Analytic Hierarchy Process

AHP

Learning to choose is hard. Learning to choose well is harder. And learning to choose well in a world of unlimited possibilities is harder still, perhaps too hard. Barry Schwartz

- Making decisions in complex environments: there is no unique solution

Paradox of choice

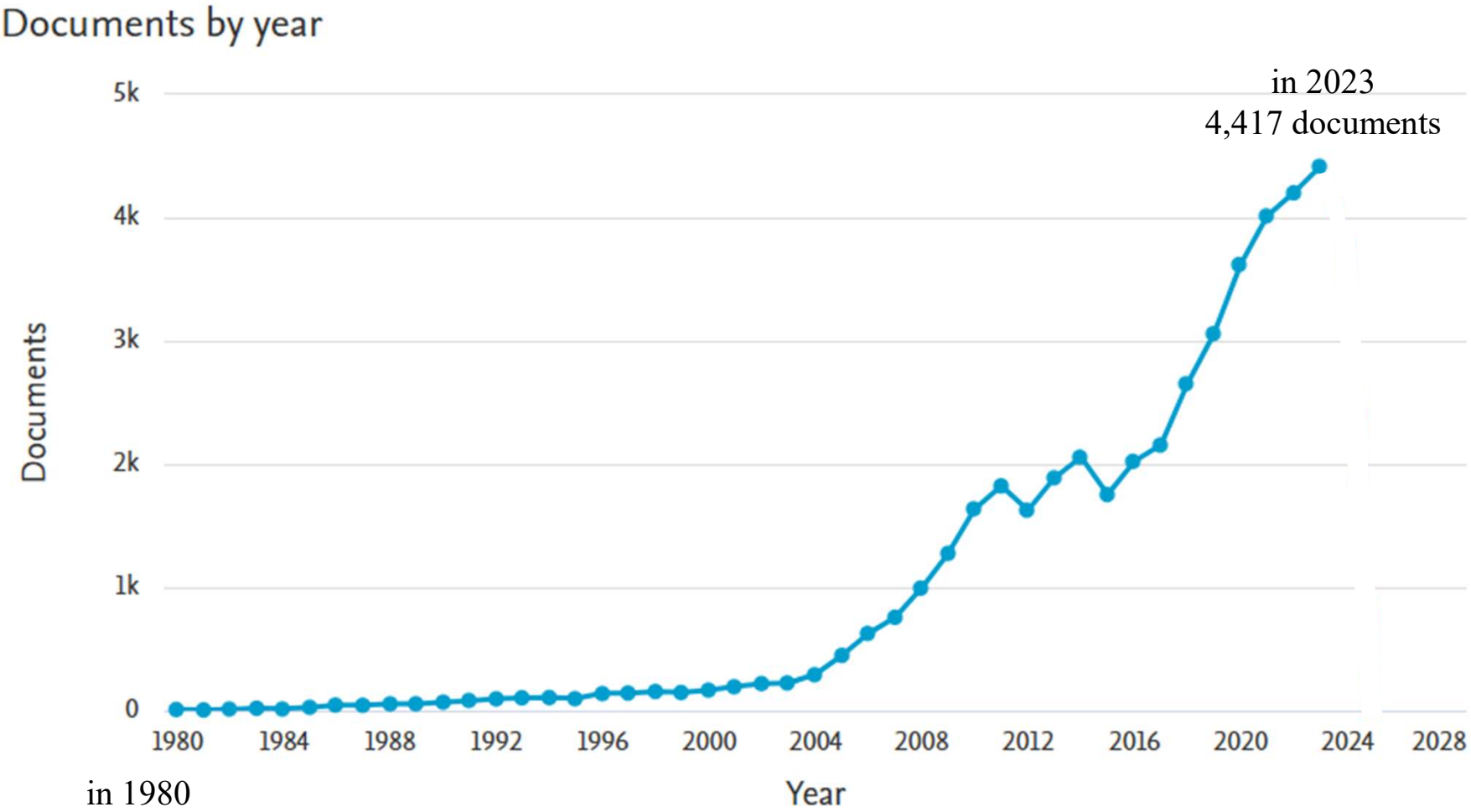
- In making simple choices, an alternative is better than the others
- When problems are complex, an alternative can be better than the others in some situations, but not in other situations
- In complex problems different alternatives can be equally valid

AHP

- AHP was developed in the **late 197**0s. Today it is the most widely used MCDA method.
- AHP generates all criteria weighting and alternative preferences within each criteria by eliciting these values from the decision-maker through a series of pairwise comparisons, as opposed to utilizing numerical values directly.
- Thus, **a complex decision is reduced to a series of simpler ones**, **between pairs of alternative values within criteria or between pairs of criteria**. The decision maker's preference is always explicit. However, the decision-maker may be asked to make very many small decisions. Hence, it becomes important to generate an optimized hierarchy of criteria and alternatives, to reduce the number of pairwise decisions.

AHP

Search in Scopus: number od documents including «AHP» within abstract, title, keywords:

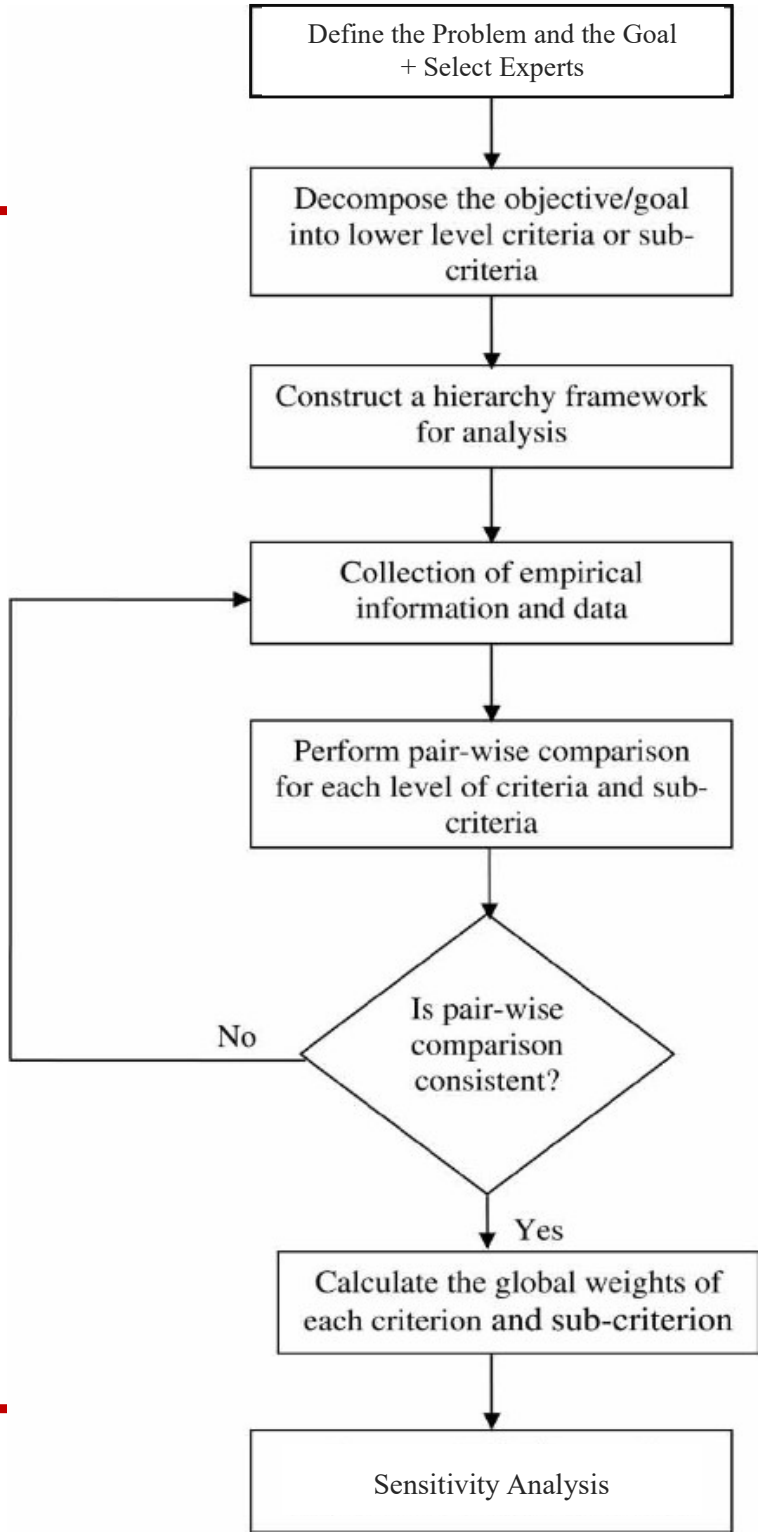


SUBJECTIVITY ≠ ARBITRARINESS

- The AHP permits the measurement of intangibles through expert judgments
- It permits to choose the ‘best’ alternative among a discrete set of alternatives, simplifying the choice
- Unlike common optimization methods, which assume the availability of ‘measures’, the AHP uses measures derived or interpreted subjectively, which are indicators of preference
- (Judgments are influenced by past experience)

AHP – Flow Chart

- Analytic: breaks down the problem into its components
- Hierarchy: structures the problem components in a hierarchical way with respect to the main objective and sub-objectives
- Process: processes judgments and data in order to reach the final result



