{y-1} = \frac{y-1+1}{y-1} = \frac{\cancel{y-1}}{\cancel{y-1}} + \frac{1}{y-1} = 1 + \frac{1}{y-1}$$

$$\int \frac{y}{y-1}\,dy = \int 1\,dy + \int \frac{1}{y-1}\,dy =$$

$$= y + \lg|y-1| + c$$

$$\int_a^b f(x)\,dx = \text{area con segno della regione compresa tra il grafico di } f \text{ e asse } x \text{ tra } x=a \text{ e } x=b.$$

$[a,b]$ INTERVALLO CHIUSO E LIMITATO

f CONTINUA in $[a,b]$. (f ha massimo e minimo in $[a,b]$)

posso cercare di estendere l'integrale in 2 modi

1) prendendo INTERVALLI ILLIMITATI

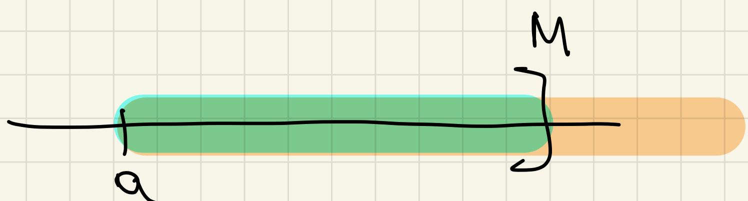
2) prendendo f continua solo in $(a,b]$ o $[a,b)$

Questa estensione si chiama
INTEGRALE GENERALIZZATO.

$$\boxed{\text{ESTENSIONE al caso di INTERVALLI ILLIMITATI}}$$

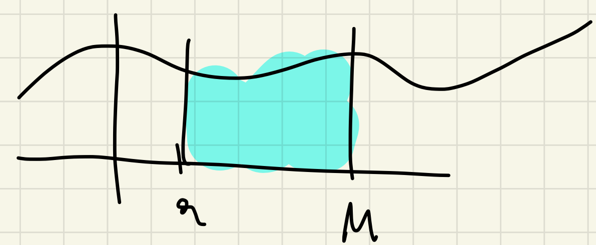
$f : [a, +\infty) \to \mathbb{R}$ f continua

$\to$ f continua su ogni intervallo chiuso e limitato



$\underbrace{[a, M]}$ $M > a$

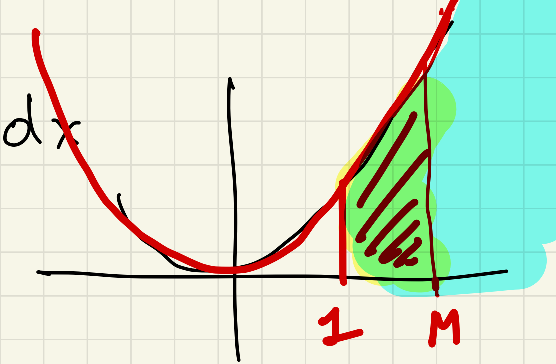
$\forall M > a$ calcolo $\int_a^M f(x)dx$



$$\boxed{\int_a^{+\infty} f(x)dx = \lim_{M \to +\infty} \int_a^M f(x)dx}$$

Es

$$\int_1^{+\infty} x\,dx = \lim_{M\to+\infty} \int_1^{M} x\,dx$$



$$\int_1^M x\,dx = \frac{1}{2}M^2 - \frac{1}{2}\cdot 1^2 = \frac{1}{2}M^2 - \frac{1}{2}$$

$$\int x\,dx = \frac{1}{2}x^2 + c$$

$$\int_1^{+\infty} x\,dx = \lim_{M\to+\infty} \int_1^M x\,dx = \lim_{M\to+\infty} \frac{1}{2}M^2 - \frac{1}{2} = +\infty$$

