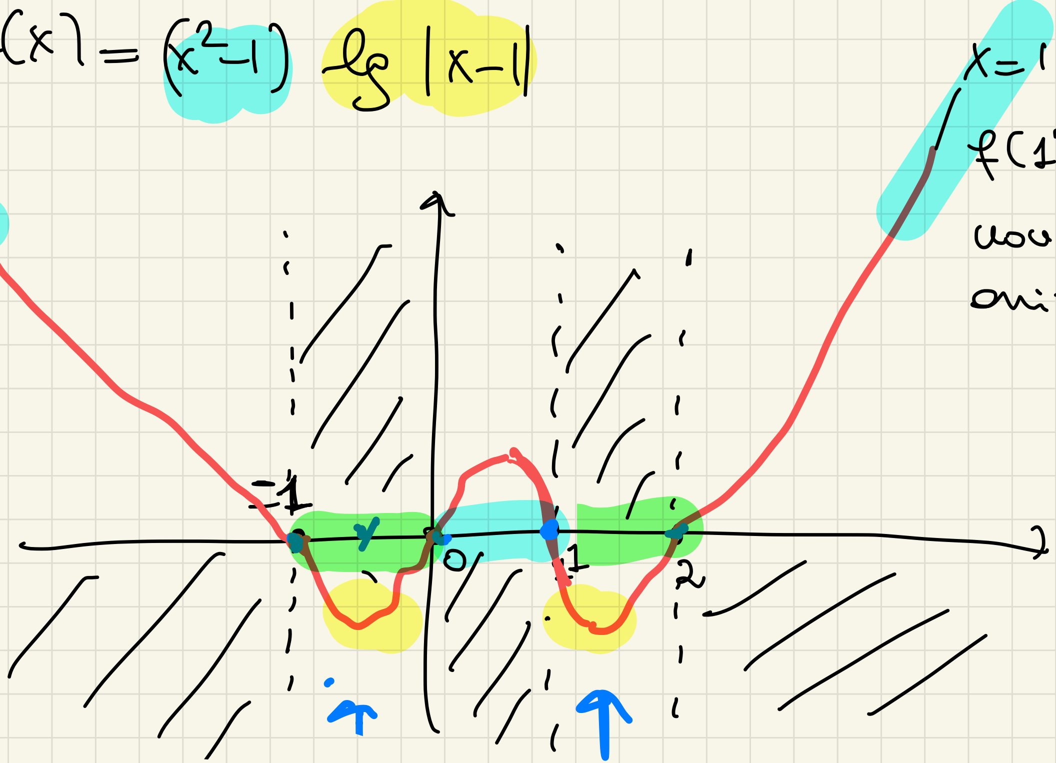
$f(x) = (x^2-1)\lg|x-1|$

$x=1$ sing. eliminabile

$f(1)=0$

non ci sono asint. orizz. né obliqui



in $(-1,0)$ e in $(1,2)$ ci sarà un pto di min. locale (e uno dei 2 globali)

in $(0,1)$ ci sarà un pto di max locale

Calcolo derivata

$$f(x) = (x^2-1)\,\lg|x-1|$$

derivata della funzione
$|x-1|$ NON È
DEFINITA in $x=1$

Calcolo prima la derivata di f per $x \neq 1$

$$|x-1| = \begin{cases} x-1 & \text{se } x>1 \\ -(x-1) = -x+1 & \text{se } x<1 \end{cases}$$

$x>1$ $\quad f(x) = (x^2-1)\,\lg(x-1)$

$x<1$ $\quad f(x) = (x^2-1)\,\lg(-x+1) = (x^2-1)\,\lg[-(x-1)]$

$x > 1$

$$f(x) = (x^2-1)\ \lg(x-1)$$

derivata logaritmo

$$f'(x) = (2x-0)\cdot \lg(x-1) + (x^2-1)\cdot \frac{1}{x-1}\cdot(1-0)$$

$$= 2x\ \lg(x-1) + \frac{x^2-1}{x-1} =$$

$$= 2x\ \lg(x-1) + \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 2x\ \lg(x-1) + x+1$$

