



Università degli Studi di Padova

The Relational Model

Basi di Dati

Bachelor's Degree in Computer Engineering Academic Year 2024/2025



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- My office: Laboratorio basi di dati 305 DEI/G
- Material: the reference books you used for the first part of the course + slides and notes taken during the course
- Topics: Relational Model, Relational Algebra, Exercises



Constructs of the relational model

Integrity constraints in the relational model

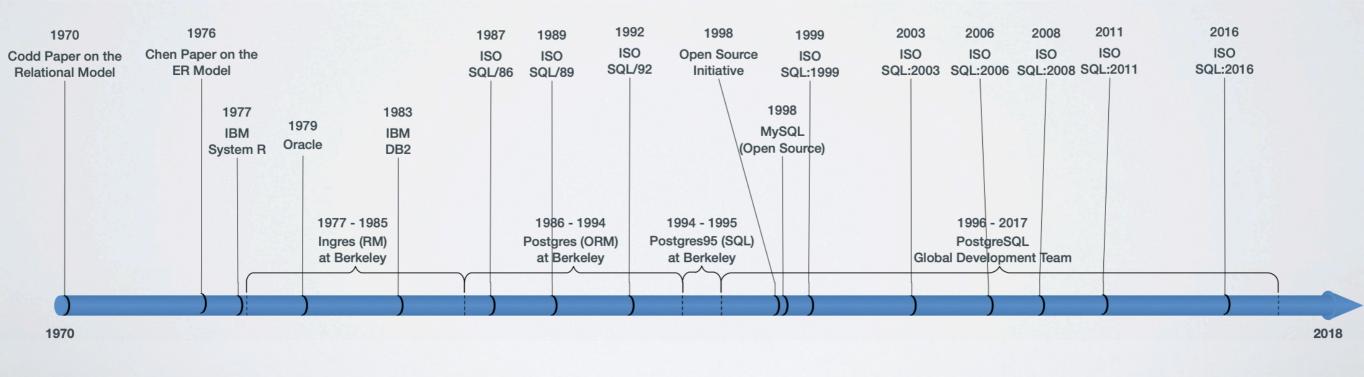
Mapping from the Entity-Relationship model to the relational model



The Relational Model



- The relational model has been proposed by E. F. Codd at IBM in 1970 to promote the data independence
- It is based on the notion of (mathematical) relation but with important differences
 - Relations are naturally represented by means of **tables**
- The relational models adopts a **value-based approach**: links and references among data in different structures (relations) are realized by means of comparison among values (and not pointers)



Relation



Given the sets D_1, D_2, \ldots, D_n the cartesian product is the set of the ordered n-uples:

 $D_1 \times D_2 \times \ldots \times D_n = \{(d_1, d_2, \ldots, d_n) | d_1 \in D_1, d_2 \in D_2, \ldots, d_n \in D_n\}$

A relation (mathematical) is a subset of the cartesian product:

$$R \subseteq D_1 \times D_2 \times \ldots \times D_n$$

 D_1, D_2, \ldots, D_n are the **domains** of the relation. A relation over *n* domains has **degree** *n*

The number |R| of n-ples is the **cardinality** of the relation



A (mathematical) relation is a set and therefore

- there is no ordering among its n-uples
- the n-uples are all **distinct**

Each n-uple in a relation is ordered and therefore

• the i-th value is associated to the i-th domain



$\mathsf{Game} \subseteq \mathsf{String} \times \mathsf{String} \times \mathsf{Integer} \times \mathsf{Integer}$

Juventus	Milan	3	1
Lazio	Roma	2	3
Inter	Juventus	3	2
Milan	Lazio	2	0

The structure is **positional**



$\mathsf{Game} \subseteq \mathsf{String} \times \mathsf{String} \times \mathsf{Integer} \times \mathsf{Integer}$

Inter	Juventus	3	2
Milan	Lazio	2	0
Juventus	Milan	3	1
Lazio	Roma	2	3

The structure is **positional**



Juventus	3	Milan	1
Lazio	2	Roma	3
Inter	3	Juventus	2
Milan	2	Lazio	0

The structure is **positional**



Juventus	3	Milan	1
Lazio	2	Roma	3
Inter	3	Juventus	2
Milan	2	Lazio	0

The structure is **positional**



Juventus	Milan	3	1
Lazio	Roma	2	3
Inter	Juventus	3	2
Lazio	Roma	2	3

The structure is **positional**



Juventus	Milan	3	1
Lazio	Roma	2	3
Inter	Juventus	3	2
Lazio	Roma	2	3

The structure is **positional**



- A relation in the relational model is similar to a mathematical relation but with some differences:
 - the components of a relation are called attributes
 - each attribute is characterized by a name and a set of values, called domain of the attribute

- A relation can be represented as a **table** where **attributes** correspond to **columns** and **attribute names** are used as **column headers**
 - since each component of a relation is univocally identified by an attribute, the ordering among attributes is irrelevant:
 - differently from a mathematical relation, the structure is not positional



• Let $X = \{A_1, A_2, \dots, A_n\}$ be the set of attributes and let $\mathcal{D} = \{D_1, D_2, \dots, D_n\}$ be the set of attribute domains, the function $\operatorname{dom}: X \to \mathcal{D}$ $A_i \mapsto D_i$ maps each attribute into its domain

• We call **tuple** over a set of attributes

$$t_j: \quad X \to D_i$$
$$A_i \mapsto v_{i,j} \in D_i$$

the function which maps each attribute into a value of its domain. We can use the following notation $t_j[A_i]$ or $t_j.A_i$ to indicate the value of an attribute A relation over a set of attributes X is a set of tuples over X



Home	Visitor	Home Score	Visitor Score
Juventus	Milan	3	1
Lazio	Roma	2	3
Inter	Juventus	3	2
Milan	Lazio	2	0

- There are 4 attributes in this relation and, in the tabular representation, they correspond to the columns of the table; the **tuples** (n-ples) correspond to the rows.
- The attribute domains are String for Home and Visitor, and Integer for Home Score and Visitor Score. Typically, they are not indicated in the tabular representation of a relation



Home	Visitor	Home Score	Visitor Score
Inter	Juventus	3	2
Milan	Lazio	2	0
Juventus	Milan	3	1
Lazio	Roma	2	3

- There are 4 attributes in this relation and, in the tabular representation, they correspond to the columns of the table; the **tuples** (n-ples) correspond to the rows.
- The attribute domains are String for Home and Visitor, and Integer for Home Score and Visitor Score. Typically, they are not indicated in the tabular representation of a relation



Home	Home Score Visitor		Visitor Score	
Juventus	3	Milan	1	
Lazio	2	Roma	3	
Inter	3	Juventus	2	
Milan	2	Lazio	0	

- There are 4 attributes in this relation and, in the tabular representation, they correspond to the columns of the table; the **tuples** (n-ples) correspond to the rows.
- The attribute domains are String for Home and Visitor, and Integer for Home Score and Visitor Score. Typically, they are not indicated in the tabular representation of a relation



Home	Visitor	Home Score	Visitor Score
Juventus	Milan	3	1
Lazio	Roma	2	3
Inter	Juventus	3	2
Lazio	Roma	2	3

- There are 4 attributes in this relation and, in the tabular representation, they correspond to the columns of the table; the **tuples** (n-ples) correspond to the rows.
- The attribute domains are String for Home and Visitor, and Integer for Home Score and Visitor Score. Typically, they are not indicated in the tabular representation of a relation



Home	Visitor	Home Score	Visitor Score
Juventus	Milan 🔶	3	1
Lazio	Roma	2	3
Inter	Juventus	3	2
Lazio	Roma	2	3

- There are 4 attributes in this relation and, in the tabular representation, they correspond to the columns of the table; the **tuples** (n-ples) correspond to the rows.
- The attribute domains are String for Home and Visitor, and Integer for Home Score and Visitor Score. Typically, they are not indicated in the tabular representation of a relation





A relation schema R(T) consists of a relation name R and a relation type T denoted by $R(A_1: D_1, A_2: D_2, \ldots, A_n: D_n)$ where the attributes are A_1, A_2, \ldots, A_n and the attribute domains are D_1, D_2, \ldots, D_n .

