

$$\text{Ese} \quad f(x) = \sqrt{x^2 - 4} - x$$

segno

determinare dominio, simmetrie e limiti agli estremi del dominio, eventuali punti in cui la funzione si può estendere per continuità (singolarità cliniche), e asintoti.

$$D: \quad x^2 - 4 \geq 0 \quad \Rightarrow \quad x \geq 2 \quad \cup \quad x \leq -2$$



$$D = (-\infty, -2] \cup [2, +\infty)$$

$$x \in D \rightarrow -x \in D$$

$$f(-x) = \sqrt{(-x)^2 - 4} - (-x) = \sqrt{x^2 - 4} + x \neq f(x)$$
$$\neq -f(x)$$

f n'est pas une fonction

seulement

$$f(x) \geq 0$$

$$x \in D = [-\infty, -2] \cup [2, +\infty)$$

$$\sqrt{x^2 - 4} \geq x$$

$$\sqrt{x^2 - 4} \geq 0 \quad \forall x \in D$$

$$\sqrt{x^2 - 4} - x \geq 0$$

$$\sqrt{x^2 - 4} \geq 0 > x$$

$\forall x \in D \text{ et } x < 0$
 $\exists x \in (-\infty, -2]$

les diseg. est sempre
vérifiée $\rightarrow f(x) \geq 0$

$$\& x > 0 \quad x \in D$$

$$\sqrt{x^2 - h} \geq x$$

↓ elos ol speedus

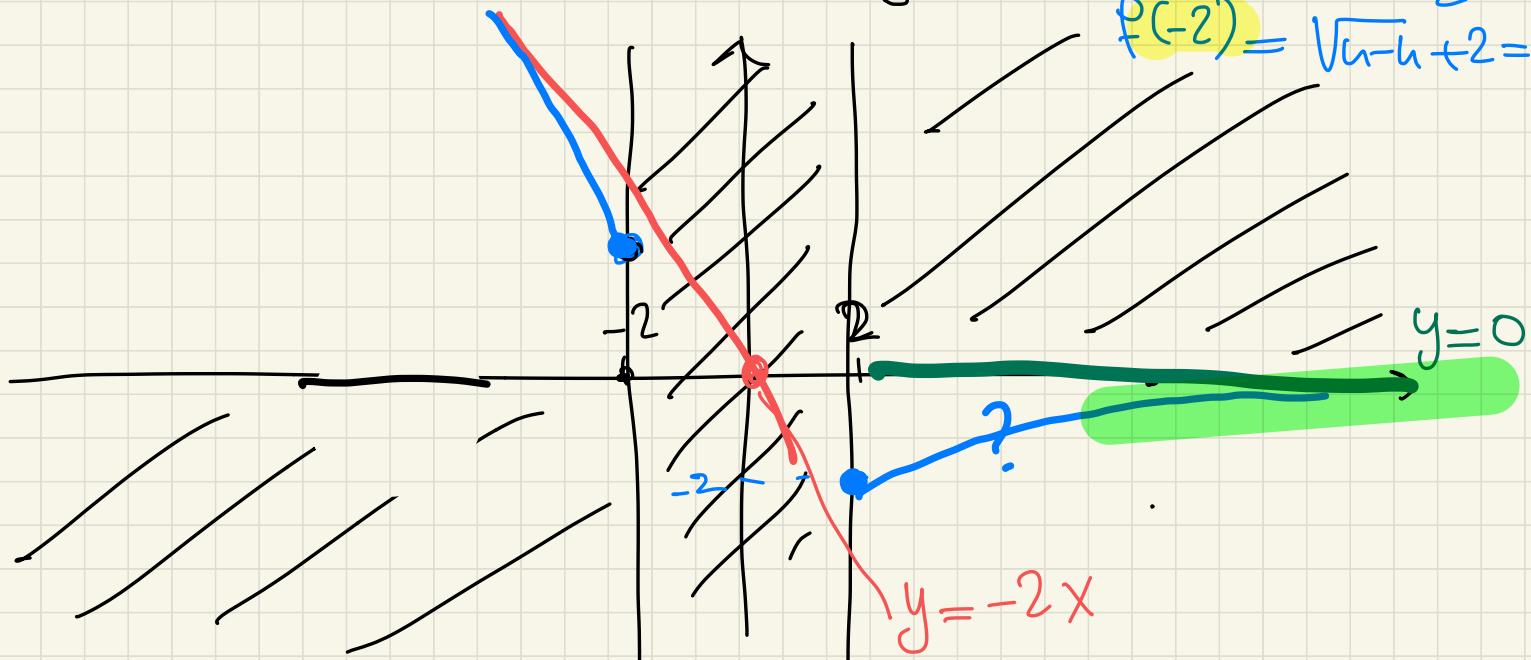
$$x^2 - h \geq x^2$$

$-h \geq 0$ impossible

$$f(x) \geq 0 \iff x \in (-\infty, -2]$$

$$f(2) = \sqrt{h-h-2}$$

$$f(-2) = \sqrt{h-h+2} = 2$$



$$D = (-\infty, -2] \cup [2, +\infty)$$

$$f(-2) = 2$$

$$f(+2) = -2$$

gli unici limiti da calcolare sono a $\frac{+\infty}{-\infty}$

le funzioni NON HA ASINTOTI VERTICALI

$$x = x_0 \text{ è as. verticale} \quad \text{line} \quad f(x) = \frac{+\infty}{-\infty} \quad x \rightarrow x_0^+$$

line

$$x \rightarrow -\infty$$

$$\sqrt{x^2 - 4}$$

$$-x = +\infty + \infty = +\infty$$

$$x \rightarrow -\infty$$

$$x^2 \rightarrow (-\infty)^2 = +\infty$$

$$x^2 - 4 \rightarrow +\infty - 4 = +\infty$$

$$\sqrt{x^2 - 4} \rightarrow \sqrt{+\infty} = +\infty$$

$$x \rightarrow -\infty$$

$$-x \rightarrow -(-\infty) = +\infty$$

$$\lim_{x \rightarrow +\infty}$$

$$\frac{\sqrt{x^2 - 4}}{-x} = +\infty - \infty$$

Forces
(indeterminate)

$$\left(\frac{\sqrt{x^2 - 4}}{-x} \right) \cdot \left(\frac{\sqrt{x^2 - 4} + x}{\sqrt{x^2 - 4} + x} \right) =$$

