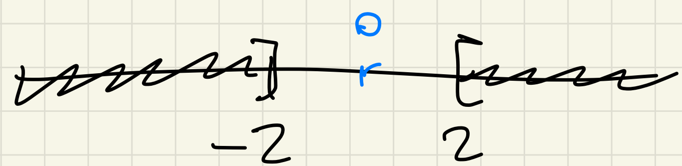
Es $f(x) = \sqrt{x^2-4} - x$

determinare dominio, simmetrie, segno, limiti agli estremi del dominio, eventuali pti in cui la funzione si può estendere per continuità (singolarità eliminabili), e asintoti.

D: $x^2 - 4 \geq 0 \implies x \geq 2$ o $x \leq -2$

$$\sqrt{x^2} \geq \sqrt{4}$$
$$\Downarrow$$
$$|x| \geq 2$$



$$D = (-\infty, -2] \cup [2, +\infty)$$

$x \in D \rightarrow -x \in D$ $\qquad f(x) = \underbrace{\sqrt{x^2-4}} - x$

$$f(-x) = \sqrt{\underbrace{(-x)^2}-4} - (-x) = \underbrace{\sqrt{x^2-4}} + x \neq f(x)$$

$$\neq -f(x)$$

f né pari né dispari

segno $\quad f(x) \geq 0 \qquad \underbrace{\sqrt{x^2-4} - x \geq 0}$

$\underline{x \in D} = (-\infty, -2] \cup [2, +\infty)$

$$\underbrace{\sqrt{x^2-4}}_{\geq 0} \geq x$$

$\sqrt{x^2-4} \geq 0 \quad \forall x \in D$

$\sqrt{x^2-4} \geq 0 > x$

[se $x \in D$ e $x < 0$

cioè $x \in (-\infty, -2]$]