Two relation types are equal if they have the same **degree**, that is the number of attributes n, the attributes themselves and the domains of the attributes with the same name.

The order of the attributes is irrelevant

• We can also use the compact notation

$$R(A_1, A_2, \ldots, A_n)$$



A relation instance r of the relation schema $R(A_1, A_2, ..., A_n)$, denoted by r(R), is a set of tuples $r = \{t_1, t_2, ..., t_m\}$ where each tuple is a labeled list of values $t_i = \langle A_1 : v_{1,i}, A_2 : v_{2,i}, ..., A_n : v_{n,i} \rangle$

• The value of attribute A_i in tuple t_j is indicated as

$$t_j[A_i] = v_{i,j} \in D_i = \operatorname{dom}(A_i)$$
$$t_j.A_i = v_{i,j} \in D_i = \operatorname{dom}(A_i)$$

• The cardinality |r| = m of a relation is the number of its tuples



It may not be always possible to associate an appropriate value with an attribute of a tuple

To overcome this issue, we introduce a special value, called NULL, which does not belong to any domain, and we extend the

$$t_j[A_i] = v_{i,j} \in D_i \lor \mathsf{NULL}$$

NULL is not one of the "ordinary" values of a domain D

- "unused values" may not exist
- "unused values" may change over time
- you need to treat such "unused values" as special cases in the applications
- "unused values" would be different from domain to domain





Person

Name	Surname	Age	MsC	HomePhone
Mario	Rossi	32	NULL	0498271234
Giovanni	Verdi	NULL	Law	0498275678
Marta	Bruni	28	Medicine	NULL

Value undefined: an appropriate value for a given instance does not exist

- Master Degree (MsC) does not apply to those people who have not attended university
- Value not available: an appropriate value for a given instance exists but it is not known in a given moment
 - You may not know the Age of a Person
- **Value unknown**: an appropriate value for a given instance may or may not exist
 - A Person may or may not have an home phone number because the contract is still in progress



Relational Model: Value-based Approach

Student

BadgeNumber	Name	Surname	BirthDate
6554	Mario	Rossi	05/12/1982
8765	Paolo	Neri	07/04/1979
9283	Luisa	Verdi	22/07/1983
3456	Marta	Rossi	14/02/1981

Student	Mark	Course
3456	30	03
3456	24	02
9283	28	01
6554	26	01

Course	Code Title		Teacher
	01	Analysis	Bianchi
	02		Bruni
	03	Physics	Rossi

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Student

BadgeNumber

6554

03

Relational Model: Value-based Approach

Name

Mario

sed Approach				
Surname	BirthDate			
Rossi	05/12/1982			
Neri	07/04/1979			
Verdi	22/07/1983			

	8765	Paolo	Neri	07/04/1979
	9283	Luisa	Verdi	22/07/1983
	3456	Marta	Rossi	14/02/1981
				Evom
	Student	Mark	Course	Exam
	3456	30	03	
	3456	24	02	
	9283	28	01	
	6554	26	01	
Course	Code	Title	Teacher	
	01	Analysis	Bianchi	
	02	Chemistry	Bruni	

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Physics

Rossi



Independence from physical data structures, which may change

Data are more **portable** from one system to another one

Values allow for bi-directionality (while physical pointers are not)



A database schema (relational schema) consists of a set of relation schemas $R_i(T_i)$ with different names and denoted by $\mathbf{R} = \{R_1(T_1), R_2(T_2), \dots, R_n(T_n)\}$

The integrity constraints (discussed in the following) are integral part of the database schema



A database instance r of the database schema $\mathbf{R} = \{R_1(T_1), R_2(T_2), \dots, R_n(T_n)\}$ is a set of relation instances $\mathbf{r} = \{r_1, r_2, \dots, r_n\}$

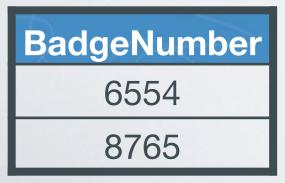


$\label{eq:R} \mathbf{R} = \big\{ \texttt{Student}(\texttt{BadgeNumber}, \texttt{Name}, \texttt{Surname}, \texttt{BirthDate}), \\ \texttt{Worker}(\texttt{BadgeNumber}) \big\}$

Student

BadgeNumber	Name	Surname	BirthDate
6554	Mario	Rossi	05/12/1982
8765	Paolo	Neri	07/04/1979
9283	Luisa	Verdi	22/07/1983
3456	Marta	Rossi	14/02/1981

Worker



Integrity Constraints



An integrity constraint is a condition expressed on the database schema (intensional level) and it must be complied with by the database instances (extensional level) which represent a correct state for the application

- Each constraint is a boolean predicate which maps each database instance into true if the constraint is complied with or false otherwise
- We add a set of constraints to the database schema and we consider correct only the database instances which comply with all the constraints



Intra-relational, concerning a single relation:

- domain constraint
- tuple constraint
- key constraint

Inter-relational, concerning two or more relations:

referential integrity constraint



Integrity Constraints: Example



Student

BadgeNumber	Name	Surname	BirthDate
6554	Mario	Rossi	05/12/1982
6554	Paolo	Neri	07/04/1979
9283	Luisa	Verdi	22/07/1983
3456	Marta	Rossi	14/02/1981

Student	Mark	Laude	Course
3456	30	TRUE	03
3456	32		02
9283	28	TRUE	01
7899	26		01