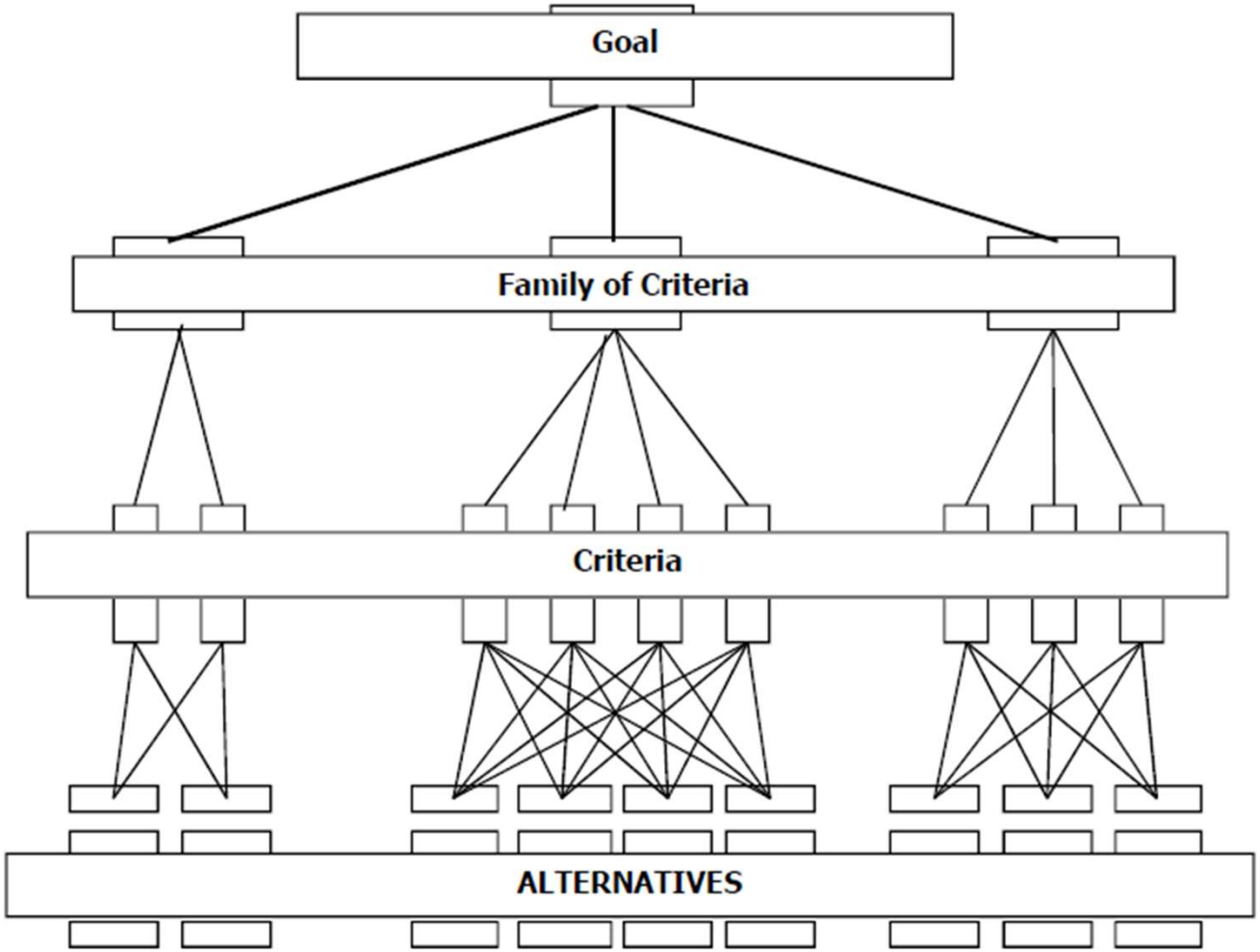
AHP – Simplified Flow Chart

First phase: Construction of the Hierarchy

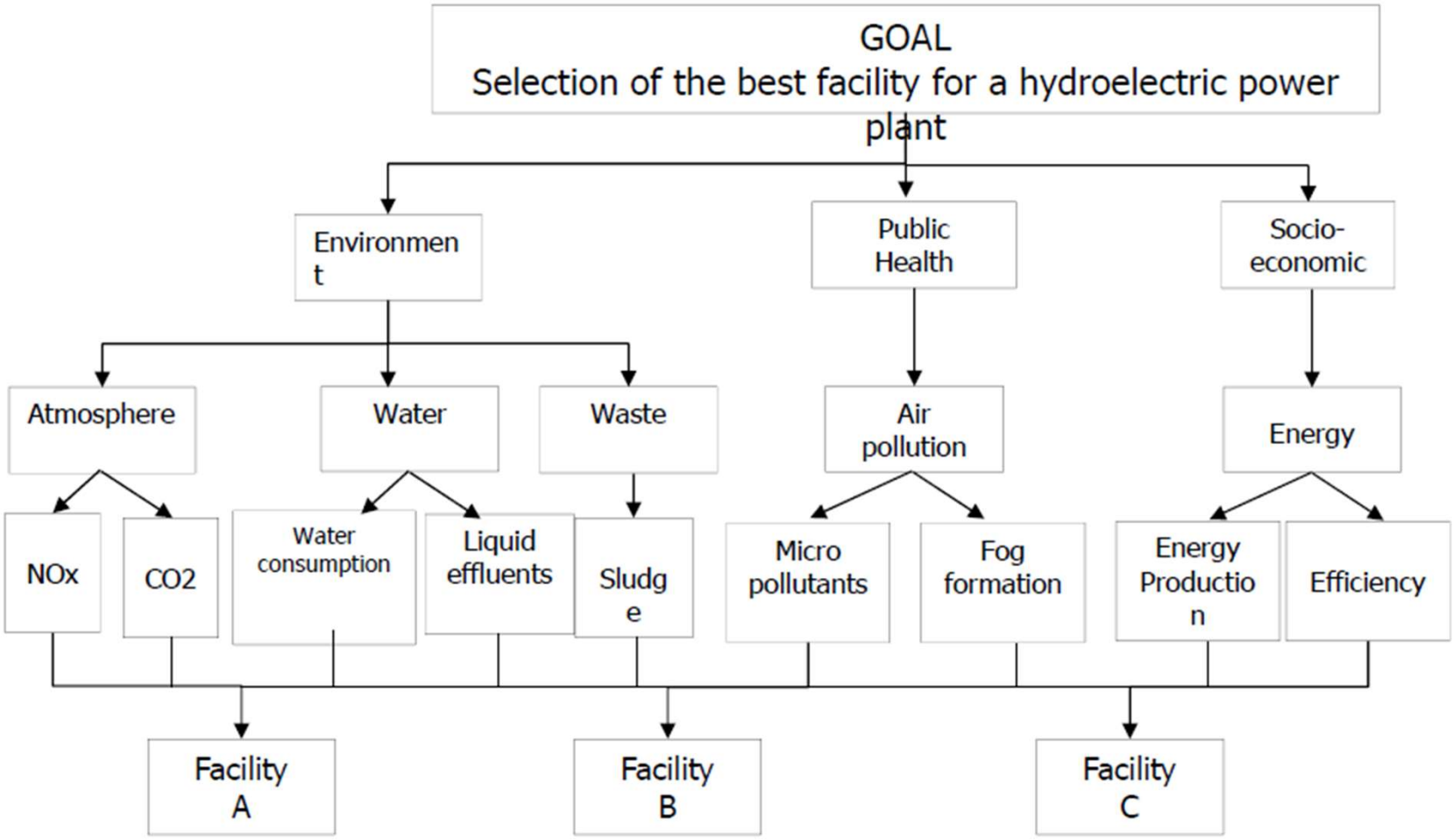
Second phase: Pairwise Comparisons

Third phase: Inconsistency Index Calculation

GERARCHIA

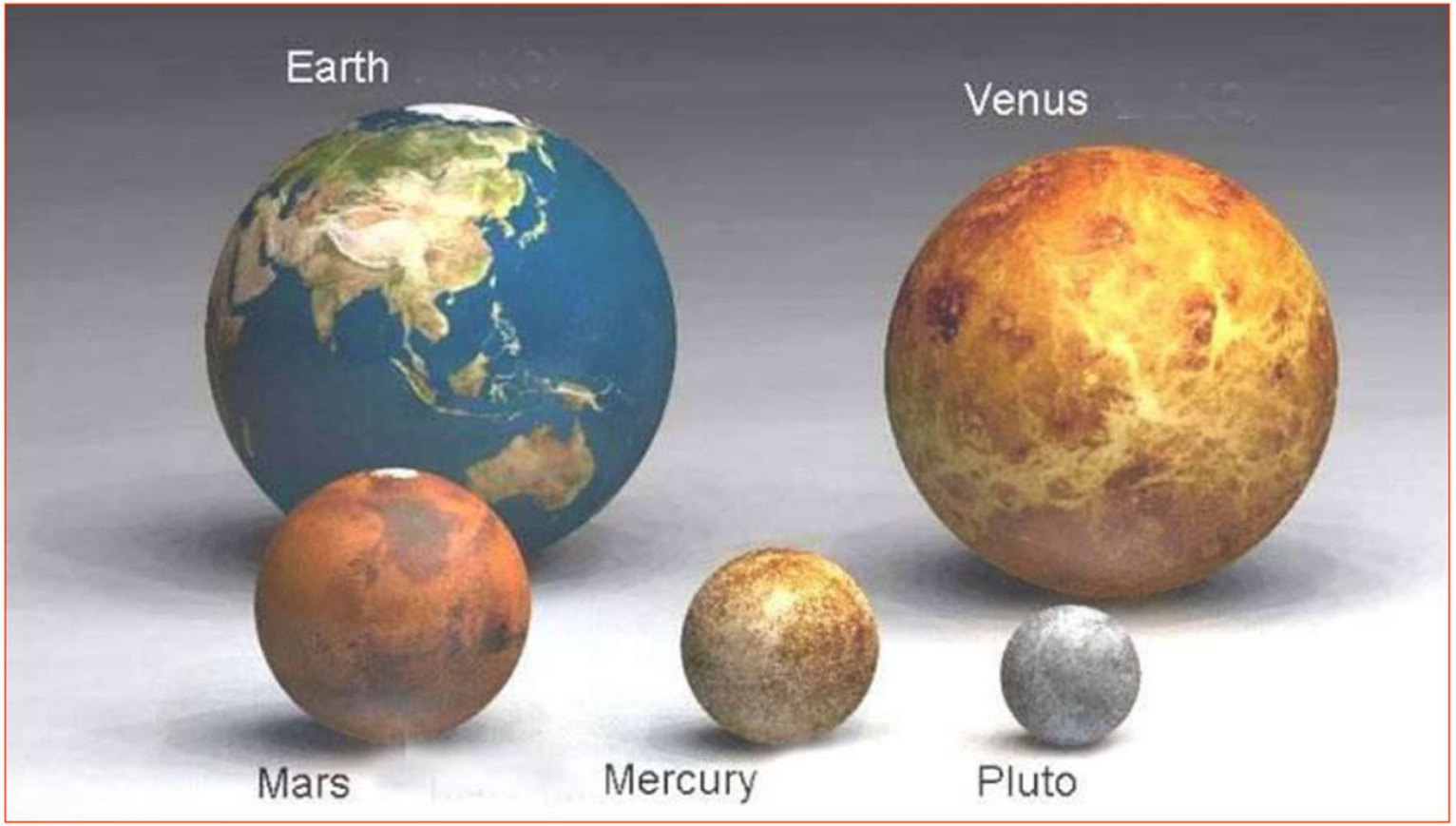


GERARCHIA (Relative Model)