$$= \frac{x^2 - 4 - x^2}{\sqrt{x^2 - 4} + x} =$$

$$= \frac{-4}{\sqrt{x^2 - 4} + x} \xrightarrow{+\infty} \frac{-4}{+\infty} \rightarrow 0$$

$$(A-B)(A+B) = A^2 - B^2$$

$$A = \sqrt{x^2 - 4}$$

$$B = x$$

$\lim_{x \rightarrow +\infty} f(x) = 0$
 $y = 0$ AS. ORIZONTALE
 $a + \infty$

ancho apunta b obliqueo $a = \infty$

$$\sqrt{x^2} = |x|$$

line $f(x) = +\infty$ (NON HA ASINTOTO
 $x \rightarrow -\infty$ ORIZONTALE)

$$m = \frac{f(x)}{x} = \frac{\sqrt{x^2 - u} - x}{x}$$

$x \rightarrow -\infty$ $\frac{+\infty}{-\infty}$

NUM

$$\begin{aligned}\sqrt{x^2 - u} - x &= \sqrt{x^2 \cdot \left(1 - \frac{u}{x^2}\right)} - x = \\&= \sqrt{x^2} + \sqrt{1 - \frac{u}{x^2}} - x = |x| \sqrt{1 - \frac{u}{x^2}} - x \\&\quad \checkmark \sqrt{1 - \frac{u}{x^2}} = \sqrt{1} = 1\end{aligned}$$

$$= |x| \sqrt{1 - \frac{4}{x^2}} - x$$

$$= -x \sqrt{1 - \frac{4}{x^2}} - x =$$

$$= x \left[-\sqrt{1 - \frac{4}{x^2}} - 1 \right]$$

mo lho coloado
ciente per $x \rightarrow -\infty$

$$x < 0$$

$$x = -50'000.000$$

$$-1'000'000.000$$

$$|x| = -x$$

$$-\sqrt{1 - \frac{4}{\infty}} - 1 = -\sqrt{1 - 1} = -1 - 1 = -2$$

line

$$x \rightarrow -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - 4} - x}{x} = \lim_{x \rightarrow -\infty}$$

$$\cancel{x} \left[-\sqrt{1 - \frac{4}{x^2}} - 1 \right]$$

$$\lim_{x \rightarrow -\infty}$$

$$-\sqrt{1 - \frac{4}{x^2}} - 1 = \underline{-2} = m$$

$$q = \lim_{x \rightarrow -\infty}$$

$$f(x) - m \cdot x$$

$$\lim_{x \rightarrow -\infty}$$

$$\sqrt{x^2 - 4} - x - (-2) \cdot x =$$

$$\lim_{x \rightarrow -\infty}$$

$$\sqrt{x^2 - 4} - x + 2x$$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2 - 4} + x$$

line

$x \rightarrow -\infty$

$$\sqrt{x^2 - 4} + x = +\infty - \infty$$

Diagram showing the behavior of the function as $x \rightarrow -\infty$. The term $\sqrt{x^2 - 4}$ is circled in blue and has a blue arrow pointing down to $+\infty$. The term $+x$ is circled in red and has a red arrow pointing down to $-\infty$.