Course

Exam

Code	Title	Teacher
01	Analysis	Bianchi
02	NULL	NULL
03	Physics	Rossi

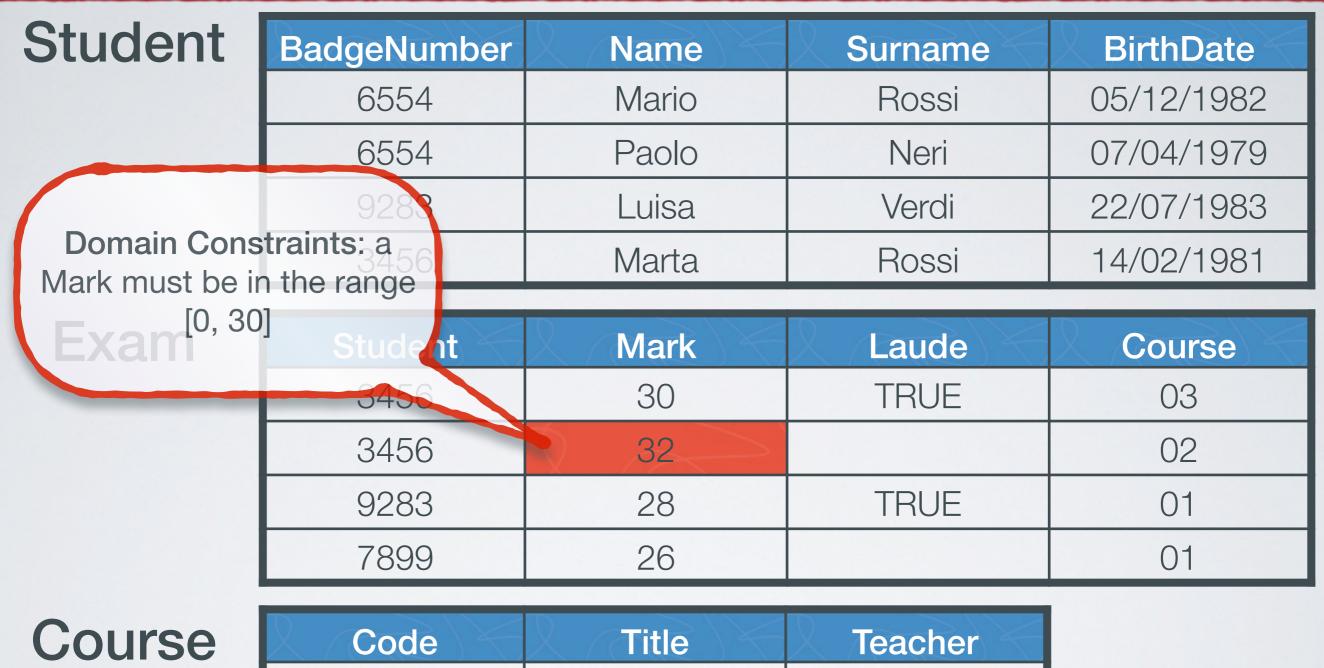
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Integrity Constraints: Example





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Analysis

NULL

Physics

Bianchi

NULL

Rossi

01

02

03



Integrity Constraints: Example



Student	BadgeNumber	Name	Surname	BirthDate
	6554	Mario	Rossi	05/12/1982
	6554	Paolo	Neri	07/04/1979
	9283	Luisa	Verdi	22/07/1983
	3456	Marta	Rossi	14/02/1981
Evom				
Exam	Student	Mark	Laude	Course
	3456	30	TRUE	03
	3456	32		02
	9283	28	TRUE	01
Domain Constra	1039	26		01
and Teacher car	nnot be			
Course	Code	Title	Teacher	
	01	Analysis	Bianchi	
	02	NULL N	NULL	
	03	Physics	Rossi	

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Integrity Constraints: Example



Student	BadgeNumber	Name	Surname	BirthDate
	6554	Mario	Rossi	05/12/1982
	6554	Paolo	Neri	07/04/1979
	9283	Luisa	Verdi	22/07/1983
Tuple Constrain	t: Laude is	Marta	Rossi	14/02/1981
possible if and o	nly if Mark			
Examis 30	Student	Mark	Laude	Course
	3466	30	TRUE	03
	3456	32		02
	9283	28	TRUE	01
	7899	26		01
Course	Code	Title	Teacher	

Code	Title	Teacher
01	Analysis	Bianchi
02	NULL	NULL
03	Physics	Rossi

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Integrity Constraints: Example



Student	BadgeNumber	Name	Surname	BirthDate
	6554	Mario	Rossi	05/12/1982
	6554	Paolo	Neri	07/04/1979
	9283	Luisa	Verdi	22/07/1983
	3456	Marta	Rossi	14/02/1981
Exam	St.dent	Mark	Laude	Course
Key Constraint: t not exist two tup	there do	30	TRUE	03
the same value		32		02
attribute Badge	Number 83	28	TRUE	01
	7899	26		01
Course	Code		Teacher	
	01	Analysis	Bianchi	
	02	NULL	NULL	
	03	Physics	Rossi	

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Integrity Constraints: Example



Student	BadgeNumber	Name	Surname	BirthDate
	6554	Mario	Rossi	05/12/1982
	6554	Paolo	Neri	07/04/1979
	9283	Luisa	Verdi	22/07/1983
	3456	Marta	Rossi	14/02/1981
Exam	Student	Mark	Laude	Course
	3456	30	TRUE	03
	3456	32		02
	9283	28	TRUE	01
	7899	26		01
Course	Code	Title	Referential Integ	rity Constraint:
	01	Analysis	there is not tuple	
	02	NULL	relation with valu attribute Bac	
	03	Physics	Rossi	

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A tuple constraint defines conditions on the values of each tuple of a relation, independently from all the other tuples. A tuple constraint concerning only one attribute is called a domain constraint

• A tuple constraint on a relation *R* is a boolean sentence, using AND, OR and NOT to compose atomic boolean predicates comparing values of or expressions on the attributes of the relation *R*



Tuple Constraints: Example



Student	Mark	Laude	Course
3456	30	TRUE	03
3456	32		02
9283	28	TRUE	01
7899	26		01

Domain Constraint

$$(Mark \ge 18) AND (Mark \le 30)$$

Tuple Constraint

(Mark = 30) OR NOT (Laude = TRUE)

Mark = 30	Laude = TRUE	(Mark = 30) OR NOT (Laude = TRUE)
TRUE	TRUE	TRUE
FALSE	TRUE	FALSE
TRUE	FALSE	TRUE
FALSE	FALSE	TRUE

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Let R(T) be a relation schema and let SK be a sub-set of the attributes $T \cdot SK$ is a super-key for R if, for a relation instance r(R), it holds $\forall t_1, t_2 \in r, \quad t_1[SK] \neq t_2[SK]$

In the relation instance there cannot be two different tuples which have the same values on all the attributes of the super-key

Since relations are sets, a relation cannot contain the same tuple twice. As a consequence, there always exists at least one super-key for each relation, that is the whole set of attributes T



A super-key SK of a relation schema R(T) is a key K (with K = SK) if, for a relation instance r(R) and for each proper subset K' of K, it holds $\exists t_1, t_2 \in r \mid t_1[K'] = t_2[K']$

• A key is a **minimal super-key**, i.e. if you remove only one of its attributes, it is no more a key

 The same relation may have more than one key (superkey)





Student

Badge	Name	Surname	BirthDate	Course
6554	Mario	Rossi	05/12/1982	Analysis
8765	Mario	Neri	07/04/1979	Physics
3219	Mario	Rossi	07/04/1979	Chemistry
9283	Piero	Neri	22/07/1983	Analysis
3456	Piero	Rossi	05/12/1982	Chemistry

Badge is a key

- Badge is a super-key
- It is a minimal super-key since it consists of just one attribute
- (Name, Surname, BirthDate) is a key
 - the set Name, Surname, BirthDate is a super-key
 - It is a minimal super-key since no one of its subset is a super-key

