TECNICA di INTEGRAZIONE per PARTI

si basa sulla regola di calcolo della derivata del prodotto

f, g 2 funzioni continue in I

$$\underbrace{\left(f(x)\,g(x)\right)'} = \underbrace{f'(x)\,g(x) + f(x)\,g'(x)}$$

$$\int \left(f(x)g(x)\right)'\,dx = \text{primitiva di } \left(f(x)g(x)\right)' = f(x)\,g(x) + c$$

(qual è la funzione la cui derivata è $\left(f(x)g(x)\right)'$

la primitiva di $(f(x)\,g(x))'$ è $f(x)g(x)$

$$(f(x)g(x))' = \boxed{f'(x)\,g(x) + f(x)g'(x)}$$

quindi

$$\int f'(x)g(x) + f(x)g'(x)\,dx = f(x)g(x) + c$$

$$\downarrow$$

$$\int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx = f(x)g(x) + c$$

$$\boxed{\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx + C}$$

FORMULA DI INTEGRAZIONE per parti

es. $I=(0,+\infty)$

$$\int x \cdot \log x \, dx$$

$\downarrow$

prodotto di 2 funzioni

di una delle 2 funzioni ($g(x)$ devo calcolare la DERIVATA)

di una delle 2 funzioni $f'(x)$ devo calcolare la primitiva

$$\int x \lg x \, dx = \underbrace{\frac{1}{2}x^2 \cdot \lg x}\ - \underbrace{\int \frac{1}{2}\cancel{x^2} \cdot \frac{1}{\cancel{x}} dx}\ + C$$

di x → calcolo una PRIMITIVA $\frac{1}{2}x^2$ $\quad f(x) = x \quad F(x) = \frac{1}{2}x^2$

di $\lg x$ calcolo la DERIVATA → $\frac{1}{x}$ → $g(x) = \lg x$

$g'(x) = \frac{1}{x}$

$$= \frac{1}{2}x^2 \lg x - \underbrace{\int \frac{1}{2} x \, dx}\ + C =$$

$$= \frac{1}{2}x^2 \lg x - \frac{1}{2} \cdot \frac{1}{2}x^2 + C = \frac{1}{2}x^2 \lg x - \frac{1}{4}x^2 + C$$

RISCRIVO la formula in modo più utilizzabile

$$\int \underbrace{f(x)}\,\underbrace{g(x)}\,dx = \underbrace{F(x)}\,g(x) - \underbrace{\int F(x)\cdot g'(x)\,dx} + c$$

sia $F(x)$ una primitiva di f $\quad (\underline{F'(x)=f(x)})$

sia $g'(x)$ la derivata di g

$$\int F'(x)\,g(x)\,dx = F(x)\,g(x) - \int F(x)\,g'(x)\,dx + c$$

$$\int \lg x\,dx = \int \overset{f}{1}\cdot \overset{g}{\lg x}\,dx = x\cdot \lg x - \int \cancel{x}\,\frac{1}{\cancel{x}}\,dx + c$$

$$\begin{cases} f(x) = 1 \rightarrow F(x) = x \\ g(x) = \lg x \longrightarrow g'(x) = \dfrac{1}{x} \end{cases}$$

$$= x \lg x - \int 1\,dx + c = x \lg x - x + c$$

Es

$$\int_0^{\pi/2} x \sin x \, dx = \left[\cancel{\overset{0}{-\cos\frac{\pi}{2}}\cdot\frac{\pi}{2}} + \overset{1}{\sin\frac{\pi}{2}}\right] - \left[\cancel{\overset{1\cdot 0}{-\cos 0\cdot 0}} + \cancel{\overset{0}{\sin 0}}\right]$$

$$= 1.$$

$$\int \overset{g}{\underbrace{x}} \, \overset{f}{\underbrace{\sin x}} \, dx = F(x)\, g(x) - \int F(x)\, g'(x) + c =$$

$$f(x) = \sin x \longrightarrow F(x) = -\cos x$$

$$g(x) = x \longrightarrow g'(x) = 1$$

$$\int x \sin x \, dx = (-\cos x)\cdot x - \int (-\cos x)\cdot 1 \, dx + c$$

$$= -\cos x \cdot x + \underbrace{\int \cos x \, dx} + c = \underbrace{-\cos x \cdot x + \operatorname{sen} x} + c$$