$x < 1$

$$f(x) = (x^2-1)\ \lg(-x+1)$$

$$f'(x) = 2x\cdot \lg(-x+1) + (x^2-1)\frac{1}{(-x+1)}\cdot(-1+0) =$$

$$= 2x\ \lg(-x+1) + \frac{\cancel{(x-1)}(x+1)\cdot 1}{\cancel{(-1)}\cdot\cancel{(x-1)}}\cdot\cancel{(-1)}$$

$$= 2x\ \lg(-x+1) + x+1$$

$x > 1$ $\quad f'(x) = 2x \lg(x-1) + x+1$

$x < 1$ $\quad f'(x) = 2x \lg(-(x-1)) + x+1$

$$x \neq 1 \quad f'(x) = 2x \lg|x-1| + x+1$$

f è derivabile per ogni $x \neq 1$

$x = 1$?

$$\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} \underbrace{2x \cdot \lg(x-1)}_{2x \to 2,\ \lg 0^+ = -\infty} + \underbrace{x+1}_{\to 2} = 2\cdot(-\infty)+2 = -\infty$$

$$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} 2x \lg(-(x-1)) + x+1 = 2\cdot(-\infty)+2 = -\infty$$

$2x \to 2$, $\lg(-0^-) = \lg(0^+) = -\infty$, $x+1 \to 2$

$x = 1$ PUNTO A TANGENTE VERTICALE

per trovare il pto di max locale e i 2 pti di minimo (locale (uno anche globale

devo risolvere $f'(x) = 0$

Ⓔ $2x \lg|x-1| + x + 1 = 0$ $\quad x \neq 0$

$$\lg|x-1| = -\left(\frac{x+1}{2x}\right)$$

$\exists$ almeno 3 soluzioni

$x_1, x_2, x_3 \qquad -1 < x_1 < 0 \qquad 0 < x_2 < 1$

$1 < x_3 < 2$.

Es $f(x) = e^{-x^2}$ (f. GAUSSIANA)

dove, simmetrie, segno, limiti, asintoti, max, min e grafico qualitativo.

$$-x^2 \neq (-x)^2$$

$$x \longrightarrow x^2 \longrightarrow -x^2$$

$-x^2 \leq 0 \quad \forall x \in \mathbb{R}$

$D = \mathbb{R}$ $\quad f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$

PARI

$f(x) > 0 \quad \forall x \in D$

$$\lim_{x \to +\infty} e^{-x^2} = e^{-\infty} = 0$$

$y=0$
AS. ORIZZONTALE
a $+\infty$ e
$-\infty$.