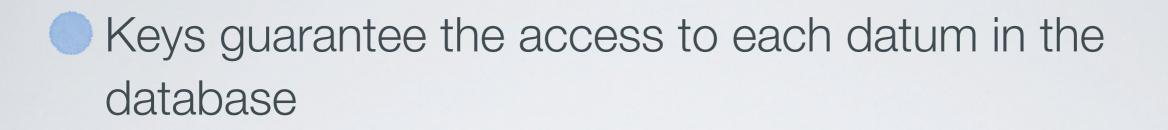
(Name, Surname, BirthDate, Course) is a super-key but not a key



A key constraint is an assertion specifying that a set of attributes constitutes a key for a relational and that all the valid relation instances must comply with that constraint

There is no limit to the number of key constraints on a relation





- Each value is univocally accessible by
 - name of the relation
 - value of the key, which identifies a specific tuple in the relation
 - name of the attribute whose value has to be accessed

As we will see in the following, keys are the main mean to link data in different relations





Student

Badge	Name	Surname	BirthDate	Course
NULL	Mario	NULL	05/12/1982	Analysis
8765	Mario	Neri	07/04/1979	Physics
3219	Mario	Rossi	07/04/1979	Chemistry
9283	Piero	Neri	NULL	Analysis
NULL	Piero	Rossi	05/12/1982	NULL

When there are NULL values, the values of the attributes constituting a key:

- o not allow us to identify tuples
- O do not allow us to link to/from other relations





A primary key constraint is a key constraint also requesting that NULL values are not allowed for the attributes constituting the key

Student

BadgeNumber	Name	Surname	BirthDate	Course
6554	Mario	Rossi	05/12/1982	Chemistry
8765	Mario	Neri	07/04/1979	Analysis
3456	Piero	Rossi	05/12/1982	Physics

The attributes constituting the primary key are <u>underlined</u>

We can define **only one** primary key for each relation

We typically chose as primary key the key with the minimal number of attributes





A set of attributes $FK = (A_1, A_2, ..., A_n)$ of a relation schema R_1 is a foreign key of R_1 referring to a relation R_2 with primary key $PK = (B_1, B_2, ..., B_n)$ if $dom(A_i) = dom(B_i), \quad 1 \le i \le n$ $\forall t_1 \in r(R_1), \ t_1[FK] = \text{NULL} \lor \exists t_2 \in r(R_2) \mid t_1[FK] = t_2[PK]$

The foreign key allows us to define the referential integrity constraint which requires that the property above is complied with by all the database instance

Note the referential integrity constraint is enforced by comparisons based on the values



Referential Integrity Constraint: Example

Student

BadgeNumber	Name	Surname	BirthDate
6554	Mario	Rossi	05/12/1982
8765	Paolo	Neri	07/04/1979
9283	Luisa	Verdi	22/07/1983
3456	Marta	Rossi	14/02/1981

<u>Student</u>	Mark	Course	Exam
3456	30	03	
3456	24	02	
9283	28	01	
6554	26	01	

Course	Code	Title	Teacher
	01	Analysis	Bianchi
	02	Chemistry	Bruni
	03	Physics	Rossi

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Referential Integrity Constraint: Example

|--|--|

Student	BadgeNumber	Name	Surname	BirthDate
	6554	Mario	Rossi	05/12/1982
	8765	Paolo	Neri	07/04/1979
	9283	Luisa	Verdi	22/07/1983
	3456	Marta	Rossi	14/02/1981
	<u>Student</u>	Mark	Course	Exam
	3456	30	03	
	3456	24	02	
	9283	28	01	
	6554	26	01	
Course				
Course		Title	Teacher	
	01	Analysis	Bianchi	
	02	Chemistry	Bruni	
	03	Physics	Rossi	4

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Mapping from ER Model to Relational Model





- The goal of the mapping is to pass from a (transformed) ER schema plus constraints to a relational schema plus constraints, representing the same information
- It is a semi-automatic process in the sense that most of the choices have been already made while transforming the ER schema
- The produced relational schema is typically a good starting point but we need to check its quality (normalization) and we may perform further transformations for performance reasons
- It consists of the following steps:
 - mapping of the entities into relations together with the respective constraints
 - mapping of the relationships into relations together with the respective constraints
 - mapping of external constraints



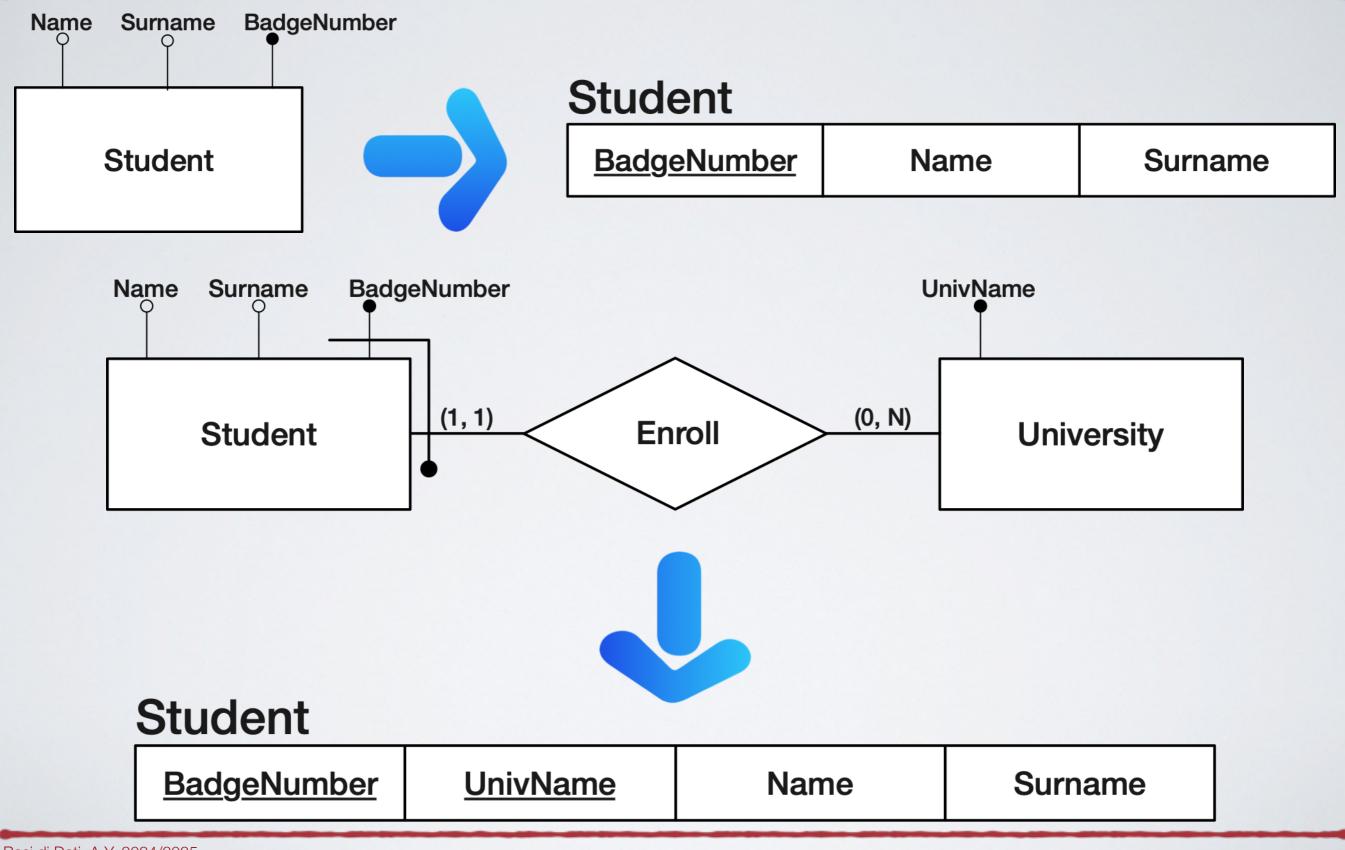
Each entity **E** of the ER schema is mapped into a relation $R_{\rm E}$ of the relational schema