$$(A+B)(A-B) = A^2 - B^2$$

$$(\sqrt{x^2 - 4} + x) \cdot (\sqrt{x^2 - 4} - x) = \frac{x^2 - 4 - x^2}{\sqrt{x^2 - 4} - x}$$

Diagram showing the simplification of the expression. The terms $\sqrt{x^2 - 4} + x$ and $\sqrt{x^2 - 4} - x$ are circled in pink. The denominator $\sqrt{x^2 - 4} - x$ is circled in light blue.

$$= \frac{-4}{\sqrt{x^2 - 4} + x} \rightarrow 0 = 9$$

Diagram showing the limit as $x \rightarrow -\infty$. The term $\sqrt{x^2 - 4}$ is circled in blue and has a blue arrow pointing down to $+\infty$. The term $-x$ is circled in red and has a red arrow pointing down to $-\infty$. The result is $-(-\infty) = +\infty$.

$$y = mx + q$$

es un hiperbole oblicua e'

$$y = -2x + 0$$

$$y = -2x \text{ e'}$$

asimptob oblicuos

a ∞

Teorema del confronto (o dei due condizionieri).

① Assumiamo di avere

$$\underbrace{h(x)}_{\text{fondo}} \leq f(x) \leq \underbrace{g(x)}_{\text{soffitto}} \quad \forall x$$

Se $\lim_{x \rightarrow x_0^+} h(x) = \lim_{x \rightarrow x_0^+} g(x) = L \stackrel{\text{G.R.}}{\implies}$

$$x \rightarrow x_0^+$$

$$\begin{matrix} x_0^- \\ +\alpha \\ -\alpha \end{matrix}$$

$$x \rightarrow x_0^+$$

$$\begin{matrix} x_0^- \\ +\infty \\ -\infty \end{matrix}$$

Se $\lim_{x \rightarrow x_0^+} f(x) = L$

$$x \rightarrow x_0^+$$

$$\begin{matrix} x_0^- \\ +\alpha \\ -\alpha \end{matrix}$$

②

$$h(x) \leq f(x) \quad \forall x$$

e che linee $f(x) = +\infty$ \Rightarrow linee $f(x) = +\infty$

$$\begin{array}{l} x \rightarrow x_0^+ \\ x_0^- \\ \nearrow \\ -\infty \end{array}$$

③

$$f(x) \leq g(x) \quad \forall x$$

linee $g(x) = -\infty$ \Rightarrow linee $f(x) = -\infty$

$$x \rightarrow \dots$$

1^a applicazione

se il reale $f(x) = 0$ e $h(x)$ è LIMITATA

$$x \rightarrow x_0$$
$$\begin{matrix} +\infty \\ -\infty \end{matrix}$$

$$\exists c > 0$$

$$-c \leq h(x) \leq +c \quad \forall x \in D$$

il reale $f(x) \cdot g(x) = 0$

$$x \rightarrow x_0$$
$$\begin{matrix} +\infty \\ -\infty \end{matrix}$$

prodotto tra una funzione che ha limite 0
e una funzione che NON HA LIMITE ma è
LIMITATA \Rightarrow ha limite 0.

es. $\lim_{x \rightarrow +\infty}$

$$\frac{\cos x}{x} = \lim_{x \rightarrow +\infty}$$

$\cos x$

$\frac{1}{x} \rightarrow 0$

NON HA LIMITE
perché è periodico
 $-1 \leq \cos x \leq 1 \quad \forall x$
LIMITATA

$$\left| -\frac{1}{x} \right| \leq |\cos x| \cdot \frac{1}{x} \leq \left| \frac{1}{x} \right|$$

Límite notável

lime
 $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

f-inedet

$$\frac{0}{0}$$

$$\frac{\sin 0}{0} = \frac{0}{0}$$

OSS 1

$$\frac{\sin(-x)}{-x} \stackrel{\downarrow}{=} \frac{-\sin x}{-x} = \frac{\sin x}{x}$$

$$\begin{array}{c} \rightarrow \\ + \\ \hline 0 \\ \leftarrow \end{array}$$

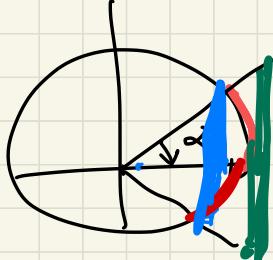
$f(x) = \frac{\sin x}{x} \in f. \text{pari}$