la diseg. è sempre verificata $\rightarrow f(x) \geq 0$

& $x>0$ $x \in D$

$\sqrt{x^2-4} \geq x$

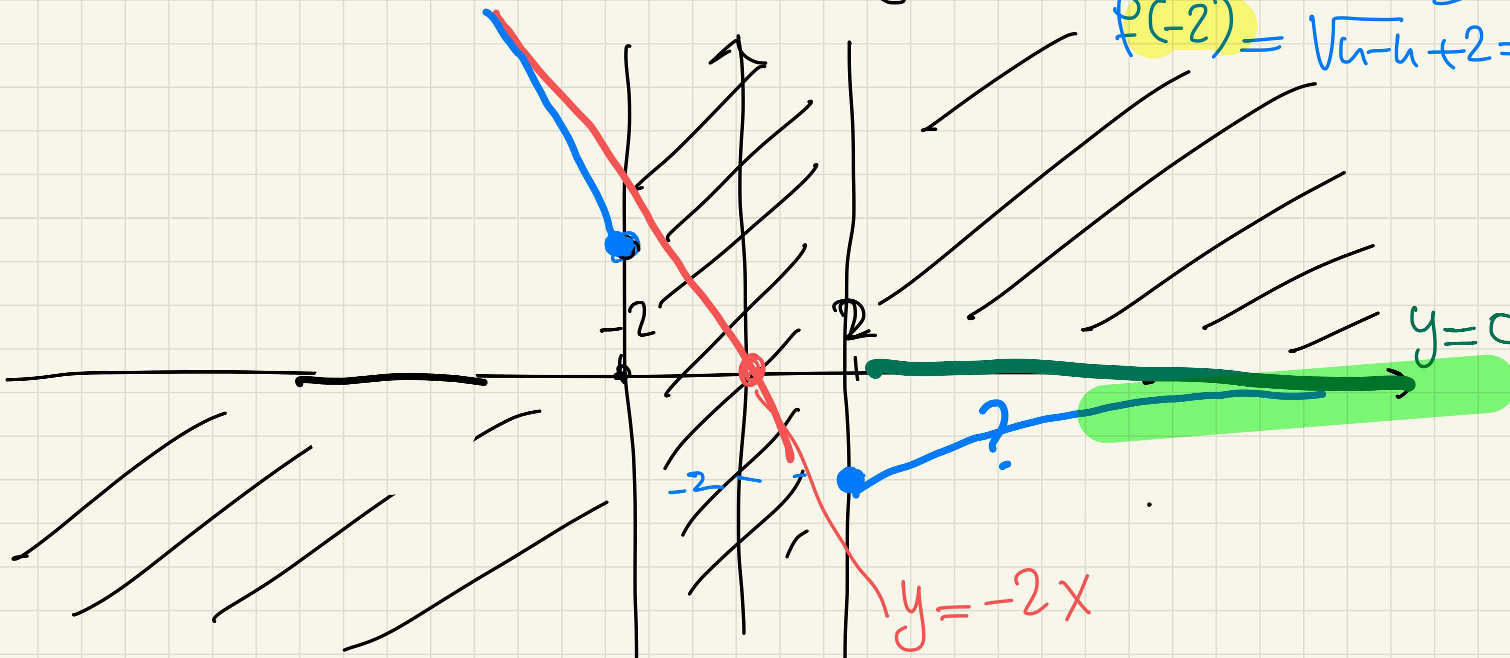
$\Downarrow$ elevo al quadrato

$\cancel{x^2}-4 \geq \cancel{x^2}$ $-4 \geq 0$ impossibile

$f(x) \geq 0 \iff x \in (-\infty,-2]$

$f(2) = \sqrt{4-4} - 2$
$= -2$

$f(-2) = \sqrt{4-4} + 2 = 2$



$D = (-\infty, -2] \cup [2, +\infty)$

$f(-2) = 2$

$f(+2) = -2$

gli unici limiti da calcolare sono a $\pm\infty$

la funzione NON HA ASINTOTI VERTICALI

$x = x_0$ è as. verticale dx $\lim_{x \to x_0^+} f(x) = \pm\infty$

$$\lim_{x \to -\infty} \underbrace{\sqrt{x^2-4}}_{+\infty} \underbrace{-x}_{+\infty} = +\infty +\infty = +\infty$$

$x \to -\infty \quad x^2 \to (-\infty)^2 = +\infty \quad x^2 - 4 \to +\infty - 4 = +\infty$

$\sqrt{x^2-4} \to \sqrt{+\infty} = +\infty$

$x \to -\infty \quad -x \to -(-\infty) = +\infty$

$$\lim_{x\to+\infty} \sqrt{x^2-4} - x = +\infty - \infty$$

forma indeterminata

$\sqrt{x^2-4} \to +\infty$, $-x \to -\infty$

$$(A-B)(A+B) = A^2-B^2$$

$$\left(\sqrt{x^2-4} - x\right)\frac{\left(\sqrt{x^2-4}+x\right)}{\sqrt{x^2-4}+x} =$$

$A = \sqrt{x^2-4}$

$B = x$

$$= \frac{\cancel{x^2}-4-\cancel{x^2}}{\sqrt{x^2-4}+x} = \frac{-4}{\sqrt{x^2-4}+x} \to \frac{-4}{+\infty} \to 0$$

$\sqrt{x^2-4} \to +\infty$, $+x \to +\infty$

$$\lim_{x\to+\infty} f(x) = 0$$

$y=0$ AS. ORIZZONTALE a $+\infty$

cerco asintoto obliquo a $-\infty$

$$\sqrt{x^2} = |x|$$

$$\lim_{x \to -\infty} f(x) = +\infty$$

(NON HO ASINTOTO ORIZZONTALE)

$$m = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{\sqrt{x^2-4} - x}{x} \qquad \frac{+\infty}{-\infty}$$

NUM
$$\sqrt{x^2-4} - x = \sqrt{x^2 \cdot \left(1 - \frac{4}{x^2}\right)} - x =$$

$$= \sqrt{x^2} \cdot \sqrt{1 - \frac{4}{x^2}} - x = |x| \sqrt{1 - \frac{4}{x^2}} - x$$

$$\sqrt{1 - \frac{4}{+\infty}} = \sqrt{1} = 1$$

$$= |x|\sqrt{1-\frac{4}{x^2}} - x$$

$$= -x\sqrt{1-\frac{4}{x^2}} - x =$$

$$= \underset{-\infty}{x}\left[-\sqrt{1-\frac{4}{x^2}} - 1\right]$$

$$-\sqrt{1-\frac{4}{+\infty}} - 1 = -\sqrt{1} - 1 = -1-1 = -2$$

ma sto calcolando limite per $x \to -\infty$

$x<0$ $\quad x = -50'0000$

$-1'000\cdot 000\ldots$

$|x| = -x$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2-4} - x}{x} = \lim_{x \to -\infty} \frac{\cancel{x}\left[-\sqrt{1-\frac{4}{x^2}} - 1\right]}{\cancel{x}}$$