The attributes of the relation R_{E} are

- all the attributes of the entity E
- lacksquare the attributes deriving from the merging of relationships in $R_{\rm E}$ for each merged relationship ${\bf Q}$ we add
 - the attributes of the relationships **Q**
 - the identifiers of all the other entities participating in **Q**

The primary key of $R_{\rm E}$ is given by the principal identifier of **E**, that is attributes $R_{\rm E}$ and/or deriving from external identification





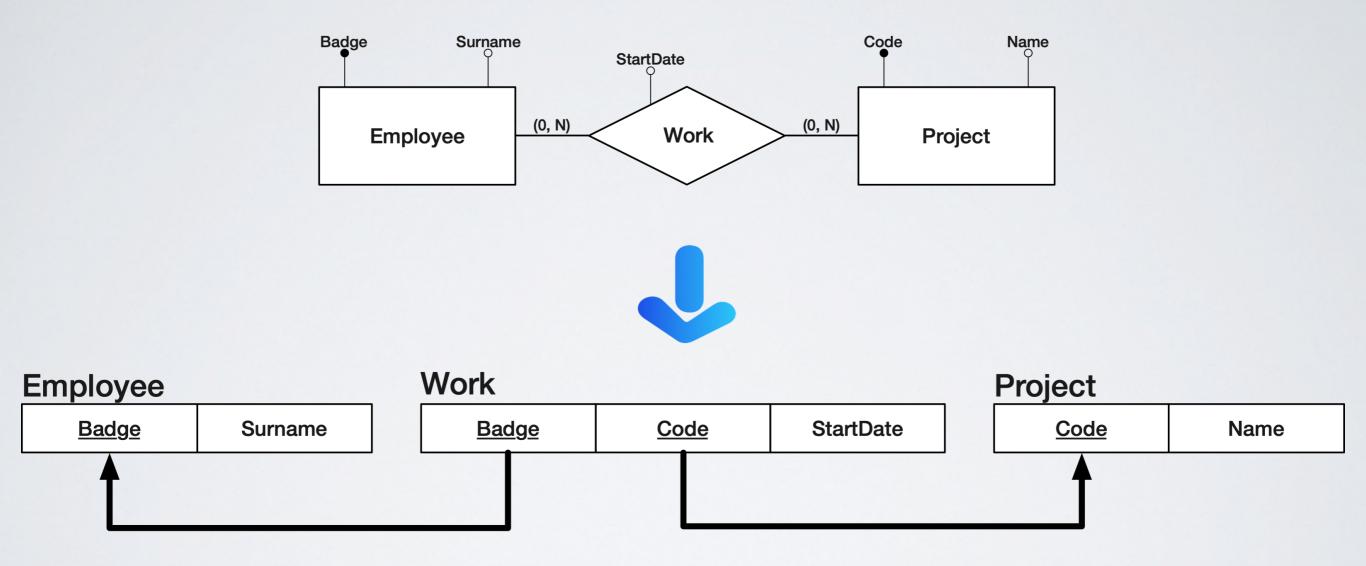


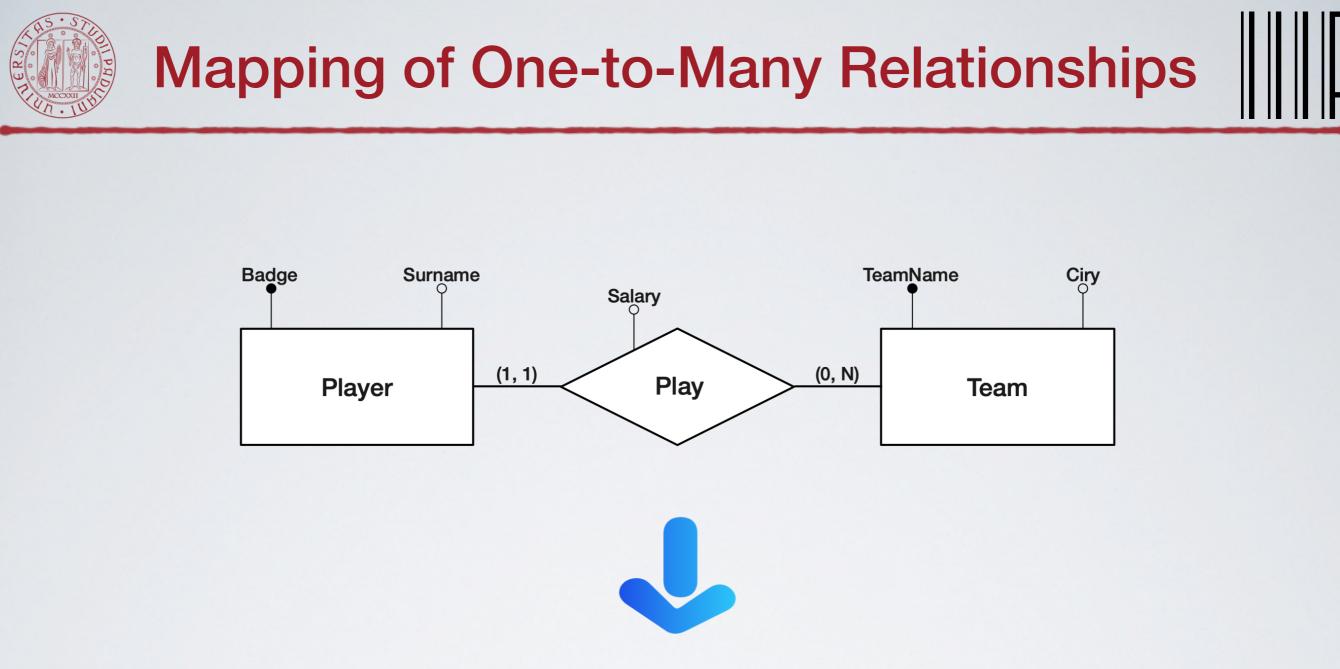
- Each relationship ${f Q}$ of the ER schema which has not been merged in the previous step is mapped to a relation $R_{f Q}$
 - The attributes of the relation R_{Q} are
 - all the attributes of the relationship Q
 - the identifier of the entities (primary keys of the relations) that participate to Q