$x^2 \to +\infty$

$-x^2 \to -\infty$

$$\lim_{x \to -\infty} e^{-x^2} = e^{-\infty} = 0$$

$(-\infty)^2 \to +\infty$

$-(-\infty)^2 \Rightarrow -\infty$

$f(x) = e^{-x^2}$ $\qquad f'(x) = e^{-x^2}(-x^2)' =$

$$= \underbrace{-e^{-x^2} \cdot 2x}$$

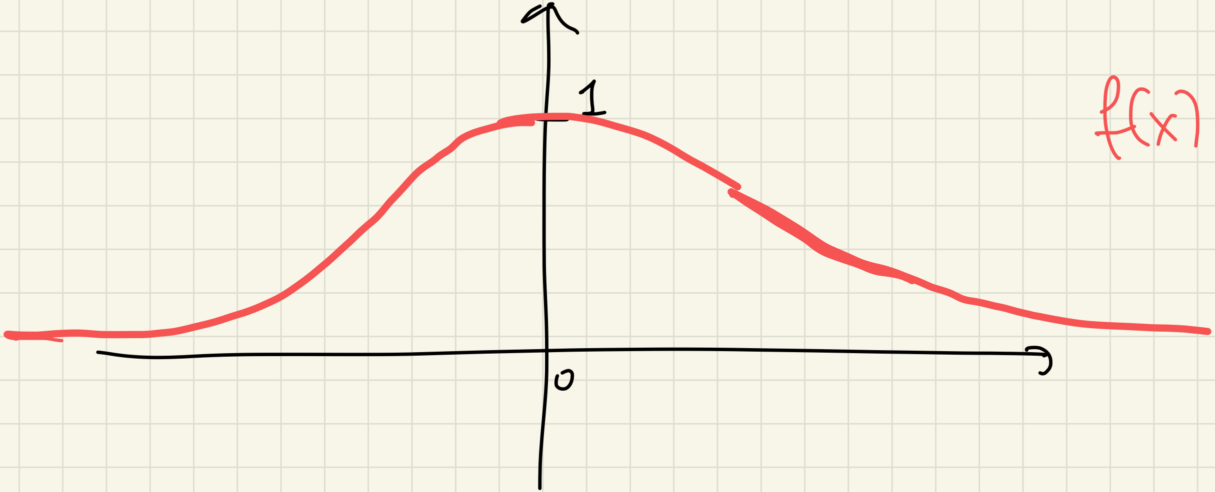
$$f'(x) = -2x\,\underbrace{e^{-x^2}}_{+}$$

$f'(x) \geq 0 \iff -2x \geq 0$
perché $e^{-x^2} > 0 \quad \forall x$

		0	
f'	$+$		$-$
f	↗		↘

f è crescente in $(-\infty, 0)$
decrescente in $(0, +\infty)$

$x = 0$ pto di MAX locale e anche GLOBALE
$f(0) = e^0 = 1$.



$$f(x) = e^{-x^2}$$

ES $f(x) = \arcsin\left(\dfrac{x^2}{x^2+2x}\right)$

DOMINIO $-1 \leq \dfrac{x^2}{x^2+2x} \leq 1$

(arcsen è definito solo se argomento è tra $[-1,1]$)

$$\begin{cases} \dfrac{x^2}{x^2+2x} \leq 1 \\ \dfrac{x^2}{x^2+2x} \geq -1 \end{cases}$$

SOLUZIONI COMUNI alle 2 DISEQUAZIONI

$$\begin{cases} \dfrac{x^2}{x^2+2x} - 1 \leq 0 \\ \dfrac{x^2}{x^2+2x} + 1 \geq 0 \end{cases} \quad \begin{cases} \dfrac{\cancel{x^2} - \cancel{x^2} - 2x}{x^2+2x} \leq 0 \\ \dfrac{x^2 + x^2 + 2x}{x^2+2x} \geq 0 \end{cases} \quad \begin{cases} \dfrac{-2x}{x^2+2x} \leq 0 \\ \dfrac{2x^2+2x}{x^2+2x} \geq 0 \end{cases}$$

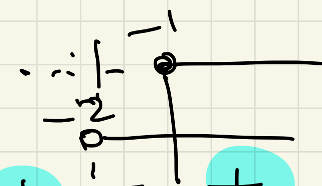
$$\begin{cases} \dfrac{-2x}{x^2+2x} < 0 \\[2ex] \dfrac{2x^2+2x}{x^2+2x} \geq 0 \end{cases} \qquad \begin{cases} \dfrac{2x}{x^2+2x} \geq 0 \\[2ex] \dfrac{2x(x+1)}{x^2+2x} \geq 0 \end{cases} \qquad \begin{cases} \dfrac{2\cancel{x}}{\cancel{x}(x+2)} \geq 0 \\[2ex] \dfrac{2\cancel{x}(x+1)}{\cancel{x}(x+2)} \geq 0 \end{cases}$$

$x \neq 0$

$$\begin{cases} ① \quad \underbrace{\dfrac{2}{x+2} \geq 0 \qquad x+2>0 \qquad x>-2} \\[3ex] ② \quad \underbrace{\dfrac{2(x+1)}{(x+2)} \geq 0} \end{cases}$$