$$= \lim_{x \to -\infty} -\sqrt{1-\frac{4}{x^2}} - 1 = \underline{-2 = m}$$

$$q = \lim_{x \to -\infty} f(x) - m \cdot x$$

$$= \lim_{x \to -\infty} \underbrace{\sqrt{x^2-4} - x} - (-2) \cdot x =$$

$$= \lim_{x \to -\infty} \sqrt{x^2-4} \underbrace{- x} \underbrace{+ 2x} = \lim_{x \to -\infty} \sqrt{x^2-4} + x$$

$$\lim_{x\to-\infty} \underbrace{\sqrt{x^2-4}}_{+\infty}\ \underbrace{+x}_{-\infty} = +\infty - \infty$$

$$(A+B)(A-B) = A^2-B^2$$

$$\left(\sqrt{x^2-4}+x\right)\cdot\frac{\left(\sqrt{x^2-4}-x\right)}{\left(\sqrt{x^2-4}-x\right)} = \frac{\cancel{x^2}-4-\cancel{x^2}}{\sqrt{x^2-4}-x}$$

$$= \frac{-4}{\underbrace{\sqrt{x^2-4}}_{+\infty}\ \underbrace{-x}_{-(-\infty)=+\infty}} = \frac{-4}{+\infty} \longrightarrow 0 = 9$$

$y = mx + q$ asintoto obliquo è

$y = -2x + 0$

$y = -2x$ è asintoto obliquo

a $-\infty$

Teorema del confronto (o dei due carabinieri).

① Assumiamo di avere

$$\underbrace{h(x)} \leq f(x) \leq \underbrace{g(x)} \qquad \forall x$$

se $\lim\limits_{x \to x_0^+} h(x) = \lim\limits_{x \to x_0^+} g(x) = L \in \mathbb{R} \implies \lim\limits_{x \to x_0^+} f(x) = L$

x_0^-, $+\infty$, $-\infty$

② $h(x) \leq f(x) \quad \forall x$

e che $\lim\limits_{\substack{x \to x_0^+ \\ x_0^- \\ +\infty \\ -\infty}} h(x) = +\infty \implies \lim\limits_{x \to \ldots} f(x) = +\infty$

③ $f(x) \leq g(x) \quad \forall x$

$\lim\limits_{x \to \ldots} g(x) = -\infty \implies \lim\limits_{x \to \ldots} f(x) = -\infty$

1° applicazione

Se $\lim_{x \to x_0, +\infty, -\infty} f(x) = 0$ e $h(x)$ è LIMITATA

$\exists\, c > 0$

$-c \leq h(x) \leq +c \quad \forall x \in D$

$$\lim_{x \to x_0, +\infty, -\infty} f(x) \cdot h(x) = 0$$

prodotto tra una funzione che ha limite 0 e una funzione che NON HA LIMITE ma è LIMITATA ha limite 0.

es. $\lim_{x \to +\infty} \frac{\cos x}{x} = \lim_{x \to +\infty} \cos x \cdot \frac{1}{x} = 0$

$\cos x$: NON HA LIMITE perché è periodico

$\frac{1}{x} \to 0$

$-1 \leq \cos x \leq 1 \quad \forall x$

LIMITATA

$$-\frac{1}{x} \leq \cos x \cdot \frac{1}{x} \leq \frac{1}{x}$$

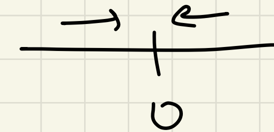
$-\frac{1}{x} \to 0$ $\qquad$ $\frac{1}{x} \to 0$

Limite notevole

$$\lim_{x\to 0} \frac{\sin x}{x}$$

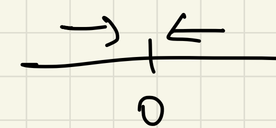
f. indet $\frac{0}{0}$ $\frac{\sin 0}{0} = \frac{0}{0}$