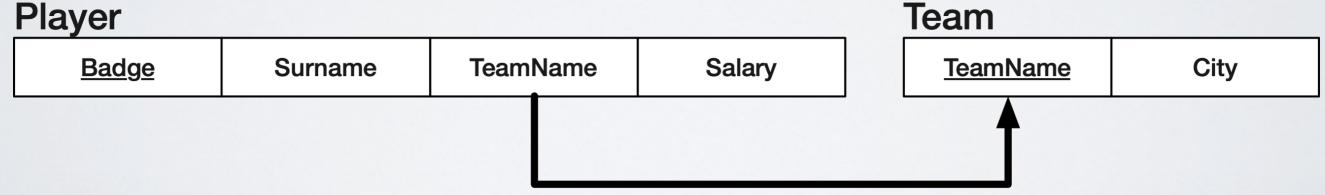
Choice of the primary key of $R_{ extsf{Q}}$

- If no entity participates to Q with maximum cardinality 1, then the primary key consists of all the primary keys of the participating entities
- Otherwise, the primary key of each entity participating to Q with maximum cardinality 1 is a key and we can choose one of these candidate keys as the primary key

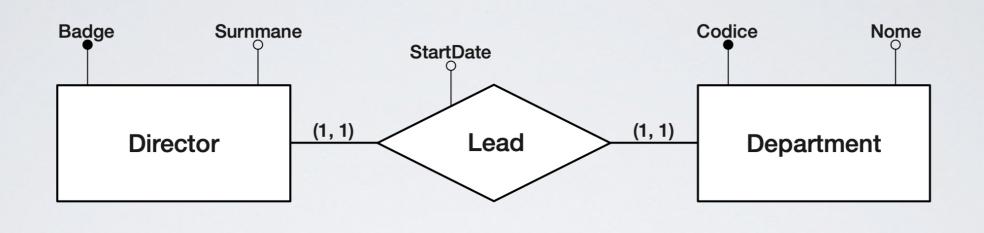
Mapping of Many-To-Many Relationships



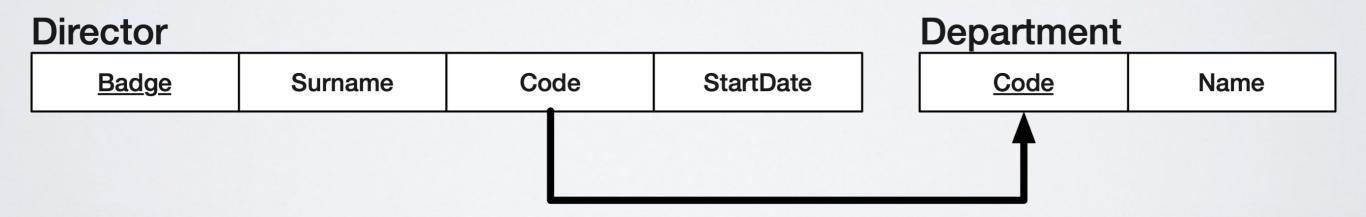






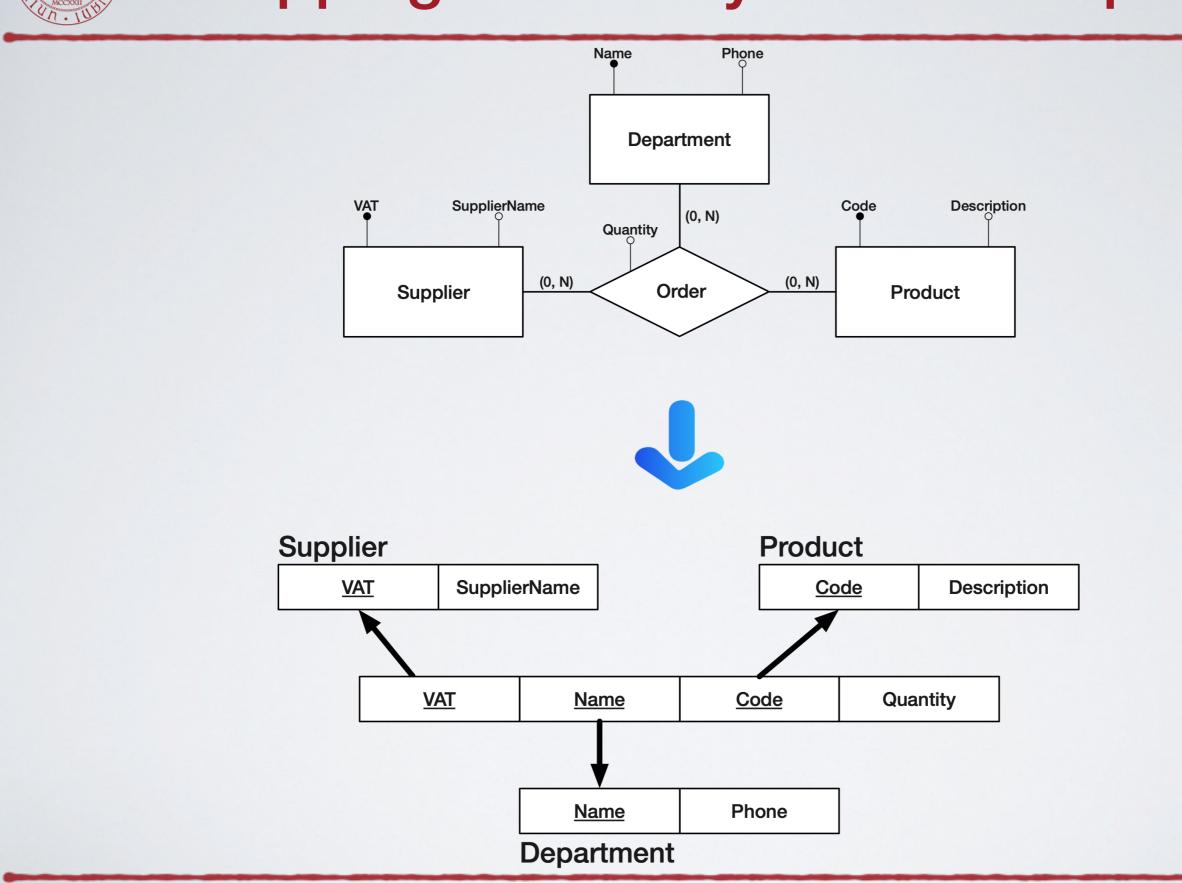






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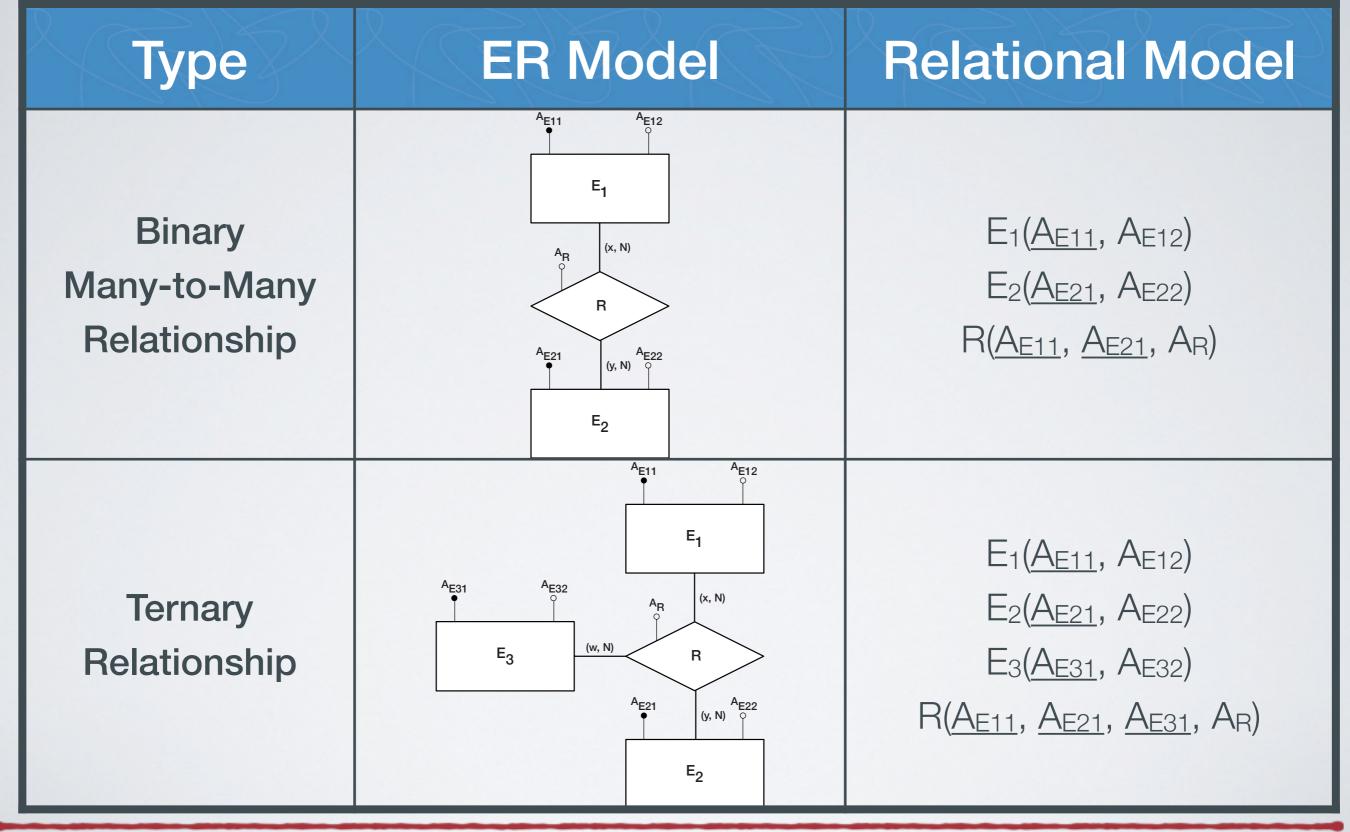
Mapping of Ternary Relationships





Mapping Summary (1/4)

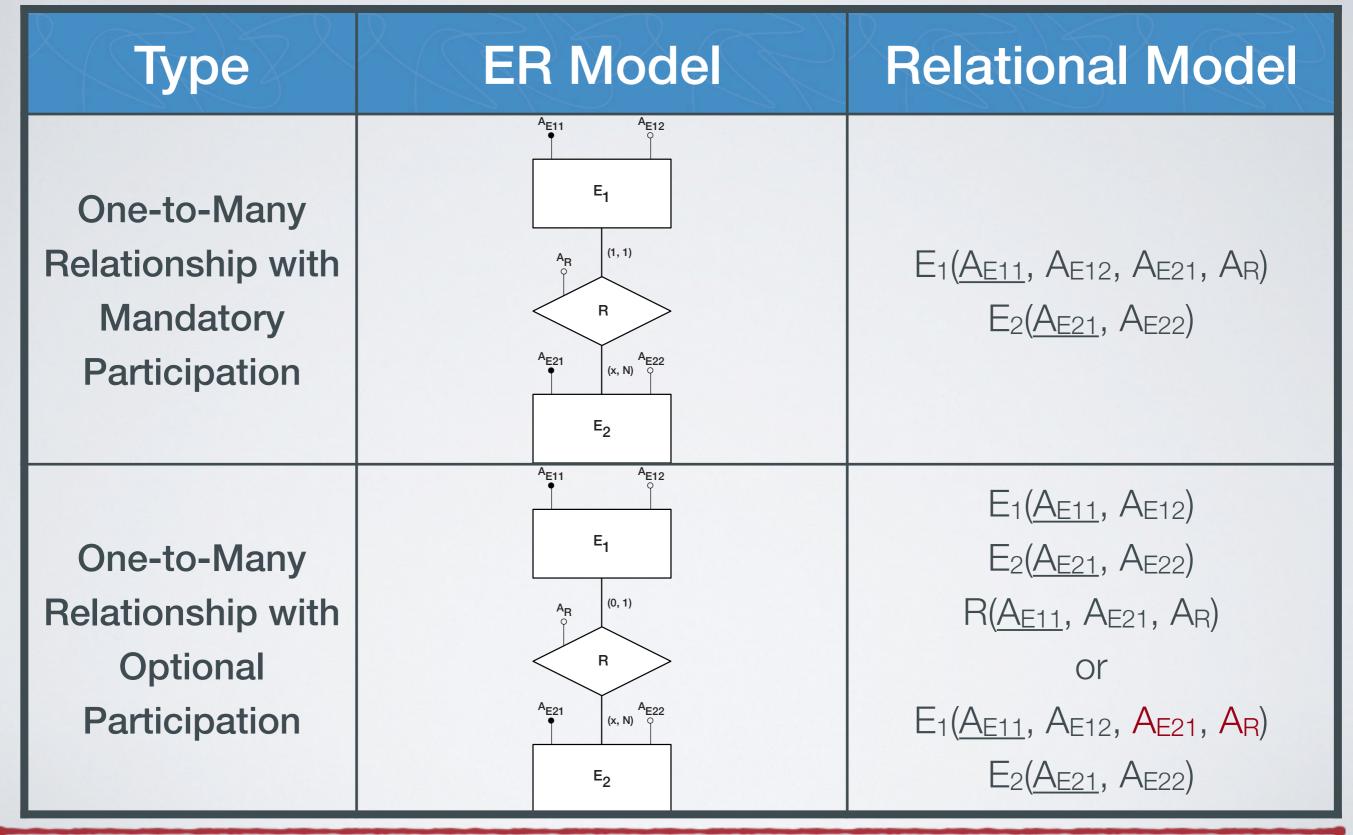






Mapping Summary (2/4)