Es

$$\int_0^{+\infty} e^{-3x}\,dx \overset{\text{per definizione}}{=} \lim_{M\to+\infty} \int_0^M e^{-3x}\,dx$$

$$G(M) \quad - \quad G(0)$$

$$\int_0^M e^{-3x}\,dx = -\frac{1}{3}e^{-3M} - \left(-\frac{1}{3}e^{-3\cdot 0}\right) =$$

$$\int e^{-3x}\,dx = -\frac{1}{3}e^{-3x} + c = G(x) + c$$

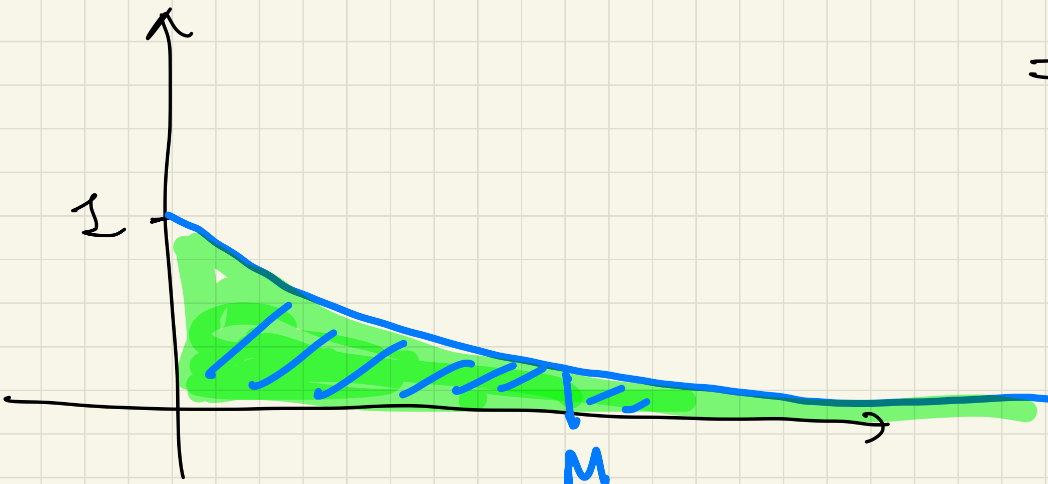
$$= -\frac{1}{3}e^{-3M} + \frac{1}{3}$$

$$\int_0^{+\infty} e^{-3x}\,dx = \lim_{M\to+\infty} \int_0^M e^{-3x}\,dx =$$

$$= \lim_{M\to+\infty} -\frac{1}{3}e^{-3M} + \frac{1}{3} = \cancel{-\frac{1}{3}e^{-\infty}} + \frac{1}{3}$$

$$\underset{0}{\shortparallel}$$

$$= \frac{1}{3}$$



Analogamente posso definire

per $f : (-\infty, b] \to \mathbb{R}$ continua

$$\boxed{\int_{-\infty}^{b} f(x)\,dx = \lim_{M\to+\infty} \int_{-M}^{b} f(x)\,dx}$$

Es

$$\int_{-\infty}^{0} e^x\,dx = \lim_{M\to+\infty} \underbrace{\int_{-M}^{0} e^x\,dx} = \lim_{M\to+\infty} 1-e^{-M} =$$

$$= 1-\cancel{e^{-\infty}} = 1$$

$$\int_{-M}^{0} e^x\,dx = e^0 - e^{-M} = 1-e^{-M}$$

Es

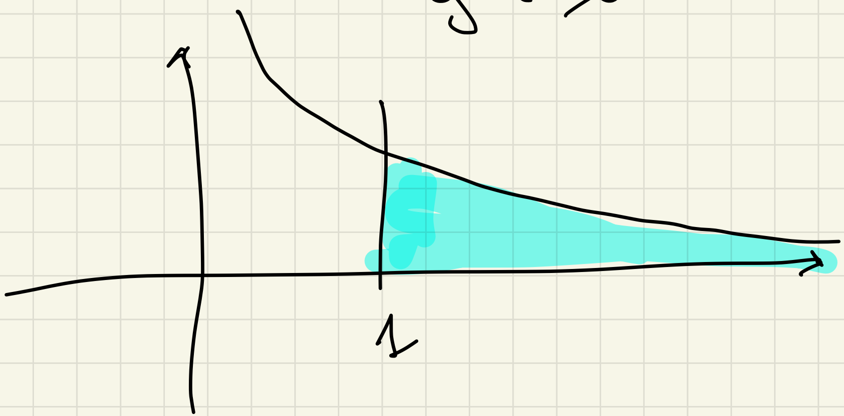
$$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{M\to+\infty} \int_1^M \frac{1}{x}\,dx = \lim_{M\to+\infty} \lg M$$