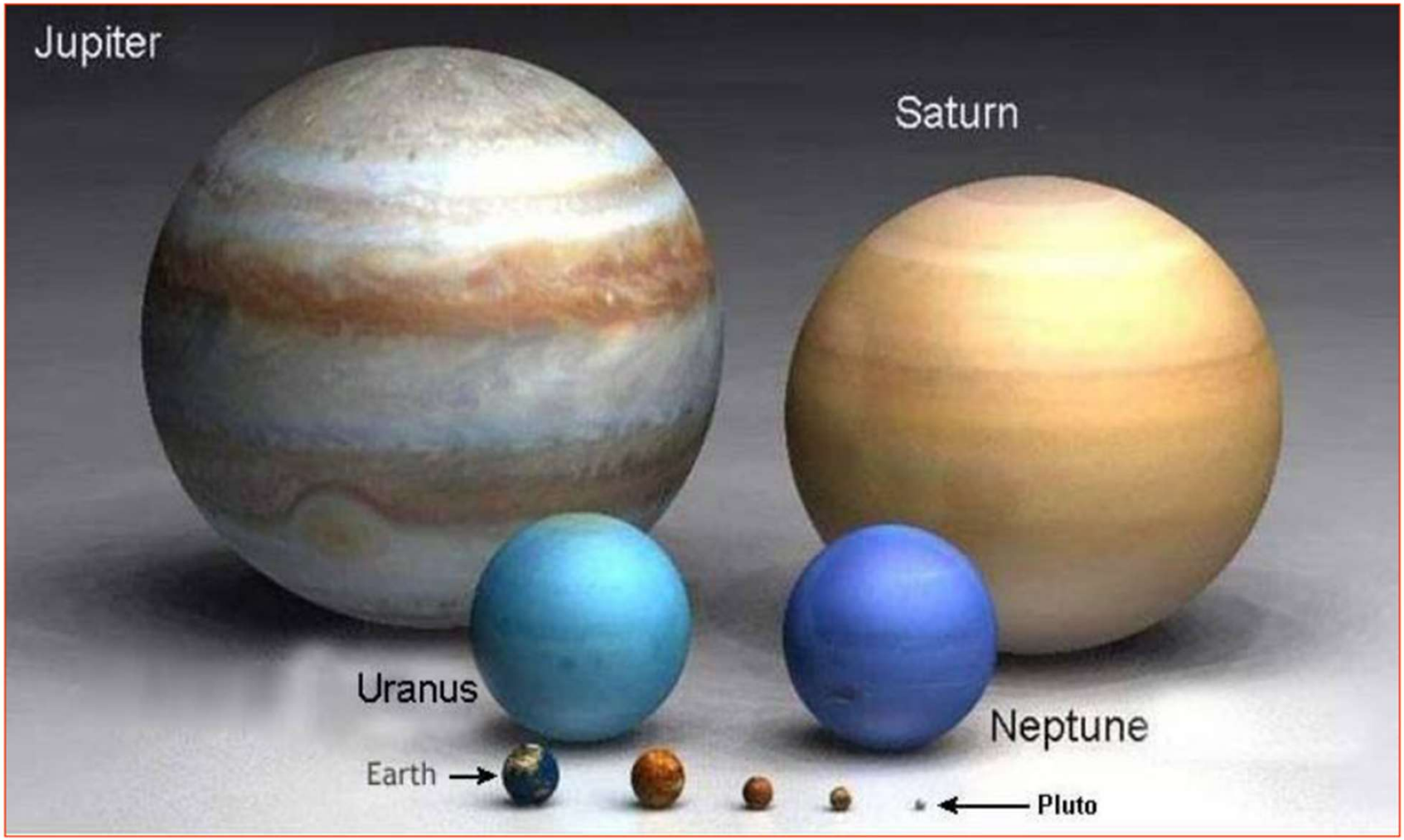


Measurement Scales

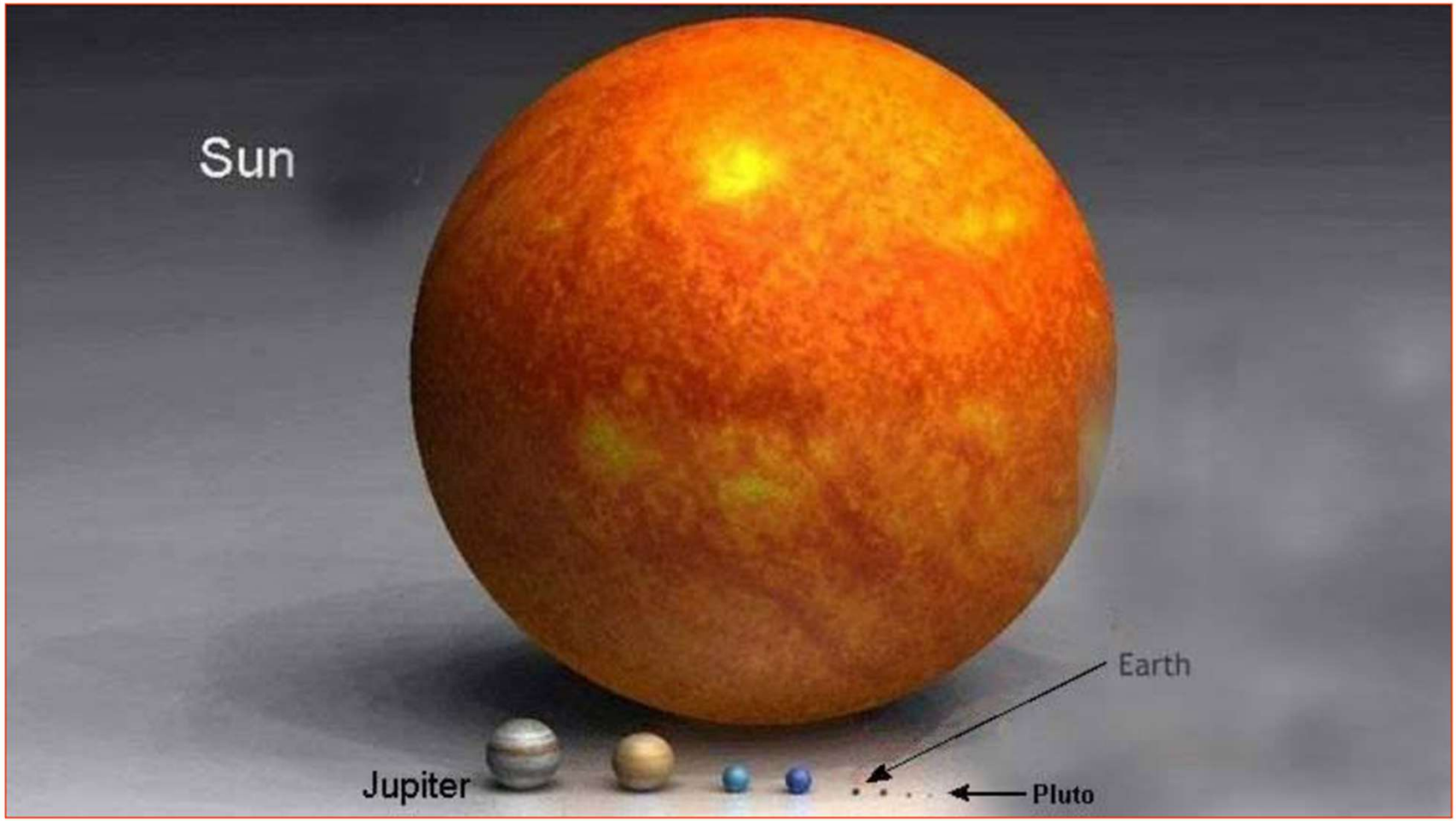
To take sound decisions, it is necessary to use appropriate scientific methods as well as appropriate measurement scales



Measurement Scales



Measurement Scales



Measurement Scales: Saaty's scale:

Saaty's semantic/fundamental scale

Numerical value	Description
1	Equal importance
3	Slight importance of one over another
5	Moderate importance of one over another
7	Very strong importance
9	Extreme importance of one over another
2,4,6,8	Intermediate values between two adjacent values

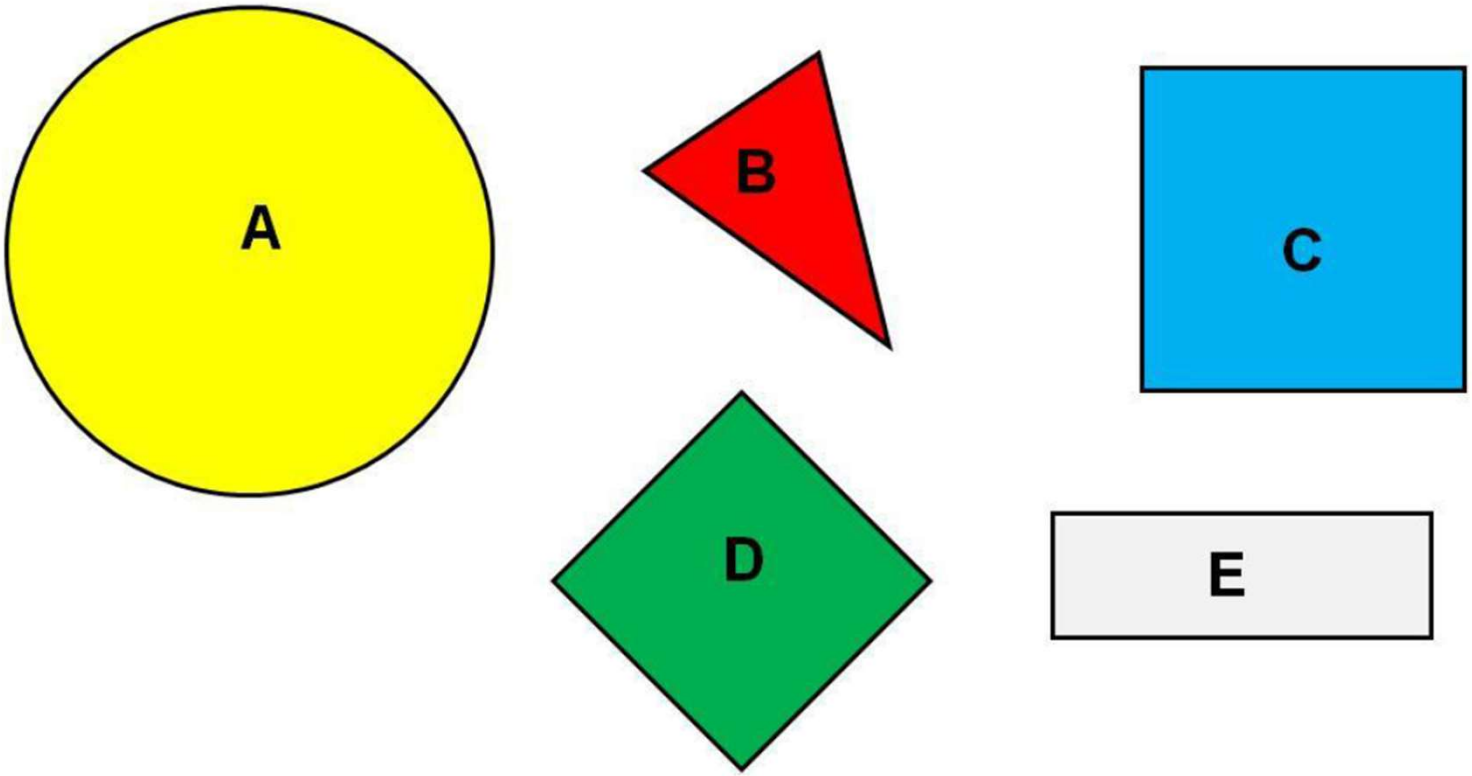
Source: Saaty (1980)

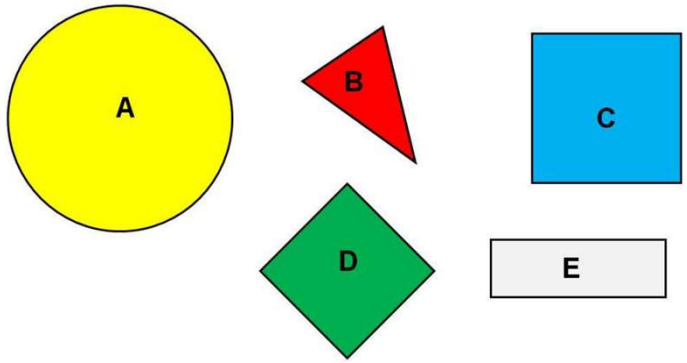
Measurement Scales: Saaty's scale:

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one
5	Strong importance	Experience and judgment strongly one actively over another
7	Very Strong Importance	An activity is favored very strongly over another its dominance demonstrated in practice
9	Extreme Importance	The evidence of favoring over another is of the highest possible area of affirmation
Reciprocal	1/2=0.500, 1/3=0.333, 1/4=0.250, 1/5=0.200, 1/6=0.1667, 1/7=0.1428, 1/8=0.125, 1/9=0.1111	If activity has one of the above non zero numbers assigned to it when compared with activity j then j has the reciprocal value which compared with i.