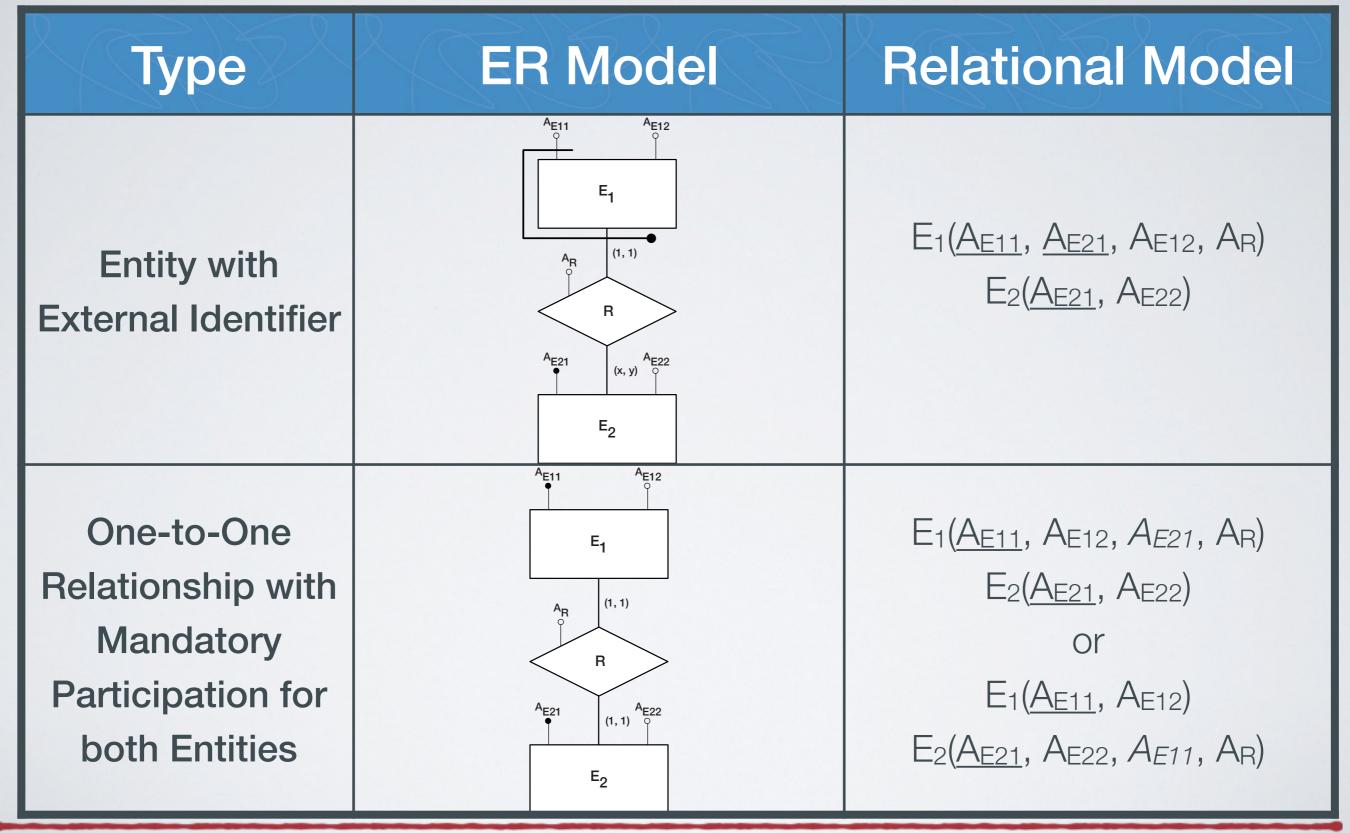






Mapping Summary (3/4)

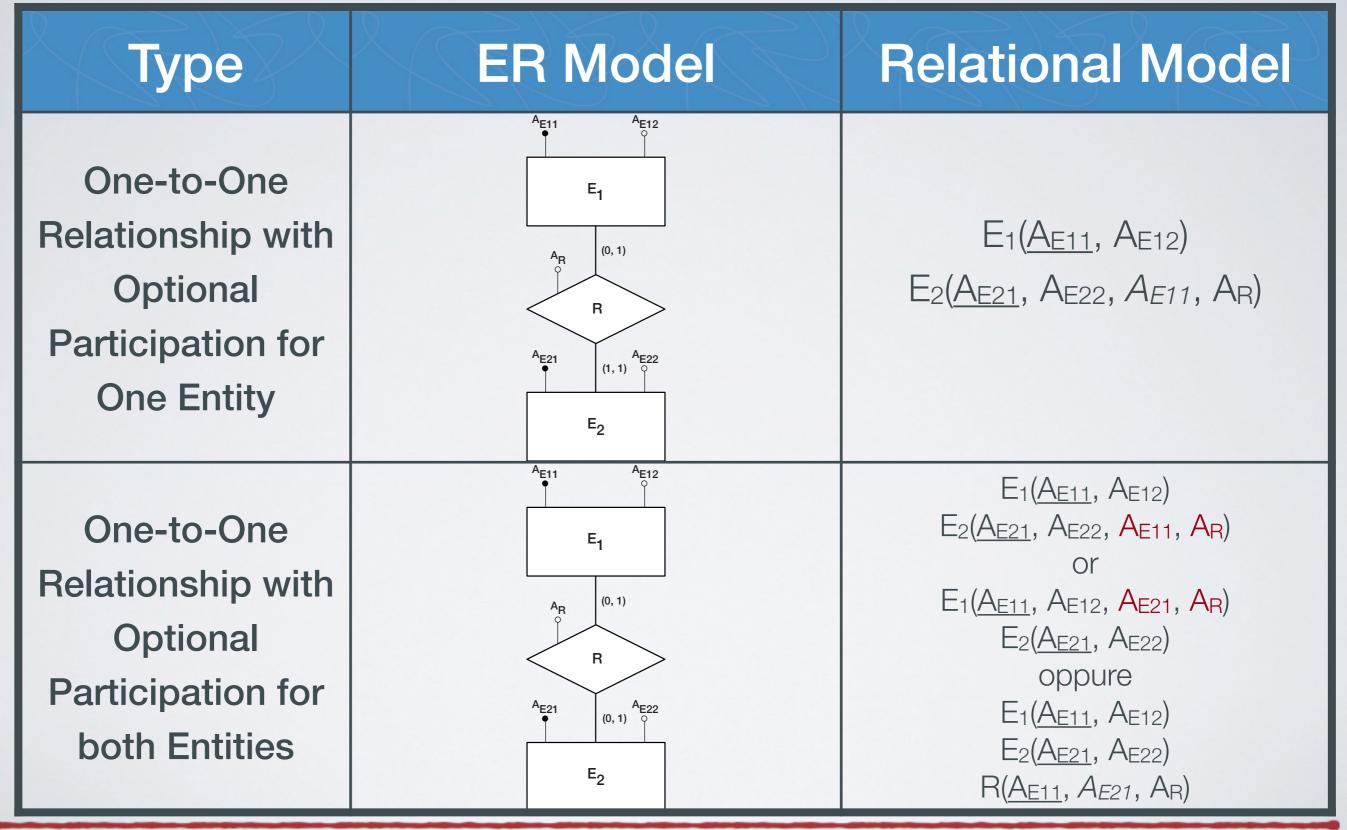






Mapping Summary (4/4)









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Questions? DilbertCartoonist@gmail.com © 2013 Scott Adams, Inc. /Dist. by Universal Uclick THE APPLICATION IS WHAT'S TAKING YOU DOES ANY OF THAT UNSTABLE BECAUSE THE SO LONG ON THE MEAN THE SAME DATA MODEL IS DRIVEN **PROJECT**? THING AS "LAZY"? BY AN OVERLY COMPLEX RELATIONAL DATABASE AND THERE WAS NO INTEGRATION TESTING. Dilbert.com 9-20-13