$$= \lg +\infty = +\infty$$

$$\int_1^M \frac{1}{x}\,dx = \lg M - \underbrace{\lg 1}_{0} =$$

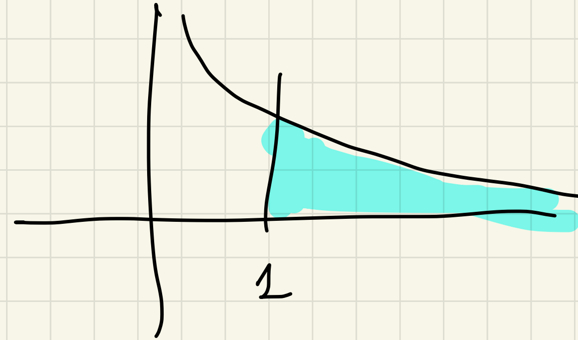
$$= \lg M - \cancel{0}$$

$$\int \frac{1}{x}\,dx = \lg |x| + c$$



? Per quali $\alpha > 0$?

$$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx < +\infty$$



per $\alpha = 1$ NO

$$\int_1^{+\infty} \frac{1}{x}\,dx = +\infty$$ (appena controllato)

$\alpha \neq 1$

$$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx = \lim_{M \to +\infty} \boxed{\int_1^{M} \frac{1}{x^\alpha}\,dx}$$

$\boxed{\alpha \neq 1}$

$$\int_1^M \frac{1}{x^\alpha}\,dx = \frac{1}{-\alpha+1} M^{-\alpha+1} - \frac{1}{-\alpha+1} 1^{-\alpha+1}$$

$-\alpha+1 = 1-\alpha$

$$= \frac{1}{-\alpha+1}\left[M^{1-\alpha} - 1\right] = \frac{1}{1-\alpha}\left[M^{1-\alpha} - 1\right]$$

$$\int \frac{1}{x^\alpha}\,dx = \int x^{-\alpha}\,dx = \frac{1}{-\alpha+1} x^{-\alpha+1} + c$$

$$\int x^k\,dx = \frac{1}{k+1} x^{k+1} + c \qquad k = -\alpha$$

$\alpha \neq 1$

$$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx = \lim_{M\to+\infty} \int_1^M \frac{1}{x^\alpha}\,dx =$$

$$= \lim_{M\to+\infty} \frac{1}{1-\alpha}\left[M^{1-\alpha} - 1\right]$$

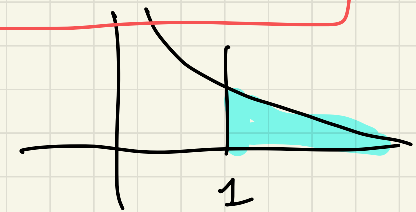
se $\alpha > 1$ $\quad 1-\alpha < 0 \quad M^{1-\alpha} \to \left(\frac{1}{\infty}\right)^{\cdot\cdot} \to 0$

$$\lim_{M\to+\infty} \frac{1}{1-\alpha}\left[\underset{\downarrow\, 0}{\cancel{M^{1-\alpha}}} - 1\right] = \frac{-1}{1-\alpha} = \frac{1}{\alpha-1} > 0$$

se $\alpha < 1$ $\quad 1-\alpha > 0 \quad M^{1-\alpha} \to (+\infty)^{1-\alpha} \to +\infty$

$$\lim_{M\to+\infty} \frac{1}{1-\alpha}\left(M^{1-\alpha} - 1\right) = +\infty$$

$$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx = \begin{cases} +\infty & \text{se} \quad \alpha \le 1 \\ \dfrac{1}{\alpha-1} & \text{se} \quad \alpha > 1 \end{cases}$$

