$h(x) \neq 0 \quad \forall x \in I$

$$\int \frac{h'(x)}{h(x)}\, dx = \log |h(x)| + c \qquad \left(\log |h(x)|\right)' = \frac{1}{h(x)} \cdot h'(x) = \frac{h'(x)}{h(x)}$$

Formula di cambio di variabile

f funzione continua in I $\rightarrow$ F una sua primitiva in I (cioè $F'(x) = f(x)\ \forall x \in I$)

g continua in I

$F(g(x))$ funzione composta

$$\left(F(g(x))\right)' = F'(g(x)) \cdot g'(x) \qquad F'(x) = f(x)$$

$$= \underbrace{f(g(x))\ g'(x)}$$

FORMULA DI CAMBIO VARIABILE

$$\boxed{\int f(g(x))\ g'(x)\,dx = F(g(x)) + c}$$

↓

cambio di variabile $\quad y = g(x)$

$$\underbrace{\int f(\underline{\underline{g(x)}})\ \underline{\underline{g'(x)dx}}} = \underbrace{\int f(y)\,\underline{\underline{dy}}} = F(y) + c = F(g(x)) + c$$

$$\underbrace{y = g(x)} \rightarrow \text{"} dy = g'(x)\,dx \text{"}$$

Es $\displaystyle\int_0^1 \frac{e^x}{e^{2x}+3e^x+2}\,dx$

$$\int \frac{e^x}{\underbrace{e^{2x}+3e^x+2}}\cdot dx = \int \frac{1}{e^{2x}+3e^x+2}\, e^x\, dx$$

$$e^{2x}+3e^x+2 = (e^x)^2+3e^x+2$$

$\underline{y = e^x}$ $\qquad e^{2x}+3e^x+2 \longrightarrow y^2+3y+2$

$y = g(x) \qquad dy = g'(x)\,dx$

$y = e^x \qquad dy = (e^x)'\,dx = e^x\,dx$

$$\int \frac{1}{y^2+3y+2}\,dy$$

$$\int \frac{e^x}{e^{2x}+3e^x+2}\,dx = \boxed{\int \frac{1}{y^2+3y+2}\,dy} \qquad y=e^x$$

$\downarrow$

$$y^2+3y+2=0 \rightarrow y_{1,2} = \frac{-3\pm\sqrt{9-8}}{2} = < \begin{matrix} -1 \\ -2 \end{matrix}$$

$$1\cdot y^2+3y+2 = (y-(-1))(y-(-2)) = (y+1)(y+2)$$

$$ay^2+by+c = a(y-y_1)(y-y_2)$$

$$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases}$$

$$\frac{0\cdot y+1}{y^2+3y+2} = \frac{A}{y+1} + \frac{B}{y+2} = \frac{Ay+2A+By+B}{(y+1)(y+2)}$$

$$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases} \qquad \begin{cases} B=-A \\ 2A-A=1 \end{cases} \qquad \begin{aligned} B&=-1 \\ A&=1 \end{aligned}$$

$$\frac{1}{y^2+3y+2} = \frac{A}{y+1} + \frac{B}{y+2} = \frac{1}{y+1} - \frac{1}{y+2}$$

$$\int \frac{1}{y^2+3y+2}\,dy = \int \frac{1}{y+1}\,dy - \int \frac{1}{y+2}\,dy =$$

$$= \lg|y+1| - \lg|y+2| + c =$$

$$= \lg \frac{|y+1|}{|y+2|} + c$$

$$\int \frac{e^x}{e^{2x}+3e^x+2}\,dx = \int \frac{1}{y^2+3y+2}\,dy =$$

$$= \log \frac{|y+1|}{|y+2|} + C \qquad y = e^x$$

$$= \log \frac{|e^x+1|}{|e^x+2|} + C = \underline{\log\left(\frac{e^x+1}{e^x+2}\right)} + C$$

$$\int_0^1 \frac{e^x}{e^{2x}+3e^x+2}\,dx = \log\left(\frac{e^1+1}{e^1+2}\right) - \log\left(\frac{e^0+1}{e^0+2}\right) =$$

$$= \log\left(\frac{e+1}{e+2}\right) - \log\left(\frac{2}{3}\right) = \log\left(\frac{e+1}{e+2}\cdot\frac{3}{2}\right)$$