Source: Saaty (1980)

Measurement Scales: Saaty's scale:



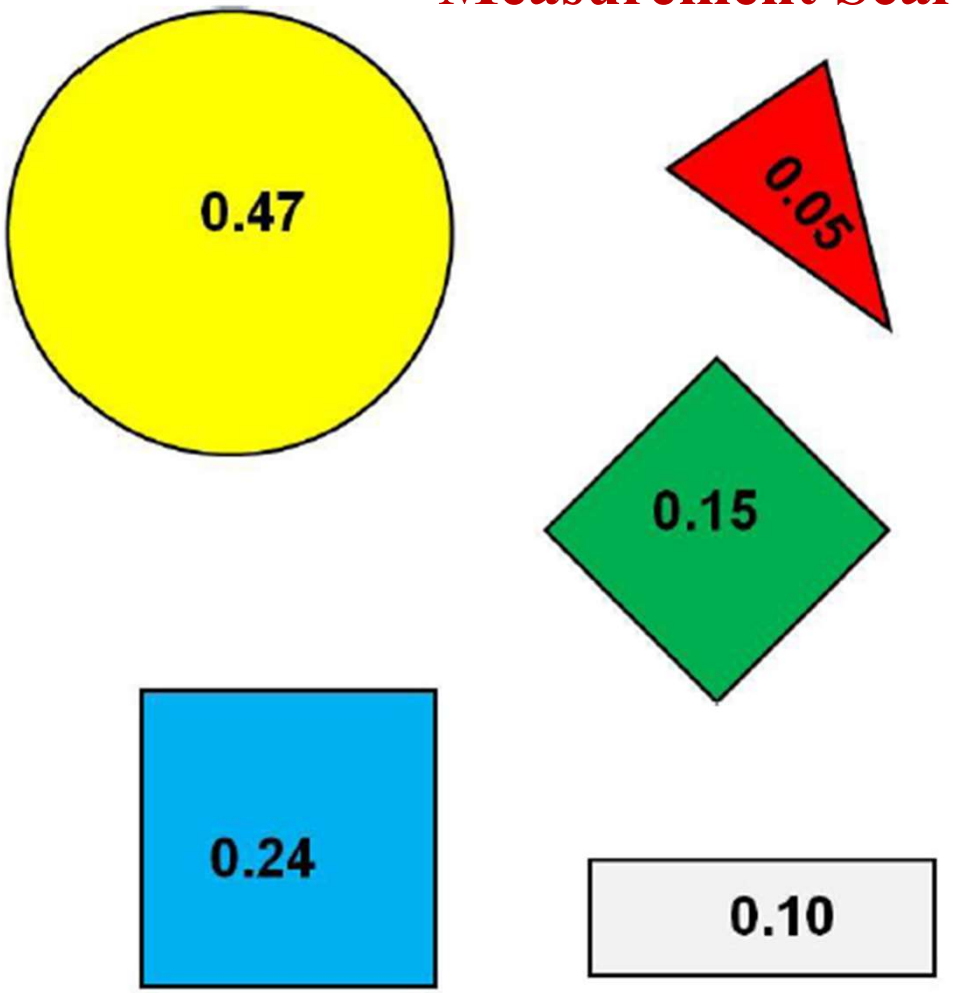


Measurement Scales: Saaty's scale:

		Circle	Triangle	Square	Diamond	Rectangle	PRIORITY VECTOR
		A	B	C	D	E	
Circle	A	1	9	2	4	5	0,48
Triangle	B	1/9	1	1/5	1/3	1/2	0,049
Square	C	1/2	5	1	2	3	0,25
Diamond	D	1/4	3	1/2	1	2	0,138
Rectangle	E	1/5	2	1/3	1/2	1	0,085

AREA ESTIMATION

Measurement Scales: Saaty's scale:



If we calculated the areas via mathematical rules we obtain: 0.47; 0.05; etc.

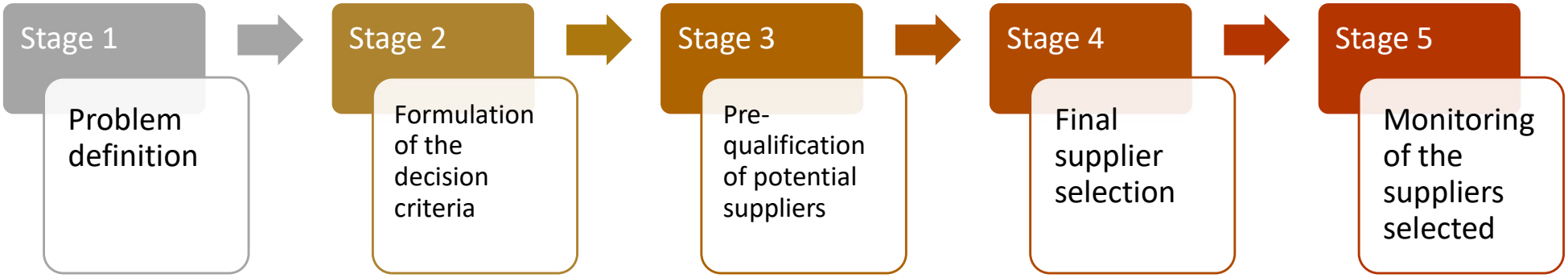
The aim of the example is to show that whenever judgments are sound and consistent results are reliable

The largest is the area of the circle, the smallest is the area of the triangle

Example

New assembly line. Some components must be purchased from the factory

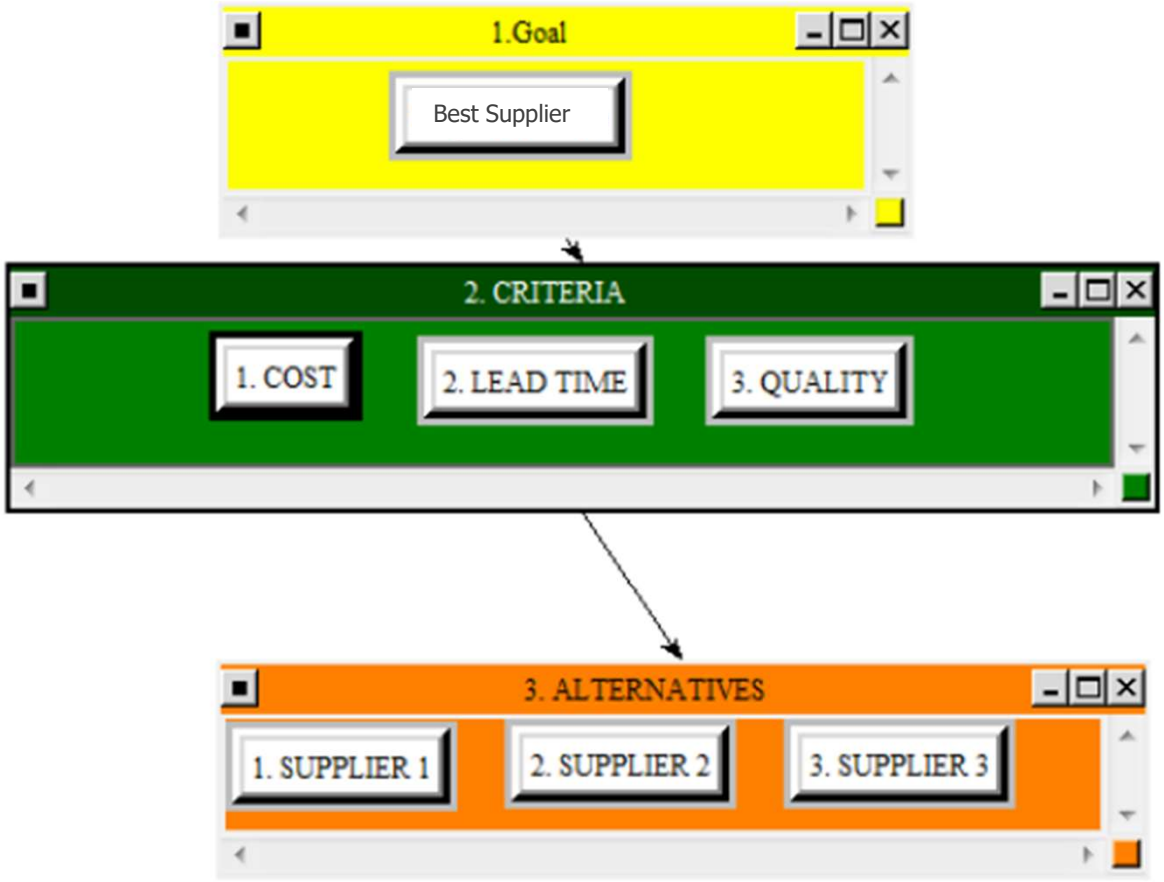
PURPOSE: Identify the best supplier



CRITERIA: Cost, Lead Time (LT), Quality, Efficiency, R&D initiatives

Example

Hierarchy



PAIRWISE COMPARISON

- Elements at the same hierarchical level are pairwise compared with respect to their parental node

- Elements are compared to elicit which element is (relatively) more important with respect to their parental node and how much

- Dominance coefficients a_{ij} represent the relative importance of a specific criterion, sub-criterion or action in comparison to another criterion, subcriterion or action

$$A = \begin{vmatrix} a_{11} \ldots\ldots a_{1n} \\ \\ a_{n1} \ldots\ldots a_{nn} \end{vmatrix}$$

PAIRWISE COMPARISON

- Dominance coefficient a_{ij} identifies the relative importance of the component on row **i** over the component on column **j**
- Elements' relative importance is determined through pairwise comparisons expressed in semantic judgments
- This scale was developed taking into account studies on the ability of the human brain to classify a finite number of elements
- The larger the number of variables, the more inconsistent the results (Saaty, 1980) as the probability to maintain the same hierarchy among variables decreases

PAIRWISE COMPARISON

- The decision-maker can easily answer to questions that require a qualitative judgment such as: *'Are they equally important? Is it much more important?...'*
- Semantic judgments are converted into numerical values according to Saaty's fundamental scale
- It is therefore possible to compile the pairwise comparison matrix using the qualitative judgments of the decision maker

N.B.: **the result of the comparison is the dominance coefficient a_{ij}** which represents an estimate of the dominance of the first element (**i**) with respect to the second (**j**). **The analysis involves the conversion of the dominance coefficients into relative scores** ($a_{ij} = w_i / w_j$).