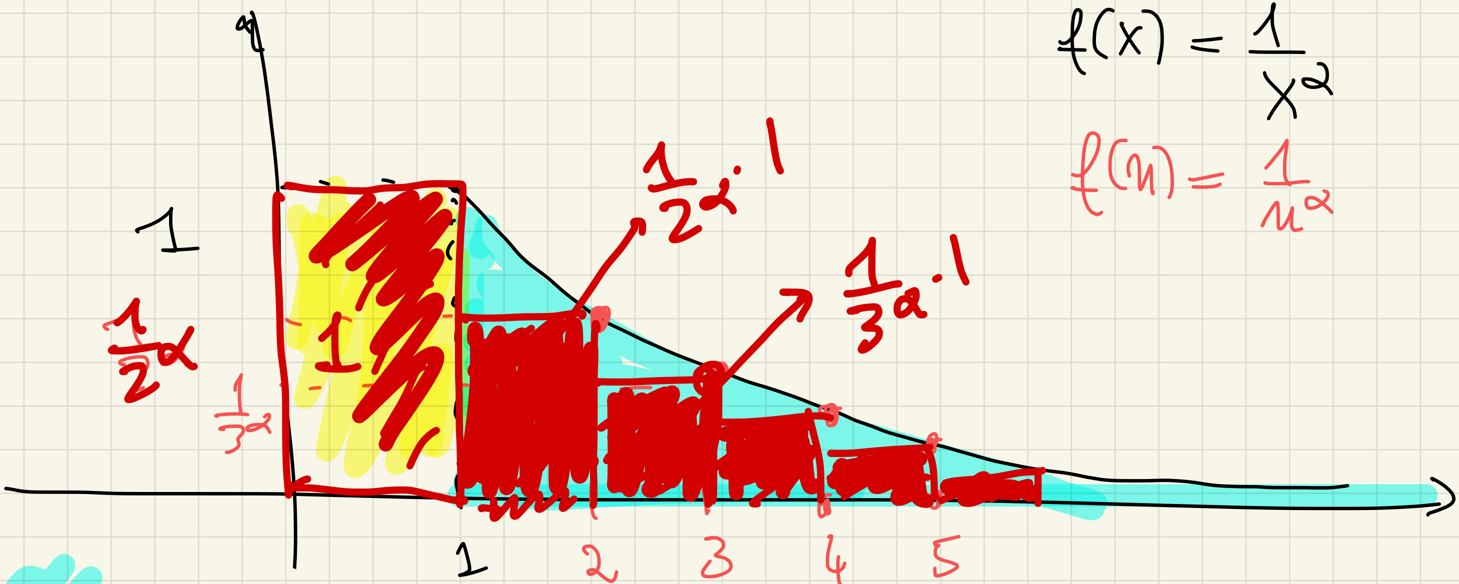


$$\int_1^{+\infty} \frac{1}{x^2}\,dx = \frac{1}{2-1} = 1 \qquad \alpha = 2$$

$$\sum_{n=1}^{+\infty} \frac{1}{n^\alpha} = \begin{cases} +\infty & \alpha \le 1 \\ \text{CONVERGENTE} & \alpha > 1 \end{cases}$$

$f(x) = \frac{1}{x^\alpha}$

$f(n) = \frac{1}{n^\alpha}$



$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx =$ area compresa tra grafico di $\frac{1}{x^\alpha}$ e asse x tra 1 e $+\infty$

$$\cancel{1} + \frac{1}{2^\alpha} + \frac{1}{3^\alpha} + \frac{1}{4^\alpha} + \dots = \sum_{n=1}^{+\infty} \frac{1}{n^\alpha} \leq 1 + \int_1^{+\infty} \frac{1}{x^\alpha}\,dx$$

AREA ROSSA

$$\sum_{n=1}^{+\infty} \frac{1}{n^\alpha} \le 1 + \int_1^{+\infty} \frac{1}{x^\alpha}\, dx$$