Volendo posso cambiare direttamente la variabile

$$\int_0^1 \frac{e^x}{e^{2x}+3e^x+2}\,dx = \qquad \boxed{y = e^x} \to dy = e^x\,dx$$

$$= \int_0^1 \frac{1}{e^{2x}+3e^x+2}\; \underbrace{e^x\,dx}_{dy}$$

$$\frac{1}{e^{2x}+3e^x+2} \to \frac{1}{y^2+3y+2}$$

$$y = e^x$$

$$\int_0^1 \dots dx \qquad x=0 \to y=e^0 \qquad x=1 \to y=e^1$$

$$= \int_{e^0}^{e^1} \frac{1}{y^2+3y+2}\,dy = \boxed{\int_1^e \frac{1}{y^2+3y+2}\,dy}$$

Es

$$\int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \underbrace{\cos\left(\frac{1}{x}\right)} \underbrace{\frac{1}{x^3}} \underbrace{dx}$$

$$0 < \frac{1}{\pi} < \frac{2}{\pi}$$

$$\left(y = g(x) \quad dy = g'(x)dx \right)$$

$$\boxed{x = h(y) \quad dx = h'(y)dy}$$

$y = \frac{1}{x} \rightarrow x = \frac{1}{y}$

$y = \frac{1}{x}$

$x = \frac{1}{y} \qquad dx = \left(\frac{1}{y}\right)' dy$

$$\boxed{dx = -\frac{1}{y^2}dy}$$

$\cos\left(\frac{1}{x}\right) \longrightarrow \cos y$

$\frac{1}{x^3} = \left(\frac{1}{x}\right)^3 \longrightarrow y^3$

$x = \frac{1}{\pi} \rightarrow y = \frac{1}{x} = \pi \qquad x = \frac{2}{\pi} \rightarrow y = \frac{\pi}{2}$

$$\boxed{\int_{1/\pi}^{2/\pi} \cos\left(\frac{1}{x}\right) \frac{1}{x^3}\, dx}$$

$$y = \frac{1}{x}$$

$$x = \frac{1}{y}$$

$$dx = \left(\frac{1}{y}\right)' dy =$$

$$= \left(-\frac{1}{y^2}\right) dy$$

$$\downarrow$$

$$\int_{\pi}^{\pi/2} \cos(y)\, y^3 \cdot \left(-\frac{1}{y^2}\right) dy =$$

$$= -\int_{\pi}^{\frac{\pi}{2}} \cos(y) \cdot \cancel{y^3} \cdot \frac{1}{\cancel{y^2}}\, dy = \boxed{\int_{\frac{\pi}{2}}^{\pi} (\cos y) \cdot y\, dy}$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_{\pi/2}^{\pi} (\cos y)\, y\, dy = \left(\cancel{\overset{0}{\sin \pi}}\right)\pi + \overset{-1}{\cos \pi} - \left[\overset{\to 1}{\sin\left(\frac{\pi}{2}\right)}\frac{\pi}{2} - \overset{\to 0}{\cos\frac{\pi}{2}}\right]$$

$$= -1 - \left[1 \cdot \frac{\pi}{2} - 0\right] = -1 - \frac{\pi}{2}$$

$$\int \cos(y)\, y\, dy = \sin y \cdot y - \int \sin y \cdot 1\, dy =$$

PER PARTI

$$\int f(y) g(y)\, dy = F(y) \cdot g(y) - \int F(y) g'(y)\, dy$$

$$f(y) = \cos y \rightarrow F(y) = \sin y$$

$$g(y) = y \rightarrow g'(y) = 1$$

$$= \sin y \cdot y - \underbrace{\int \sin y\, dy} = \sin y \cdot y - (-\cos y) + C$$

$$= \underline{\sin y \cdot y + \cos y + C}$$

Es $\int \arcsin x \, dx$

per parti:

$F(x)\,g(x) - \int F(x)\,g'(x)\,dx$

$$\int 1 \cdot \arcsin x \, dx = x \cdot \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$f(x) = 1 \rightarrow F(x) = x$