PAIRWISE COMPARISON

If judgments are perfectly coherent, the matrix of pairwise comparisons is symmetric, reciprocal and consistent, i.e. it satisfies the three following conditions:

$$a_{ij} = a_{ji} = 1;$$
$$a_{ij} = 1/a_{ij}$$
$$a_{ij}a_{ik}=a_{ik} \quad i,j,k=1...n$$

		j			
		A	B	C	D
i	A	1	4	3	7
	B	1/4	1	1	2
	C	1/3	1	1	2
	D	1/7	1/2	1/2	1

- **The diagonal = 1.** In fact, in the comparison with itself (A with A) there is parity, and, according to the Saaty scale, it is = to 1.
- By comparing A to B, A is preferred to B by 4; consequently, by comparing B to A, B gets ¼
- aij = 1/aij satisfies **the symmetry of value judgments**
- E.g. if A is worth twice as B (A=2B), then necessarily B is worth half of A (B = ½ A)

WEIGHT CALCULATION

The elements of the corresponding eigenvector, normalized with respect to their maximum value, represent the weights of the elements with respect to the parental node for which the pairwise comparison matrix is compiled

For each raw we calculate the 'weight', obtained by calculating the n-th root of the multiplication of the elements of each raw

		j								
		A	B	C	D	Matrix Rank	Weights (Xi)	Normalization	Coeff.	Enginvalue
i	A	1	4	3	7	4	3.027			
	B	1/4	1	1	2		0.841			
	C	1/3	1	1	2		0.904			
	D	1/7	1/2	1/2	1		0.435			
	Total Yj	**1.726**	**6.500**	**5.500**	**12.000**		**5.207**			

WEIGHT CALCULATION

Weights normalization: Σ i=5.207 = this sum must be set to 1

(e.g. 3.027/5.207 = 0.581)

		j								
		A	B	C	D	Matrix Rank	Weights (Xi)	Normalization	Coeff.	Enginvalue
i	A	1	4	3	7	4	3.027	0.581		
	B	1/4	1	1	2		0.841	0.162		
	C	1/3	1	1	2		0.904	0.174		
	D	1/7	1/2	1/2	1		0.435	0.083		
	Total Yj	**1.726**	**6.500**	**5.500**	**12.000**		**5.207**	**1.000**		

WEIGHT CALCULATION

We then calculate the coefficients (ideal weights).
The highest weight is set to 1, the others are set equal to Pi/Pmax.
In this example 0.581 is set to 1, then 0.162/0.581=0.278, ….

		j								
		A	B	C	D	Matrix Rank	Weights (Xi)	Normalization	Coeff.	Enginvalue
i	A	1	4	3	7	4	3.027	0.581	**1.000**	
	B	1/4	1	1	2		0.841	0.162	0.278	
	C	1/3	1	1	2		0.904	0.174	0.298	
	D	1/7	1/2	1/2	1		0.435	0.083	0.144	
	Total Yj	**1.726**	**6.500**	**5.500**	**12.000**		**5.207**	**1.000**		

WEIGHT CALCULATION

To calculate the eigenvalue: Xi * (total Yj)/(total Xi)

A: (3.027)x(1.726) / (5.207)=1.004

		j								
		A	B	C	D	Matrix Rank	Weights (Xi)	Normalization	Coeff.	Enginvalue
i	A	1	4	3	7	4	**3.027**	0.581	**1.000**	1.004
	B	1/4	1	1	2		0.841	0.162	0.278	1.050
	C	1/3	1	1	2		0.904	0.174	0.298	0.955
	D	1/7	1/2	1/2	1		0.435	0.083	0.144	1.002
	Total Yj	**1.726**	**6.500**	**5.500**	**12.000**		**5.207**	**1.000**		

PAIRWISE COMPARISON - Consistency Index

- Unlike other multi-criteria approaches, the AHP **tolerates** some **inconsistency in expert judgments**
- The consistency of pairwise comparison matrices is verified by determining the **consistency index CI**:

$$CI = (\lambda max - n) / (n-1)$$

- Then the consistency ratio is obtained:

$$CR = CI/RI$$

where ***RI is a random consistency index***, which depends on n.

PAIRWISE COMPARISON - Consistency Index

- **CR < 0.1 is considered acceptable**
- Whenever CR > 0.1, experts' judgments are inconsistent, and a revision of the pairwise comparison matrix is recommended
- If the pairwise comparison matrix A is perfectly consistent (judgments are perfectly coherent), then the maximum eigenvalue λmax is equal to its rank n (Perron-frobenius theorem), therefore CI=0
- When inconsistency increases, the CR increases (CI also)

Size of Matrix (n)	Random Consistency Index (RI)
1	0
2	0
3	0.52
4	0.89
5	1.11
6	1.25
7	1.35
8	1.40
9	1.45
10	1.49

PAIRWISE COMPARISON - Consistency Index

- **CI**: CI = (λmax – n) / (n-1) = (4.010-4)/3 = **0.003**

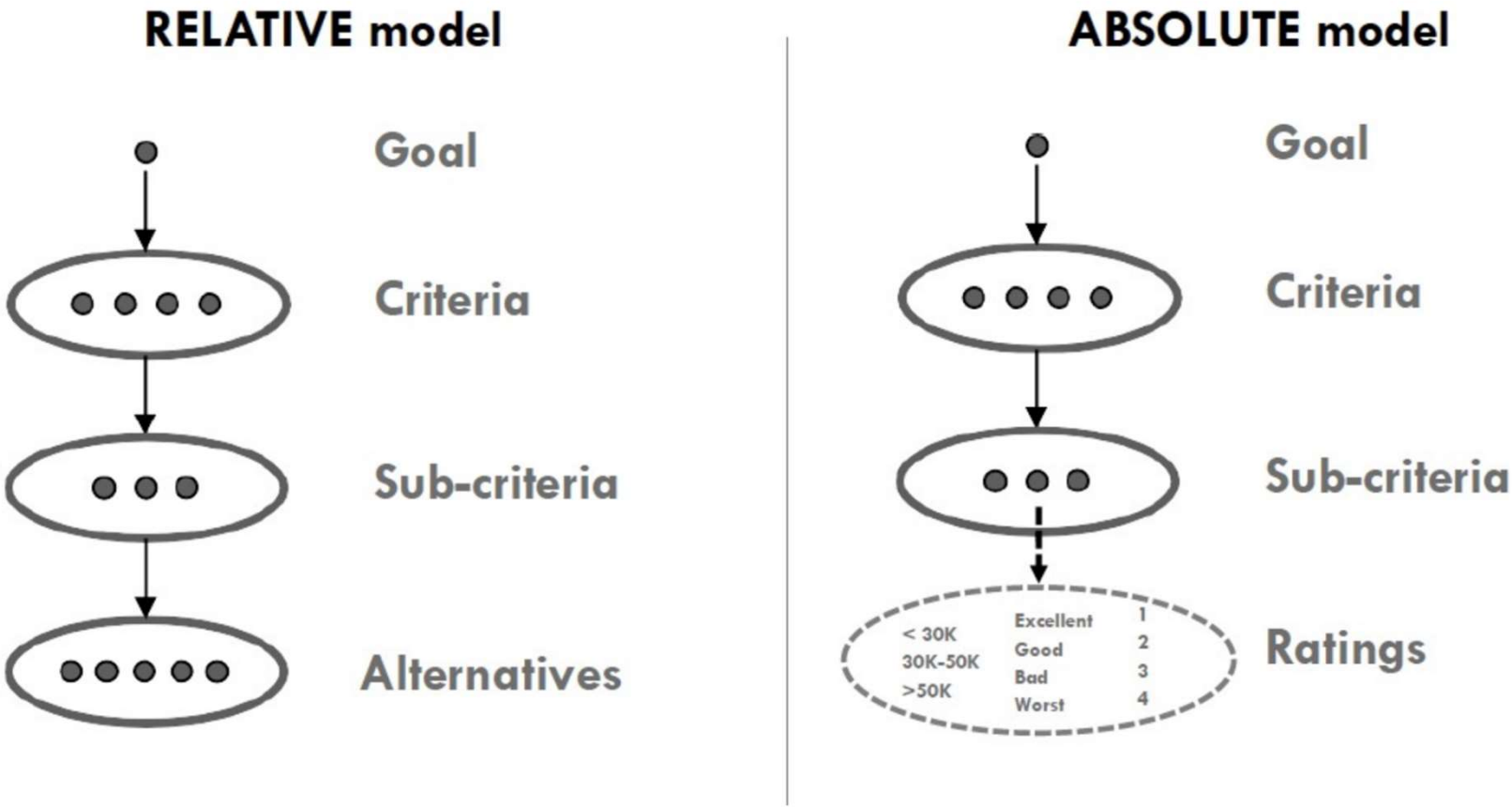
  where: λmax = 4.010; n= 4; n-1 = 3

- Then the consistency ratio is obtained **CR**=CI/RI where RI(n=4)=0.89;

  CR= **0.003 / 0.89 = 0.04 < 0.1**

		j								
		A	B	C	D	Matrix Rank	Weights (Xi)	Normalization	Coeff.	Enginvalue
i	A	1	4	3	7	4	3.027	0.581	1.000	1.004
	B	1/4	1	1	2		0.841	0.162	0.278	1.050
	C	1/3	1	1	2		0.904	0.174	0.298	0.955
	D	1/7	1/2	1/2	1		0.435	0.083	0.144	1.002
	Total Yj	1.726	6.500	5.500	12.000		5.207	1.000	**lmax**	**4.010**
									CI	**0.003**
									RI	**0.890**
									CR	**0.004**