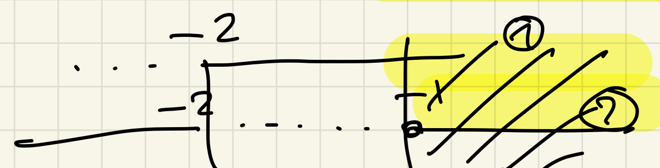
$\rightarrow$ N $x \geq -1$

D $x > -2$



$x \geq -1 \qquad x < -2$

prendo le sol. comuni



DOMINIO $\{\underbrace{x \geq -1 \,,\; x \neq 0}\}$

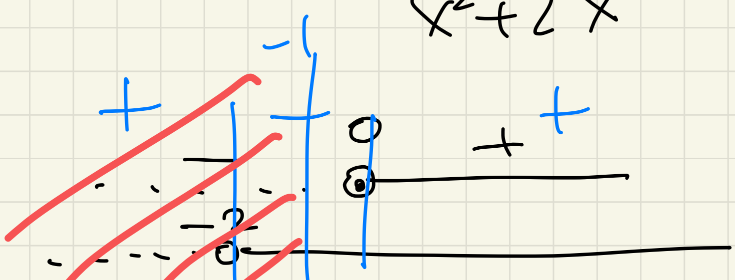
$D: [-1, 0) \cup (0, +\infty)$ No SIMM.

segno $\underbrace{\underbrace{\arcsin}\left(\frac{x^2}{x^2+2x}\right) \geq 0 = \underbrace{\arcsin}\, 0}$

$$\frac{x^2}{x^2+2x} \geq 0 \qquad x^2 \geq 0 \;\forall x$$

$$x^2+2x>0$$

$$\boxed{x(x+2)>0}$$



$$\begin{cases} f(x) > 0 & x > 0 \\ f(x) \le 0 & -1 \le x < 0 \end{cases}$$

D

$[-1,0) \cup (0,+\infty)$

$$f(-1) = \arcsin\left(\frac{(-1)^2}{(-1)^2+2(-1)}\right) = \arcsin\left(\frac{+1}{1-2}\right) =$$

$$= \arcsin\left(\frac{1}{-1}\right) = \arcsin(-1) = -\frac{\pi}{2}$$

$\boxed{0 \text{ SING ELIMINABILE}}$ $\rightarrow$ $f(0)=0$

$$\lim_{x\to 0} \arcsin\left(\frac{x^2}{\underline{\underline{x^2+2x}}}\right) = \lim_{x\to 0} \arcsin\left(\frac{\cancel{x^2}}{\cancel{x}(x+2)}\right)$$

$$= \lim_{x\to 0} \arcsin\left(\frac{x}{x+2}\right) = \arcsin 0 = 0$$

$$\lim_{x \to +\infty} \arcsin\left(\frac{x^2}{x^2+2x}\right) =$$

$$= \lim_{x \to +\infty} \arcsin\left(\frac{\cancel{x^2}^{\,1}}{\cancel{x^2}\left(1+\frac{2\cancel{x}}{x^{\cancel{2}}}\right)}\right) =$$

$$= \lim_{x \to +\infty} \arcsin\left(\frac{1}{\left(1+\underbrace{\frac{2}{x}}_{\to 0}\right)}\right) = \arcsin 1 = \frac{\pi}{2}$$

$\boxed{y = \frac{\pi}{2} \text{ è AS. ORIZZONTALE a } +\infty}$

$$f(x) = \arcsin\left(\frac{x^2}{x^2+2x}\right) = \arcsin\left(\frac{x^{\cancel{2}}}{\cancel{x}(x+2)}\right) =$$

$$= \arcsin\left(\frac{x}{x+2}\right)$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{x+2}\right)^2}} \cdot \left(\frac{x}{x+2}\right)' =$$

$$= \frac{1}{\sqrt{1-\frac{x^2}{(x+2)^2}}} \cdot \left[\frac{1\cdot(x+2) - x(1+0)}{(x+2)^2}\right] =$$

$$= \frac{1}{\sqrt{\frac{(x+2)^2 - x^2}{(x+2)^2}}} \cdot \frac{\cancel{x}+2-\cancel{x}}{(x+2)^2} =$$