Es

$$\int_0^{\pi} x^2 \cos(3x)\,dx$$

$$\int F(x)\,g(x)\,dx = F(x)\,g(x) + - \int F(x)\,g'(x)\,dx + c$$

$$\int x^2 \cos(3x)\,dx = \frac{1}{3}\sin(3x)\cdot x^2 - \int \frac{1}{3}\sin(3x)\cdot 2x\,dx + c$$

$$g(x) = x^2 \longrightarrow 2x = g'(x)$$

$$f(x) = \cos(3x) \longrightarrow F(x) = \frac{1}{3}\sin(3x)$$

$$= \frac{1}{3}(\sin 3x)\,x^2 - \frac{2}{3}\int \sin(3x)\cdot x\,dx + c =$$

$$= \frac{1}{3}\sin(3x)\cdot x^2 - \frac{2}{3}\left[\int \underbrace{(\sin 3x)\cdot x \, dx}\right] + C$$

$$f(x) = \sin(3x) \longrightarrow F(x) = -\frac{1}{3}\cos(3x)$$

$$g(x) = x \longrightarrow g'(x) = 1$$

$$= \frac{1}{3}\sin(3x)\cdot x^2 - \frac{2}{3}\left[\underbrace{-\frac{1}{3}\cos(3x)\cdot x}_{F\cdot g} - \underbrace{\int\left(-\frac{1}{3}\cos 3x\right)\cdot 1\, dx}_{F\cdot g'}\right] + C$$

$$= \frac{1}{3}\sin(3x)\cdot x^2 + \frac{2}{9}\cos(3x)\cdot x - \frac{2}{9}\underline{\int \cos(3x)\,dx} + C$$

$$= \frac{1}{3}\sin(3x)\, x^2 + \frac{2}{9}\cos(3x)\cdot x - \frac{2}{9}\cdot\left[\frac{1}{3}\sin(3x)\right] + C =$$

$$= \frac{1}{3}\sin(3x)\cdot x^2 + \frac{2}{9}\cos(3x)\cdot x - \frac{2}{27}\sin(3x) + C$$

$G(x)$

$$= \int x^2\cos(3x)\,dx$$

$$\int_0^{\pi} x^2\cos(3x)\,dx = G(\pi) - G(0) =$$

$$= \left[\cancel{\frac{1}{3}\sin(3\pi)\cdot\pi^2}^{\,0} + \frac{2}{9}\overbrace{\cos(3\pi)}^{-1}\pi - \cancel{\frac{2}{27}\cdot\sin(3\pi)}^{\,0}\right] -$$

$$- \left[\underbrace{\cancel{\frac{1}{3}\sin(0)\cdot 0^2}}_{0} + \underbrace{\cancel{\frac{2}{9}\cdot\cos(0)\cdot 0}}_{0} - \underbrace{\frac{2}{27}\sin(0)}_{0}\right] = \frac{2}{9}\cdot(-1)\cdot\pi$$

$$-\frac{2}{9}\pi$$

es

$$\int_0^1 \lg(1+x^2)\,dx$$

$$\int f g\,dx = F g - \int F g'\,dx$$

$$\int \lg(1+x^2)\,dx = \int 1 \cdot \lg(1+x^2)\,dx =$$

$$f(x) = 1 \longrightarrow F(x) = x$$

$$g(x) = \lg(1+x^2) \qquad g'(x) = \frac{1}{1+x^2} \cdot 2x = \frac{2x}{1+x^2}$$

$$= x \cdot \lg(1+x^2) - \int x \cdot \frac{2x}{1+x^2}\,dx + c =$$

$$= x \lg(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx + c$$

$$\frac{2x^2}{x^2+1} = \frac{\overbrace{2x^2+2}-2}{x^2+1} = \frac{\overbrace{2(x^2+1)}-2}{x^2+1} = \frac{2\cancel{(x^2+1)}}{\cancel{x^2+1}} - \frac{2}{x^2+1} =$$