RELATIVE VS ABSOLUTE MODELS



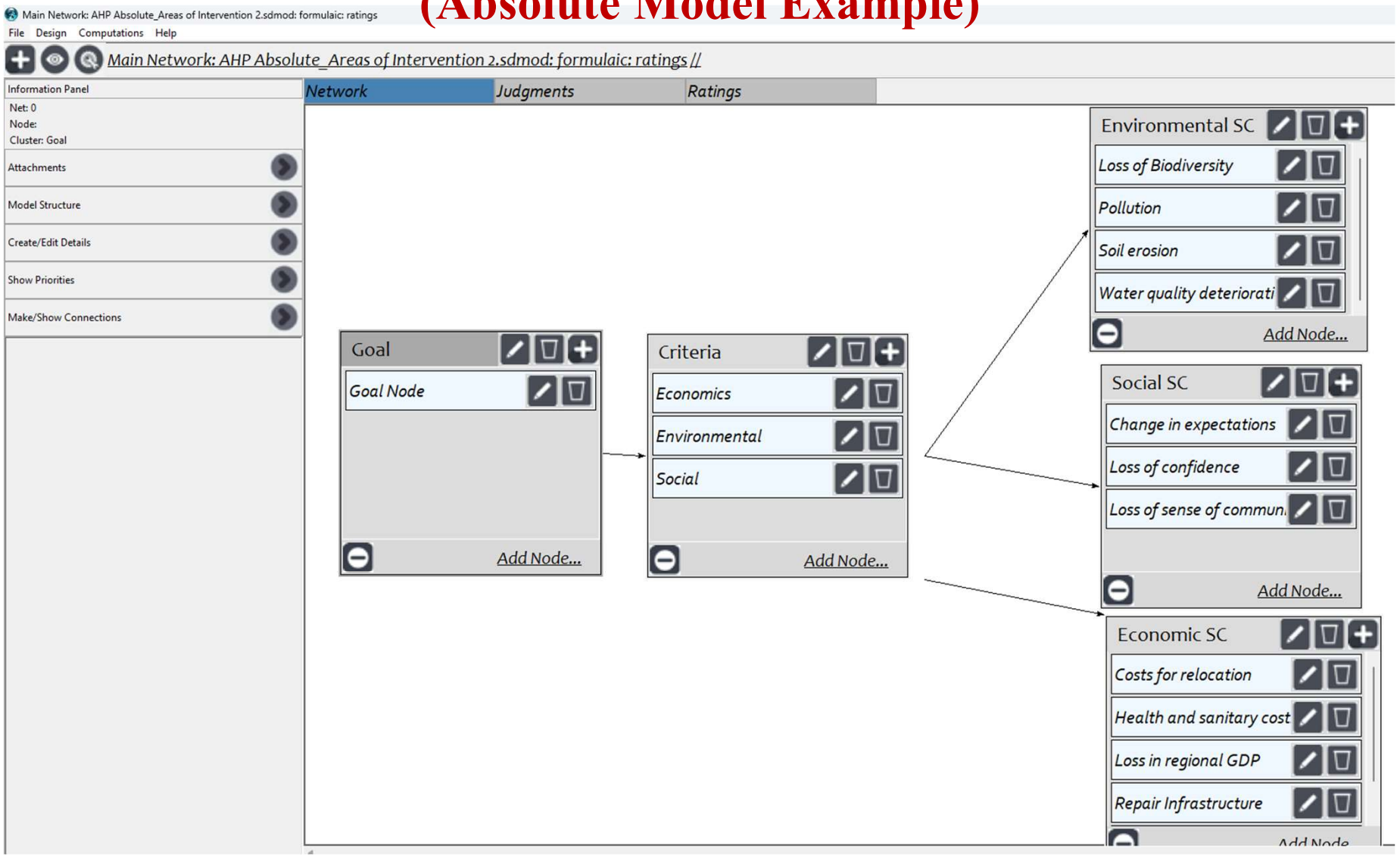
AHP – SUPER DECISION SOFTWARE

https://www.superdecisions.com/

Downloads: https://www.superdecisions.com/downloads/

AHP – SUPER DECISION SOFTWARE

(Absolute Model Example)



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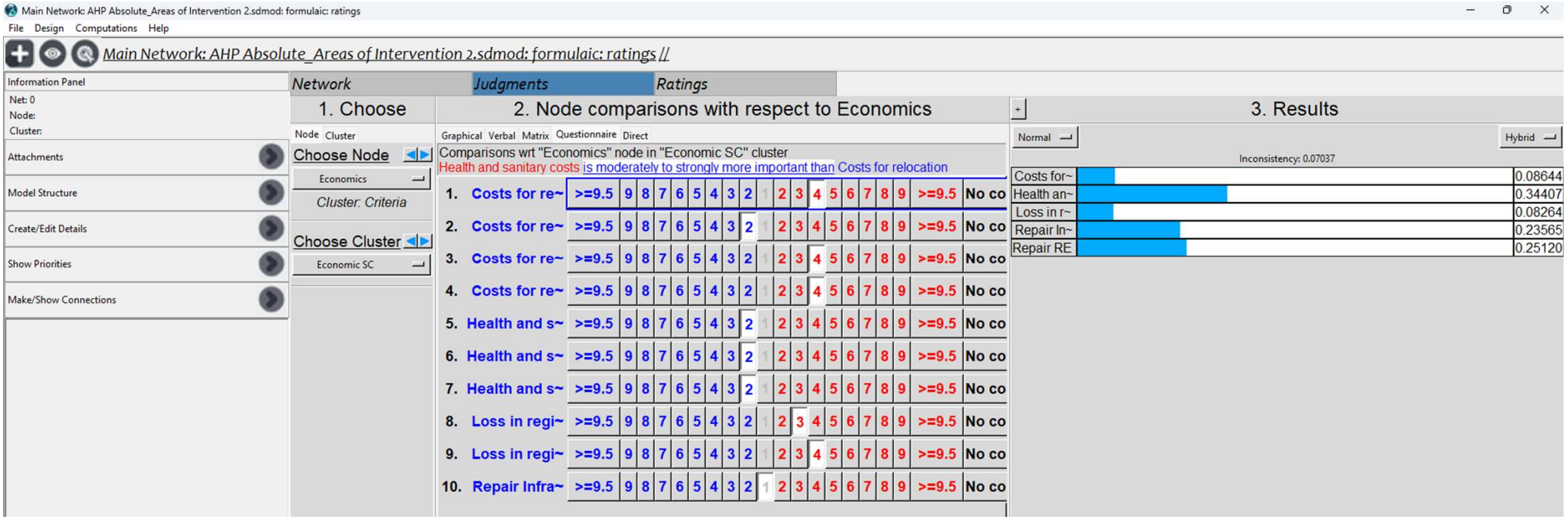
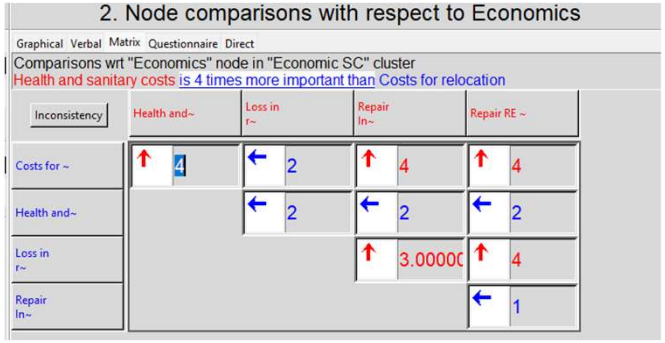
Questionnaire





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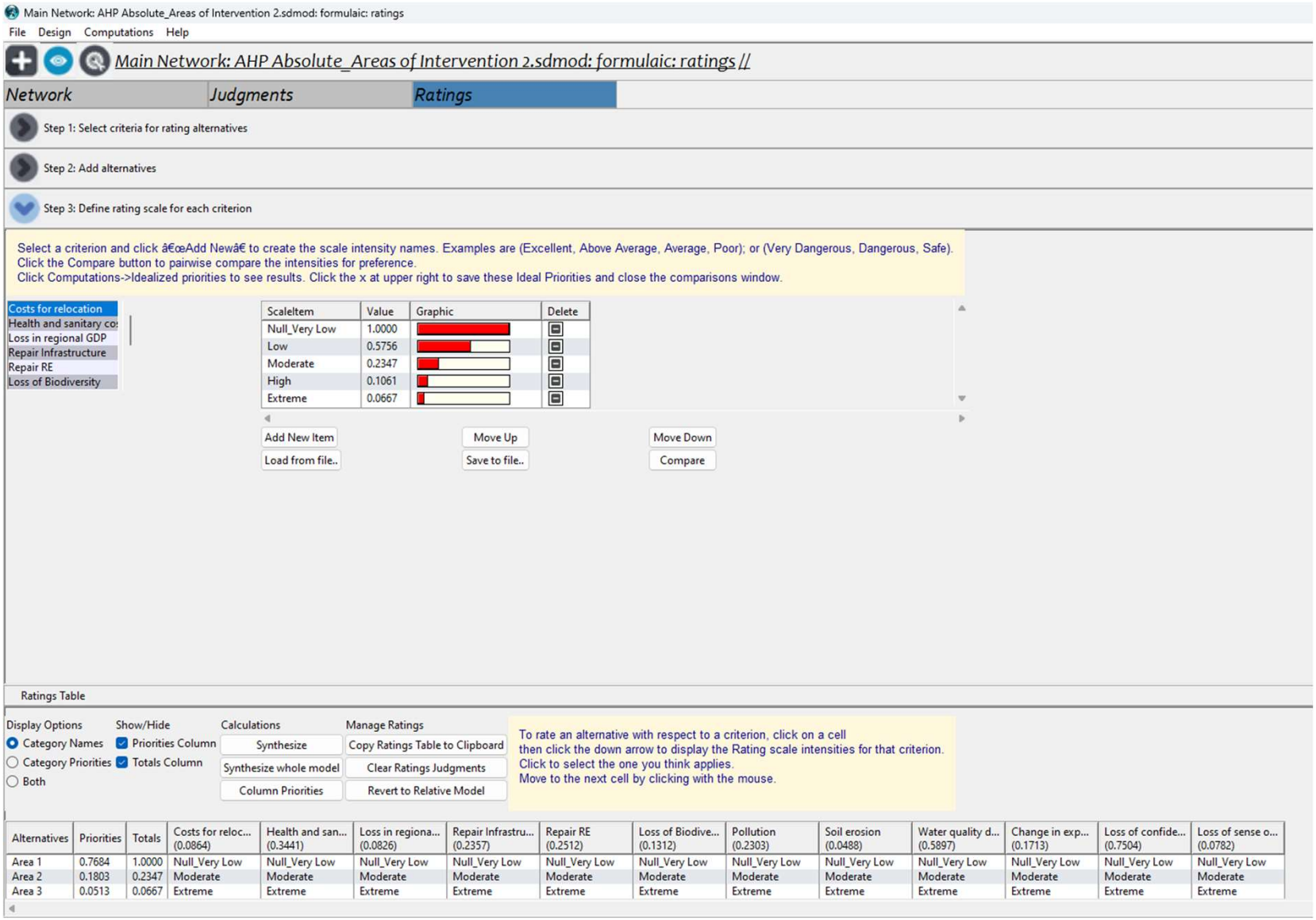
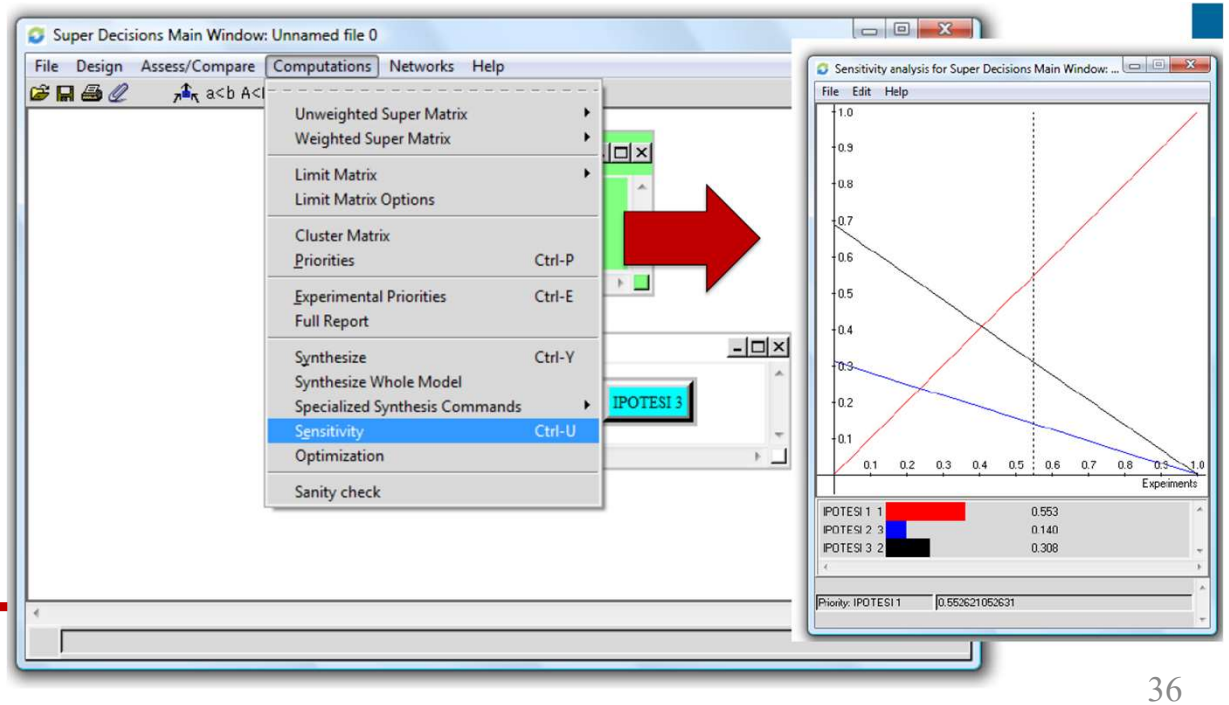