$D = [-1, +\infty)$
$\downarrow$
dominio

$$= \sqrt{\frac{(x+2)^2}{(x+2)^2 - x^2}} \cdot \frac{2}{(x+2)^2} = \frac{|x+2|}{\sqrt{\cancel{x^2}+4x+4-\cancel{x^2}}} \cdot \frac{2}{(x+2)^2}$$

$$= \frac{|x+2|}{\sqrt{4(x+1)}} \cdot \frac{2}{(x+2)^2} = \frac{\cancel{|x+2|} \cdot \cancel{2}}{\cancel{\sqrt{4}} \cdot \sqrt{x+1} \quad |x+2|^{\cancel{2}}} =$$

$$= \frac{1}{\sqrt{x+1} \quad |x+2|}$$

$\boxed{\forall x \in (-1, +\infty)}$

$f'(x) > 0 \quad \forall x \in (-1, +\infty) \implies f$ è strett. crescente

?

$x = -1$

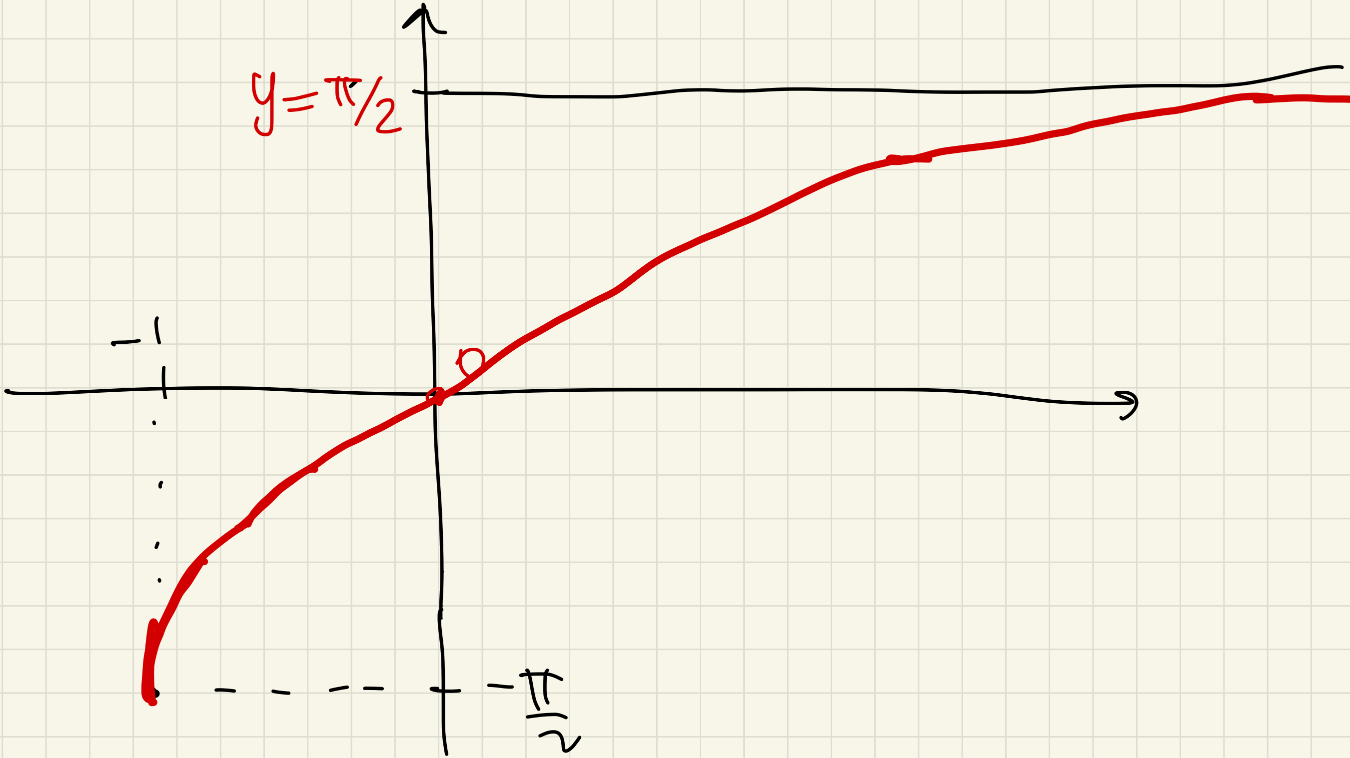
$$f'(x) = \frac{1}{\sqrt{x+1} \cdot |x+2|}$$

