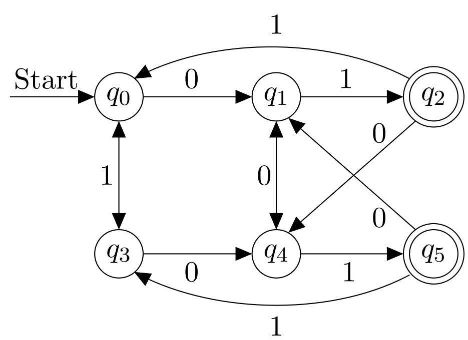
Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

September 16th, 2024

1. **[6 points]** Consider the DFA A whose transition function is graphically represented below (arcs with double direction represent two arcs in opposite directions)



   (a) Provide the definition of equivalent pair of states for a DFA.

   (b) Apply to A the tabular algorithm for detecting pairs of equivalent states, reporting **all the intermediate steps**.

   (c) Specify the minimal DFA equivalent to A.

2. **[9 points]** Let $\Sigma = \{a, b\}$. Consider the following languages over Σ

$$L_1 = \{a^p b a^q b a^r b \mid p, q, r \geq 1,\ p = q \text{ and } q = r\} ;$$
$$L_2 = \{a^p b a^q b a^r b \mid p, q, r \geq 1,\ p = q \text{ or } q = r\} .$$

   (a) Prove that L_1 is not in CFL.

   (b) Prove that L_2 is not in REG.

   (c) Prove that L_2 belongs to CFL.

3. **[5 points]** With reference to push-down automata (PDA), answer the following questions.

   (a) Provide the definition of language accepted by final state and language accepted by empty stack.

   (b) Prove that if P_F is a PDA accepting by final state, then there exists a PDA P_N accepting by empty stack such that $L(P_F) = N(P_N)$.

(please turn to the next page)

4. **[6 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) The class $\mathcal{P}$ of languages that can be recognized in polynomial time by a TM is closed under complementation.

   (b) The class $\mathcal{P}$ defined as in (a) includes the class REG.

   (c) Let L_1 be a finite language such that $|L_1| = 1$, that is, L_1 contains only one string. Let also L_2 be a language in CFL$\smallsetminus$REG. The language $L_1 \cap L_2$ is in CFL$\smallsetminus$REG.

   (d) Let L_1 and L_2 be as in (c). The language $L_1 \cup L_2$ is in CFL$\smallsetminus$REG. **Hint**: this is more difficult that the previous questions; use a proof by contradiction and known properties of the class REG.

5. **[7 points]** We say that a string w_1 is a proper infix of a string w_2 if there exist strings $u, v \neq \epsilon$ such that $w_2 = uw_1v$. Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} \;=\; \{L \mid L \in \text{RE, for no pair of strings } w_1, w_2 \in L \text{ we have that } w_1 \text{ is a proper infix of } w_2\}\ .$$

   Define $L_{\mathcal{P}} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$.

   (a) Provide an example of a finite language $L \in \mathcal{P}$.

   (b) Provide an example of an infinite language $L \in \mathcal{P}$.

   (c) Use Rice's theorem to show that $L_{\mathcal{P}}$ is not in REC.

   (d) Prove that $L_{\mathcal{P}}$ is not in RE.