$\alpha > 1$

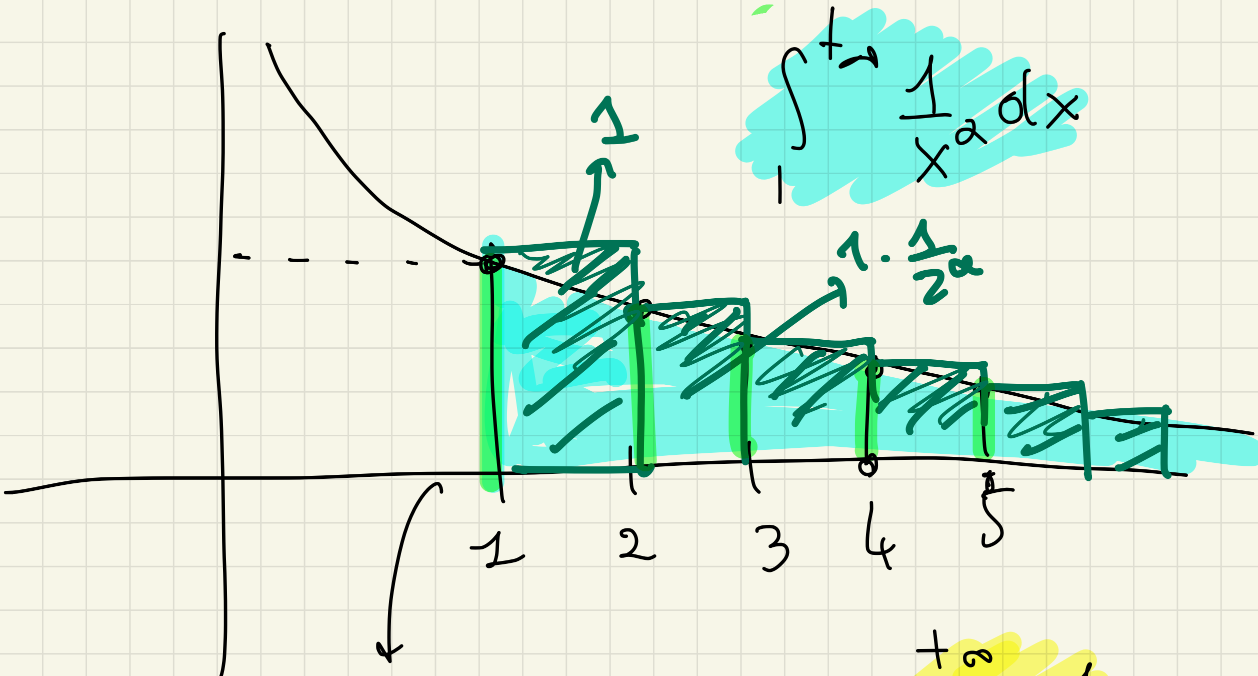
$$\boxed{\sum_{n=1}^{+\infty} \frac{1}{n^\alpha}} \le 1 + \int_1^{+\infty} \frac{1}{x^\alpha}\, dx =$$

$$= 1 + \frac{1}{\alpha-1} = \frac{\alpha \cancel{-1} \cancel{+1}}{\alpha-1} = \frac{\alpha}{\alpha-1}$$

(se $\alpha > 1$ $\sum_{n=1}^{+\infty} \frac{1}{n^\alpha}$ CONVERGE)

$$0 \le \sum_{n=1}^{\infty} \frac{1}{n^2} \le \frac{2}{2-1} = 2$$

$$f(x) = \frac{1}{x^\alpha}$$



$$1 + \frac{1}{2^\alpha} + \frac{1}{3^\alpha} + \dots = \sum_{n=1}^{+\infty} \frac{1}{n^\alpha} \geq \int_1^{+\infty} \frac{1}{x^\alpha} dx$$

$\alpha \leq 1$

$$\underbrace{\sum_{n=1}^{+\infty} \frac{1}{n^\alpha}} \geq \int_1^{+\infty} \frac{1}{x^\alpha} dx = +\infty$$

$$\boxed{\int_1^{+\infty} \frac{1}{x^\alpha}\,dx \le \sum_{n=1}^{+\infty} \frac{1}{n^\alpha} \le 1 + \int_1^{+\infty} \frac{1}{x^\alpha}\,dx}$$

per $\alpha > 1$ la serie converge e

$$\frac{1}{\alpha-1} \le \sum_{n=1}^{\infty} \frac{1}{n^\alpha} \le \frac{\alpha}{\alpha-1}$$

per $\alpha \le 1$ la serie diverge a $+\infty$.