$$\lim_{x \to (-1)^+} f'(x) = \lim_{x \to (-1)^+} \frac{1}{\sqrt{x+1}\,|x-2|} = \frac{1}{0^+} = +\infty$$

$\sqrt{0^+} = 0^+$

$|-3| = 3$

$x = -1$ il grafico si attacca con pendenza verticale



X = −1 È PUNTO DI MINIMO GLOBALE

la funzione non ha pti di max.

es

$$f(x) = \sqrt{x^2-1} + 2x$$

D: $x^2-1 \geq 0$ $\quad x \geq +1 \quad x \leq -1$

$(-\infty, -1] \cup [1, +\infty)$ $\quad f(1) = 2$

$f(-1) = -2$

$f(-x) = \sqrt{(-x)^2-1} + 2(-x) = \sqrt{x^2-1} - 2x \neq f(x)$

$\neq -f(x)$

né pari né dispari

segno $\sqrt{x^2-1} + 2x \geq 0$

se $x \in [1, +\infty)$ $\quad \sqrt{x^2-1} \geq 0 \quad 2x \geq 0 \Longrightarrow f(x) \geq 0$

& $x \in (-\infty, -1]$ $\qquad -x \geq 0\,!$

$$f(x) \geq 0 \qquad \underbrace{\sqrt{x^2-1}}_{+} \geq \underbrace{-2x}_{+}$$

$$\downarrow$$

$$x^2 - 1 \geq 4x^2$$

$$\underbrace{-3x^2}\underbrace{-1} \geq 0 \qquad \nexists\, x$$

$$f(x) \geq 0 \iff x \in [1, +\infty)$$

$$\lim_{x \to -\infty} \left(\underbrace{\sqrt{x^2-1}}_{\downarrow \, +\infty} + \underbrace{2x}_{-\infty} \right) \cdot \frac{\left(\sqrt{x^2-1} - 2x\right)}{\sqrt{x^2-1} - 2x} =$$

$$= \lim_{x \to -\infty} \frac{x^2 - 1 - 4x^2}{\sqrt{x^2-1} - 2x} =$$

$$\sqrt{x^2} = |x| = -x \quad x < 0$$

$$= \lim_{x \to -\infty} \frac{-3x^2 - 1}{\sqrt{x^2-1} - 2x} \qquad \frac{-\infty}{+\infty} =$$

$$= \lim_{x \to -\infty} \frac{x^2\left(-3 - \frac{1}{x^2}\right)}{\sqrt{x^2}\sqrt{1 - \frac{1}{x^2}} - 2x} = \lim_{x \to -\infty} \frac{x^2\left(-3 - \frac{1}{x^2}\right)}{-x\sqrt{1 - \frac{1}{x^2}} - x} =$$

$$= \lim_{x\to-\infty} \frac{\underbrace{\cancel{x^2}}_{\to -\infty}\underbrace{\left(-3-\frac{1}{x^2}\right)}_{\to -3}}{\cancel{x}\underbrace{\left[-\sqrt{1-\frac{1}{x^2}}-1\right]}_{-1-1=-2}} = -\infty$$