AHP – SUPER DECISION SOFTWARE

New synthesis for: Main Network: AHP Absolute_Areas...

Here are the overall synthesized priorities for the alternatives. You synthesized from the network Main Network: AHP Absolute_Areas of Intervention 2.sdmod: formulaic: ratings

Name	Graphic	Ideals	Normals	Raw
Area 1		1.000000	0.768413	0.768413
Area 2		0.234677	0.180329	0.180329
Area 3		0.066706	0.051258	0.051258



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UNIVERSITÀ DEGLI STUDI DI PADOVA

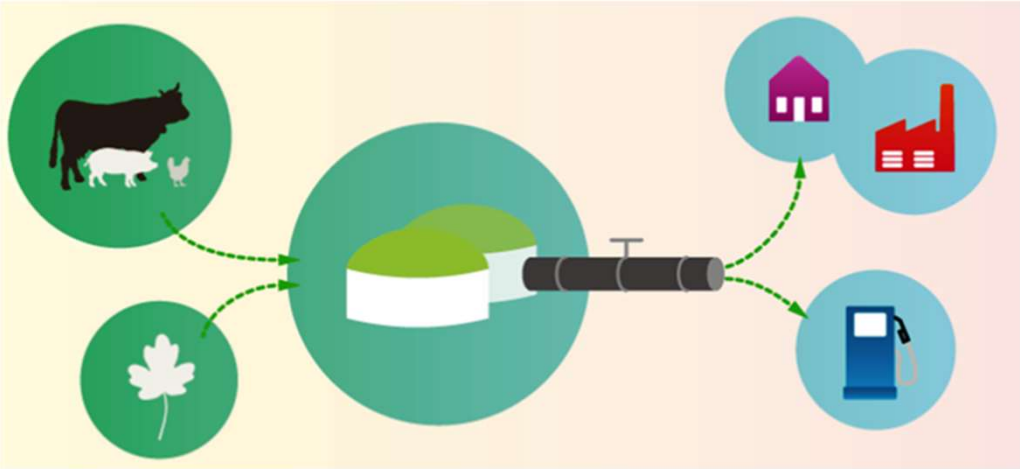
Department Of Civil, Environmental and Architectural Engineering

ICEA

CASE STUDY:

BIOGAS AND BIOMETHANE TECHNOLOGIES

AHP MODEL TO SUPPORT THE POLICY MAKER IN INCENTIVE DESIGN

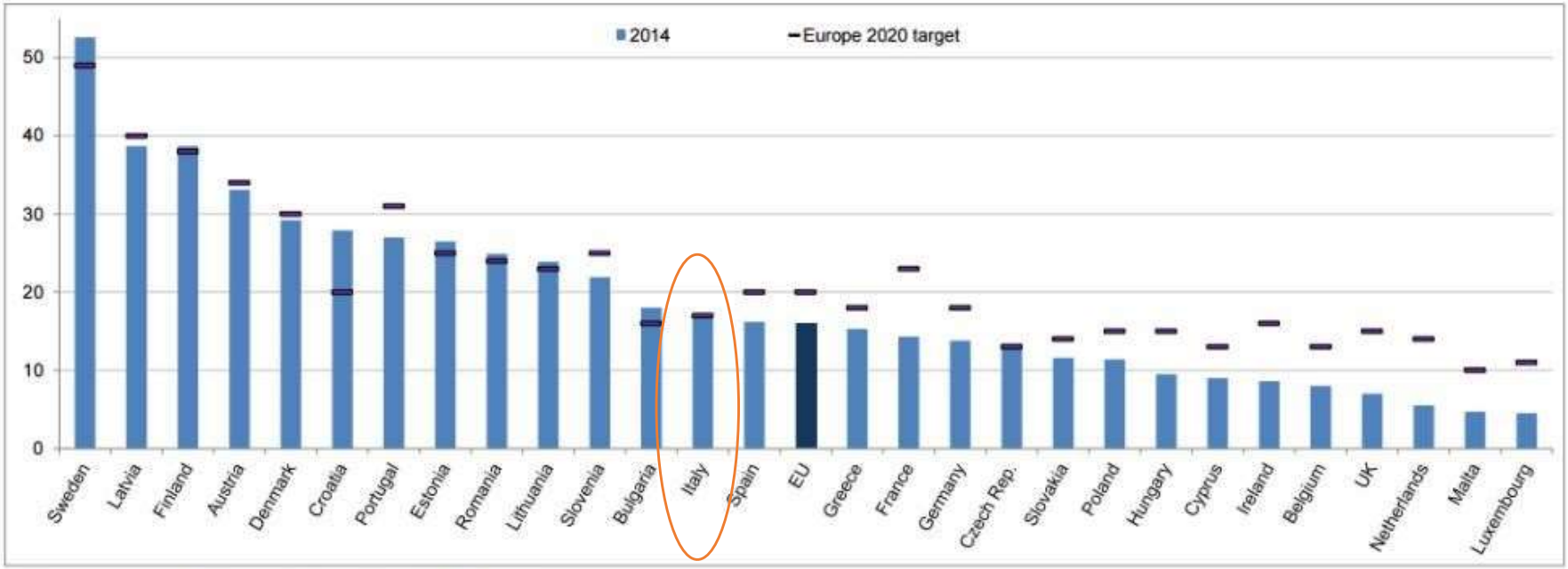


2020/30 Framework for Climate & Energy

The 2030 climate and energy framework sets three key targets for the year 2030:

- At least 40% cuts in **greenhouse gas emissions** (from 1990 levels)
- At least 27% share for **renewable energy** consumption
- At least 27% improvement in **energy efficiency**

Share of energy from renewable sources in the EU Member States, 2014
(in % of gross final energy consumption)



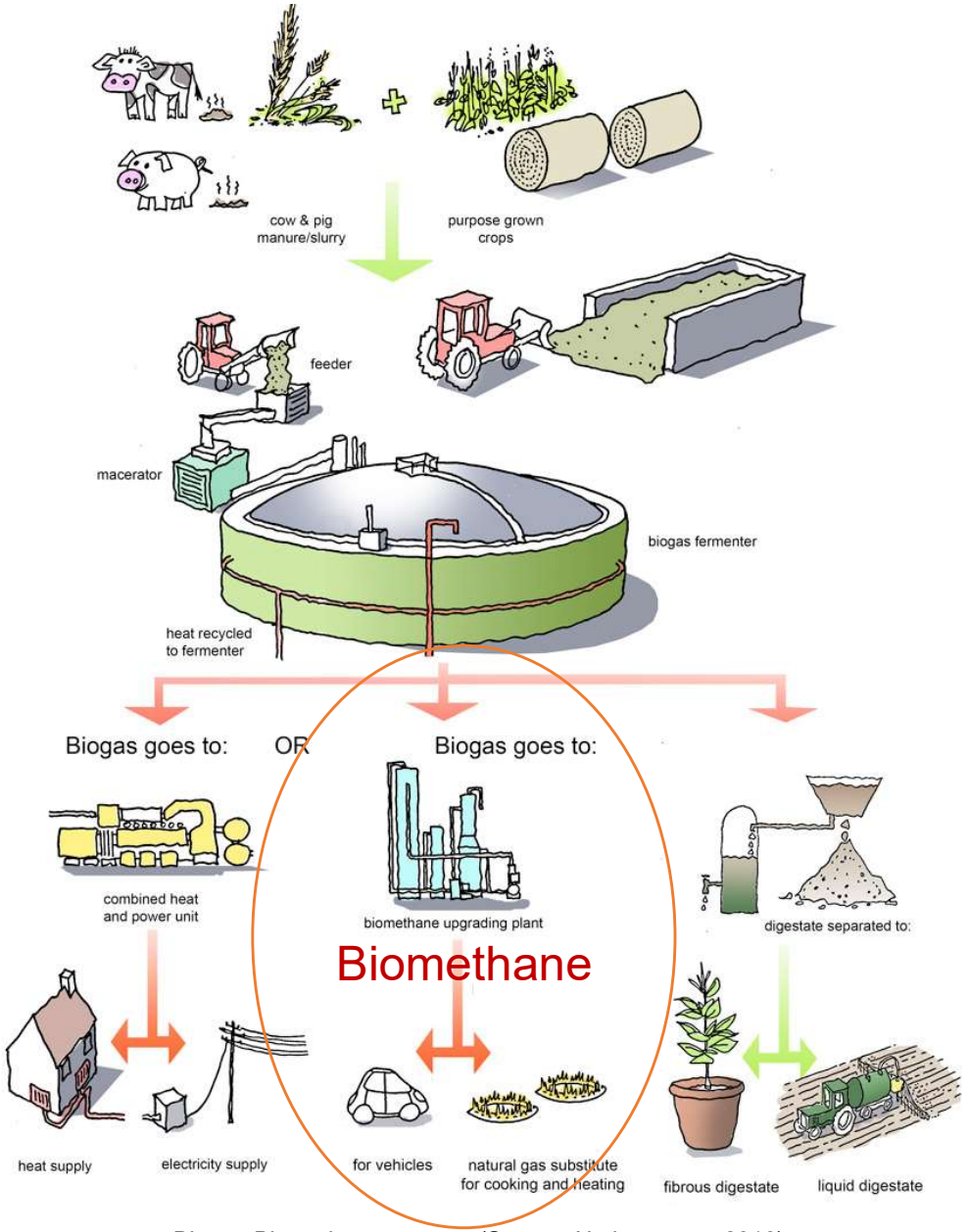
(Source: Eurostat, 2016)