$$= 2 - \frac{2}{x^2+1} \quad \left(= \frac{2x^2}{x^2+1} \right) \qquad \frac{2x^2}{x^2+1}$$

$$\int \frac{2x^2}{x^2+1}\,dx = \int \left(\overbrace{2 - \frac{2}{x^2+1}} \right) dx =$$

$$= \int 2\,dx - \int \frac{2}{x^2+1}\,dx =$$

$$= 2x - 2\,\mathrm{arctg}\,x + C$$

$$\int \lg(1+x^2)\,dx = x\lg(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx + C =$$

$$= x\lg(1+x^2) - \left[2x - 2\,\mathrm{arctg}\,x\right] + C =$$

$$= \underbrace{x\lg(1+x^2) - 2x + 2\,\mathrm{arctg}\,x} + C$$

$$\int_0^1 \lg(1+x^2)\,dx = \left[1\cdot\lg(1+1) - 2 + 2\,\mathrm{arctg}\,1\right] -$$

$$- \left[0\;\cancel{\lg(1+0)} - \cancel{2\cdot 0} + 2\,\cancel{\mathrm{arctg}\,0}\right]$$

$$= \lg 2 - 2 - 2\,\pi/4$$

Es $\displaystyle\int_0^3 \underbrace{x^2 e^{-2x}} dx$

$$\int x^2 \underbrace{e^{-2x}} dx = -\frac{1}{2} e^{-2x} \cdot x^2 - \int \left(-\frac{1}{\cancel{2}} e^{-2x}\right) \cdot \cancel{2} x \, dx + c$$

$f(x) = e^{-2x} \longrightarrow F(x) = -\frac{1}{2} e^{-2x}$

$g(x) = x^2 \longrightarrow g'(x) = 2x$

$$= -\frac{1}{2} e^{-2x} x^2 + \int \underbrace{e^{-2x}} \cdot x \, dx + c =$$

$f(x) = e^{-2x} \rightarrow F(x) = -\frac{1}{2} e^{-2x}$ $\quad g(x) = x$ $\quad g' = 1$

$$= -\frac{1}{2}e^{-2x}\cdot x^2 + \left(-\frac{1}{2}e^{-2x}\right)x - \int\left(-\frac{1}{2}e^{-2x}\right)\cdot 1\,dx + c =$$

$$= -\frac{1}{2}e^{-2x}\cdot x^2 - \frac{1}{2}e^{-2x}\cdot x + \frac{1}{2}\int e^{-2x}\,dx + c$$

$$= -\frac{1}{2}e^{-2x}\cdot x^2 - \frac{1}{2}e^{-2x}\cdot x + \frac{1}{2}\cdot\left(-\frac{1}{2}e^{-2x}\right) + c$$

$$= -\frac{1}{2}e^{-2x}x^2 - \frac{1}{2}e^{-2x}\cdot x - \frac{1}{4}e^{-2x} + c$$

$$\int_0^3 x^2 e^{-2x}\,dx = \left[-\frac{1}{2}e^{-6}\cdot 9 - \frac{1}{2}e^{-6}\cdot 3 - \frac{1}{4}e^{-6}\right] - \left[0 + 0 - \frac{1}{4}\right]$$

$$= e^{-6}\left[-\frac{9}{2}-\frac{3}{2}-\frac{1}{4}\right]+\frac{1}{4}=$$

$$= \underbrace{e^{-6}\left[-\frac{25}{4}\right]}+\frac{1}{4} > 0$$