$\sin(-x) = -\sin x$

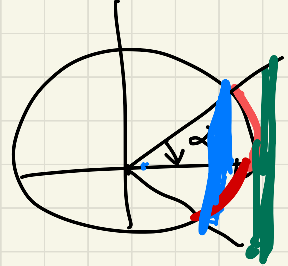


oss 1 $$\frac{\sin(-x)}{-x} = \frac{-\sin x}{-x} = \frac{\sin x}{x}$$

$f(x) = \frac{\sin x}{x}$ è f. pari $f(-x) = f(x)$



$$\boxed{\lim_{x\to 0^+} \frac{\sin x}{x} = \lim_{x\to 0^-} \frac{\sin x}{x}}$$



$0 < \alpha < \frac{\pi}{2}$

$$\cos \alpha \le \frac{\sin \alpha}{\alpha} \le 1$$

$\forall \alpha \in \left(0, \frac{\pi}{2}\right)$

se $x \to 0^+ \Rightarrow x > 0$ e $x \to 0$ $\quad x \in \left(0, \frac{\pi}{2}\right)$

$\forall\, x > 0 \quad x \to 0$

$$\cos x \le \frac{\sin x}{x} \le 1$$

$\cos 0 = 1$ $\qquad$ 1

per teorema del confronto

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$$\Rightarrow \boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{x}{\sin x}}$$ LIMITE NOTEVOLE

Limiti collegati

$$\frac{\sin x}{\cos x} \cdot \frac{1}{x} = \frac{\sin x}{\cos x \cdot x}$$

$$= \frac{\sin x}{x \cdot \cos x}$$

$\downarrow 1$

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = \frac{\operatorname{tg} 0}{0} = \frac{0}{0} \text{ f.i.}$$

$$= \lim_{x\to 0} \operatorname{tg} x \cdot \frac{1}{x} = \lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} =$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1 = \lim_{x\to 0} \frac{x}{\operatorname{tg} x}$$

$\frac{\sin x}{x} \to 1$

$\cos x \to \cos 0 = 1$

$$\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{\arcsin x}{\operatorname{sen}(\arcsin x)} =$$

$x \to 0 \qquad x \approx 0 \qquad \boxed{x = \sin(\arcsin x) \quad \forall x \in [-1,1]}$

$$x \longrightarrow \arcsin x \longrightarrow \frac{\arcsin x}{\sin(\arcsin x)}$$

$x \to 0 \qquad \arcsin x \to \arcsin 0 = 0$

$$\lim_{x\to 0} \frac{\arcsin x}{\sin(\arcsin x)} = \lim_{y\to 0} \frac{y}{\sin y} = 1$$

$$\lim_{x\to 0} \frac{x}{\arcsin x} = 1 = \lim_{x\to 0} \frac{\arcsin x}{x}$$

$$\lim_{x\to 0} \frac{x}{\operatorname{arctg} x} = 1 = \lim_{x\to 0} \frac{\operatorname{arctg} x}{x}$$

$$x = \operatorname{tg}(\operatorname{arctg} x)$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{x}{\sin x}$$

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1 = \lim_{x\to 0} \frac{x}{\operatorname{tg} x}$$

$$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}} \qquad \frac{1-\cos 0}{0^2} = \frac{1-1}{0} = \frac{0}{0}$$

$$\frac{(1-\cos x)}{x^2} \cdot \frac{(1+\cos x)}{1+\cos x} = \frac{1-(\cos x)^2}{x^2} \cdot \frac{1}{(1+\cos x)}$$

$$(\cos x)^2 + (\sin x)^2 = 1 \qquad (\sin x)^2 = 1-(\cos x)^2$$

$$= \frac{(\sin x)^2}{x^2} \cdot \underbrace{\frac{1}{1+\cos x}} = \left(\frac{\sin x}{x}\right)^2 \cdot \frac{1}{1+\cos x}$$

$$\left(\frac{\sin x}{x}\right)^2 \to 1^2 \qquad \frac{1}{1+\cos x} \to \frac{1}{1+\cos 0} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \qquad \lim_{x \to 0} \frac{\cos x - 1}{x^2} = -\frac{1}{2}$$

$$\lim_{x \to 0} \frac{x^2}{1-\cos x} = 2$$

$$\lim_{x \to 0} \frac{1-\cos x}{x} \cdot \frac{x}{x} = \lim_{x \to 0} \frac{1-\cos x}{x^2} \cdot x = \frac{1}{2} \cdot 0 = 0$$

$$\frac{1}{2} \qquad 0$$

$$\lim_{x \to 0} \frac{1-\cos x}{x} = 0$$