Biogas and Biomethane (1)

Anaerobic Digestion 's contribution to key EU policy areas:

- **European climate targets** (cut greenhouse gas emission)
- **European energy security** (locally production of biomethane)
- **Food security and resource efficiency** (recycling waste)
- **Improved air quality** (carbon sequestration, reducing PM10 and NOx emissions)
- **Bioeconomy** (green job creation)
- **Bioenergy**
- **Prevention of contamination** (reducing pathogen fertilizer production)

(Source: EBA, 2015)



Biogas-Biomethane process (Source: Nethyenergy, 2016)

New feed-in tariffs (FITs)

EU National energy policies are evaluated by:

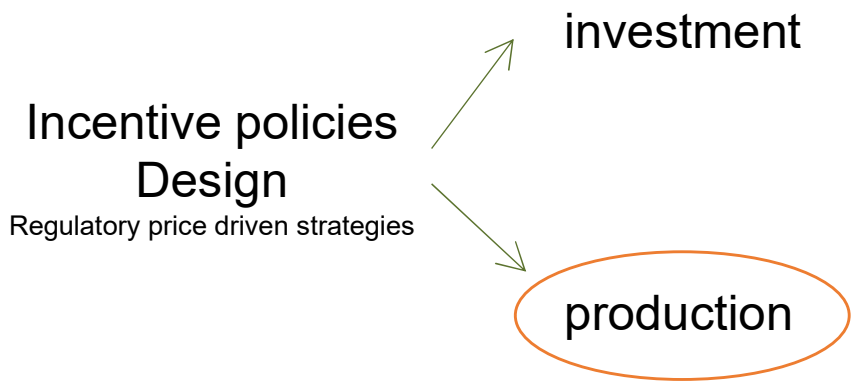
- long-term RE targets,
- increased economic and export market opportunities,
- sustainable job creation,
- enhanced use of forestry,
- enhanced use of agricultural wastes,
- development of innovative RE technologies.

(see European Commission, 2009/28/EC).

to achieve Policy goals

FITs (feed-in tariffs) differentiated by:

- technology type,
- the project size
- outputs,
- inputs,
- resource quality
- location of the project
- etc.…



The model
Overall methodology

AHP relative model, according to the following steps:

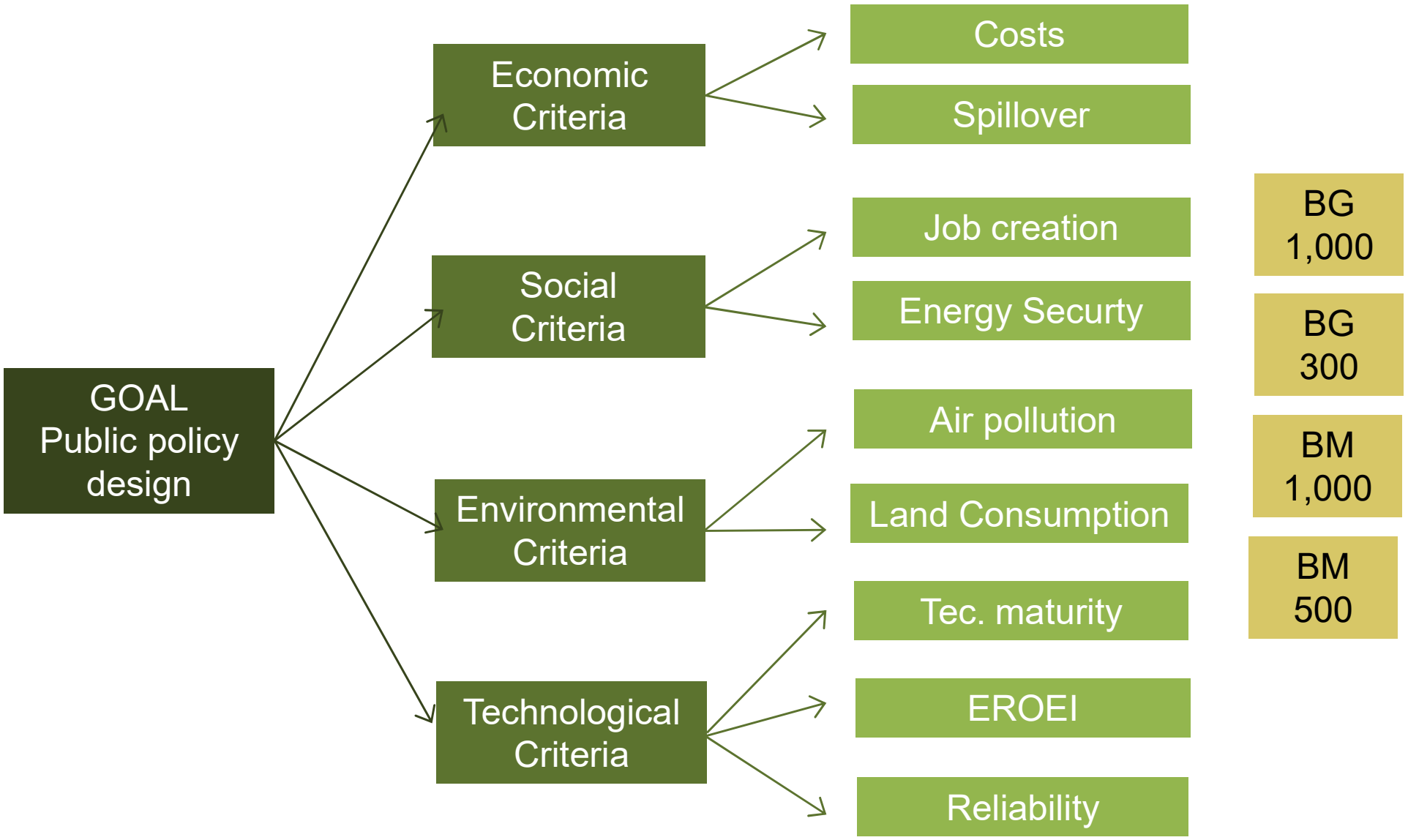
1. Defining the problem
2. Definition of the experts' team
3. Defining the Hypothesis
4. Identification of criteria and subcriteria
5. Construction of the AHP model
6. Pairwise comparison
7. Analysis of the consistency
8. Synthesize these results to determinate an overall outcome

Support the policy maker in the definition of sustainable development policies for biogas and biomethane production

20 Italian experts adressed in the survey (phone, emailed and panel discussion)

The compared alternatives:
- 2 Biogas Plants (300 – 1,000 kW),
- 2 Biomethane Plants (500 – 1,000 kW)
- Same feedstock

The model
The Hierachy



The model
Criteria and Subcriteria

Goal	Criteria	Subcriteria	Description
Public policy design	Economic	Costs	Global costs (investment cost, capex, opex) and feed-in costs
		Spillover	Generate externalities
	Social	Job creation	To estimate the employment effects resulting from the deployment of AD technologies both construction and operation phases have to be taken into account.
		Energy security	The reduction of dependence on imported energy
	Evironmental	Air pollution	Amount of CH4 emissions
		Land Consumption	This criteria is related to the dimension of the plans and to the area involved in the transformation process
	Technological	Technical maturity	Refers to the specific involved technology, defying if it has successfully passed all research stages and has been commercialized for a number of years without severe problems in the operation
		EROEI	Energy Returned On Energy Invested
		Reliability	(start of uptime - start of downtime) / days of failure

The model
Results

$$A = a_{ij} = \begin{array}{c|cccc} & A_1 & A_2 & \dots & A_m \\ A_1 & 1 & a_{12} & \dots & a_{1m} \\ A_2 & 1/a_{12} & 1 & \dots & a_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ A_m & 1/a_{1m} & 1/a_{2m} & \dots & 1 \end{array}$$

Parewise comparisons

Table 2 Criteria and subcriteria aggregation of experts' judgments (priority vectors)

Criteria	Priority vector	Subcriteria	Priority vector
Economic	0.198	Costs	0.25
		Spillover	0.75
Social	0.359	Job creation	0.66
		Energy security	0.33
Environmental	0.284	Air pollution	0.80
		Land Consumption	0.20
Technological	0.157	Technical maturity	0.21
		EROEI	0.55
		Reliability	0.24

Partial results

Spillover

Inconsistency: 0.06395

BG 1,000 ~	0.22890
BG 300 kW	0.08960
BM 1,000 ~	0.44907
BM 500 kW	0.23243

Energy Security

Inconsistency: 0.01629

BG 1,000 ~	0.19983
BG 300 kW	0.07809
BM 1,000 ~	0.52224
BM 500 kW	0.19983

Land consumption

Inconsistency: 0.03044

BG 1,000 ~	0.12727
BG 300 kW	0.47699
BM 1,000 ~	0.08460
BM 500 kW	0.31114

Reliability

Inconsistency: 0.00772

BG 1,000 ~	0.36289
BG 300 kW	0.32608
BM 1,000 ~	0.16304
BM 500 kW	0.14800

The model
Results

Prioritization of alternatives

	Name	Graphic	Ideals	Normals
4	BG 1,000 kW		0.402334	0.150705
3	BG 300 kW		0.440711	0.165080
1	BM 1,000 kW		1.000000	0.374578
2	BM 500 kW		0.826629	0.309637

Effect on incentive policy design?

Additional Prioritizations

Consider:

- Different feedstocks
- Valuable by product
- Green taxes