$g(x) = \arcsin x \rightarrow g'(x) = \frac{1}{\sqrt{1-x^2}}$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$y = 1 - x^2$

$x = \sqrt{1-y}$

$dx = (\sqrt{1-y})'\,dy$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$y = 1 - x^2$$

$$x^2 = 1 - y \qquad x = \sqrt{1-y}$$

$$dx = \left(\sqrt{1-y}\right)' dy = \left[(1-y)^{+1/2}\right]' dy = \frac{1}{2}(1-y)^{\frac{1}{2}-1}\cdot(-1)\,dy = -\frac{1}{2}\frac{1}{\sqrt{1-y}}\,dy$$

$$\int \frac{\cancel{\sqrt{1-y}}}{\sqrt{y}}\left(-\frac{1}{2}\right)\frac{1}{\cancel{\sqrt{1-y}}}\,dy$$

$$= -\frac{1}{2}\int \frac{1}{\sqrt{y}}\,dy$$

$$= -\frac{1}{2}\int y^{-\frac{1}{2}}\,dy = -\frac{1}{2}\left[\frac{1}{-\frac{1}{2}+1}\,y^{-\frac{1}{2}+1}\right] + C = -\frac{1}{\cancel{2}}\,\frac{1}{\frac{1}{\cancel{2}}}\,y^{\frac{1}{2}} + C$$

$$y = 1 - x^2 \ !$$

$$= -y^{\frac{1}{2}} + c = -(1-x^2)^{1/2} + c =$$

$$= \underbrace{-\sqrt{1-x^2}}\ + c \ = \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$\int \arcsin x \, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx =$$

$$= x \arcsin x - \left[-\sqrt{1-x^2}\right] + c$$

$$= x \arcsin x + \sqrt{1-x^2} + c$$

Es

$$\int_0^1 x^2 \underbrace{\sqrt{x^3+2}}\, dx = \int_2^3 \boxed{(y-2)^{2/3}}\,\sqrt{y}\,\boxed{\frac{1}{3}(y-2)^{-\frac{2}{3}}}dy$$

$$y = x^3+2 \rightarrow \sqrt{x^3+2} = \sqrt{y}$$

$$\boxed{x^3 = y-2} \qquad y = x^3+2$$

$$x = \sqrt[3]{y-2} = (y-2)^{1/3} \qquad x^2 = (y-2)^{2/3}$$

$$dx = \left((y-2)^{1/3}\right)' dy = \frac{1}{3}(y-2)^{\frac{1}{3}-1} dy = \frac{1}{3}(y-2)^{-\frac{2}{3}}dy$$

$$x=0 \rightarrow y=2 \qquad x=1 \rightarrow y=1^3+2=3$$

$$= \frac{1}{3} \int_2^3 \underbrace{\cancel{(y-2)^{\frac{2}{3}}} \cdot \cancel{(y-2)^{-2/3}}}_{\begin{array}{c} \| \\ (y-2)^0 = 1 \end{array}} \sqrt{y} \, dy = \frac{1}{3} \int_2^3 \sqrt{y} \, dy =$$

$$= \frac{1}{3} \int_2^3 y^{\frac{1}{2}} dy = \frac{1}{3} \cdot \left[\frac{2}{3} 3^{3/2} - \frac{2}{3} 2^{3/2} \right]$$

$$\left(\int y^{\frac{1}{2}} dy = \frac{1}{\frac{1}{2}+1} y^{\frac{1}{2}+1} + C = \frac{1}{\frac{3}{2}} y^{\frac{3}{2}} + C = \frac{2}{3} y^{\frac{3}{2}} + C \right.$$

$$= \frac{2}{9} \left(3^{3/2} - 2^{3/2} \right).$$

oss $\displaystyle\int \frac{e^x}{e^{2x}+3e^x+2}\,dx$

$dy = e^x\,dx.$

$\underbrace{y = e^x} \rightarrow x = \lg y$

$dx = \frac{1}{y}\,dy$

$$\int \frac{\cancel{y}}{y^2+3y+2}\,\frac{1}{\cancel{y}}\,dy$$