$f(-x) = f(x)$

lime
 $\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = \lim_{x \rightarrow 0^-} \frac{\sin x}{x}$

$$\begin{array}{c} \rightarrow \\ + \\ \hline 0 \\ \leftarrow \end{array}$$

$$0 < \alpha < \frac{\pi}{2}$$



$$\cos \alpha \leq \frac{\text{adjacent}}{\alpha} \leq 1$$

$$\forall \alpha \in (0, \frac{\pi}{2})$$

$$\text{per } x \rightarrow 0^+ \Rightarrow x > 0 \quad \text{e } x \rightarrow 0$$

$$\forall x > 0 \quad x \rightarrow 0$$

$$\cos x \leq \frac{\sin x}{x} \leq 1$$

↓

$\cos 0 = 1$

per lecione del
Capitolo

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$$

$$\Rightarrow \boxed{\begin{array}{l} \text{line} \\ x \rightarrow 0 \end{array} \frac{\sin x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\sin x}} \quad \begin{array}{l} \text{LIMITE} \\ \text{NOTEVOLE} \end{array}$$

Limite collegati:

$$\frac{\sin x}{\cos x} \cdot \frac{1}{x} = \frac{\sin x}{\cos x \cdot x}$$

$$= \frac{\sin x}{x \cos x}$$

$$\text{line} \quad \frac{\tan x}{x} = \frac{\tan 0}{0} = \frac{0}{0} \quad \text{f. i.}$$

$$= \text{line} \quad \frac{\tan x}{x} \quad x \rightarrow 0$$

$$\tan x \cdot \frac{1}{x} = \text{line} \quad x \rightarrow 0$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{x} =$$

$$= \text{line} \quad \frac{\sin x}{x} \quad x \rightarrow 0$$

$$\frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1 = \text{line} \quad \frac{x}{\tan x} \quad x \rightarrow 0$$

$$\lim_{\substack{x \rightarrow 0}} \frac{\arcsin x}{x} = \lim_{\substack{x \rightarrow 0}} \frac{\arcsin x}{\sin(\arcsin x)} =$$

$$x \rightarrow 0 \quad x \approx 0$$

$$x = \sin(\arcsin x) \quad \forall x \in [-1, 1]$$

$$x \rightarrow \arcsin x \rightarrow \frac{\arcsin x}{\sin(\arcsin x)}$$

$$x \rightarrow 0 \quad \arcsin x \rightarrow \arcsin 0 = 0$$

$$\lim_{\substack{x \rightarrow 0}} \frac{\arcsin x}{\sin(\arcsin x)} = \lim_{\substack{y \rightarrow 0}} \frac{y}{\sin y} = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1 = \lim_{x \rightarrow 0}$$

$$\frac{\sin x}{x}$$

$$\lim_{x \rightarrow 0} \frac{x}{\operatorname{arctg} x} = 1 = \lim_{x \rightarrow 0}$$

$$\frac{\operatorname{arctg} x}{x}$$

$$x = \operatorname{tg}(\operatorname{arctg} x)$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 = \lim_{x \rightarrow 0}$$

$$\frac{x}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1 = \lim_{x \rightarrow 0}$$

$$\frac{x}{\operatorname{tg} x}$$

line

$x \rightarrow 0$

$$\frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\frac{1 - \cos 0}{0^2} = \frac{1 - 1}{0} = \frac{0}{0}$$

$$\frac{(1 - \cos x)}{x^2} \cdot \frac{(1 + \cos x)}{1 + \cos x} = \frac{1 - (\cos x)^2}{x^2} \cdot \frac{1}{(1 + \cos x)}$$

$$(\cos x)^2 + (\sin x)^2 = 1$$

$$(\sin x)^2 = 1 - (\cos x)^2$$

$$= \frac{(\sin x)^2}{x^2} \cdot \frac{1}{1 + \cos x}$$

$$= \left(\frac{\sin x}{x} \right)^2 \cdot \frac{1}{1 + \cos x}$$

$$\frac{1}{1 + \cos 0} = \frac{1}{2}$$

$$12$$

line $x \rightarrow 0$ $\frac{1-\cos x}{x^2} = \frac{1}{2}$

line $x \rightarrow 0$ $\frac{\cos x - 1}{x^2} = -\frac{1}{2}$

line $x \rightarrow 0$ $\frac{x^2}{1-\cos x} = ?$

line $x \rightarrow 0$ $\frac{1-\cos x}{x}$ $\frac{x}{x} = \text{line } x \rightarrow 0$

line $x \rightarrow 0$ $\frac{1-\cos x}{x} = 0$

$$\frac{1-\cos x}{x^2} \cdot \frac{x}{x} = \frac{1}{2} \cdot 0 = 0$$