Dynamic market potential, $m(t)$

A generalization of the Bass Model considers a dynamic market potential, $m(t)$

$$z'(t) = m(t)\left\{ \left(p + q\frac{z(t)}{m}\right)\left(1 - \frac{z(t)}{m(t)}\right) \right\} + z(t)\frac{m'(t)}{m(t)}$$

$$\frac{z'(t)m(t) - z(t)m'(t)}{m^2(t)} = \left(\frac{z(t)}{m(t)}\right)' = \left(p + q\frac{z(t)}{m(t)}\right)\left(1 - \frac{z(t)}{m(t)}\right)$$

and, by setting $y(t) = z(t)/m(t)$, we have

$$y'(t) = p + qy(t)(1 - y(t))$$

which is a standard Bass Model.

Dynamic market potential, $m(t)$

1. Market of new products is unstable and uncertain in the first phase of diffusion: incubation
2. Advertising and promotional efforts play a central role to overcome this phase
3. These efforts influence the structure of the market potential, which depends on information on the product
4. Communication and adoption are two separate phases, needing a distinct modelling

Dynamic market potential, $m(t)$

We may notice that di $m(t)$ is 'free'

$$z(t) = m(t)F(t) = m(t)\ \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}}$$

Dynamic market potential, $m(t)$: GGM

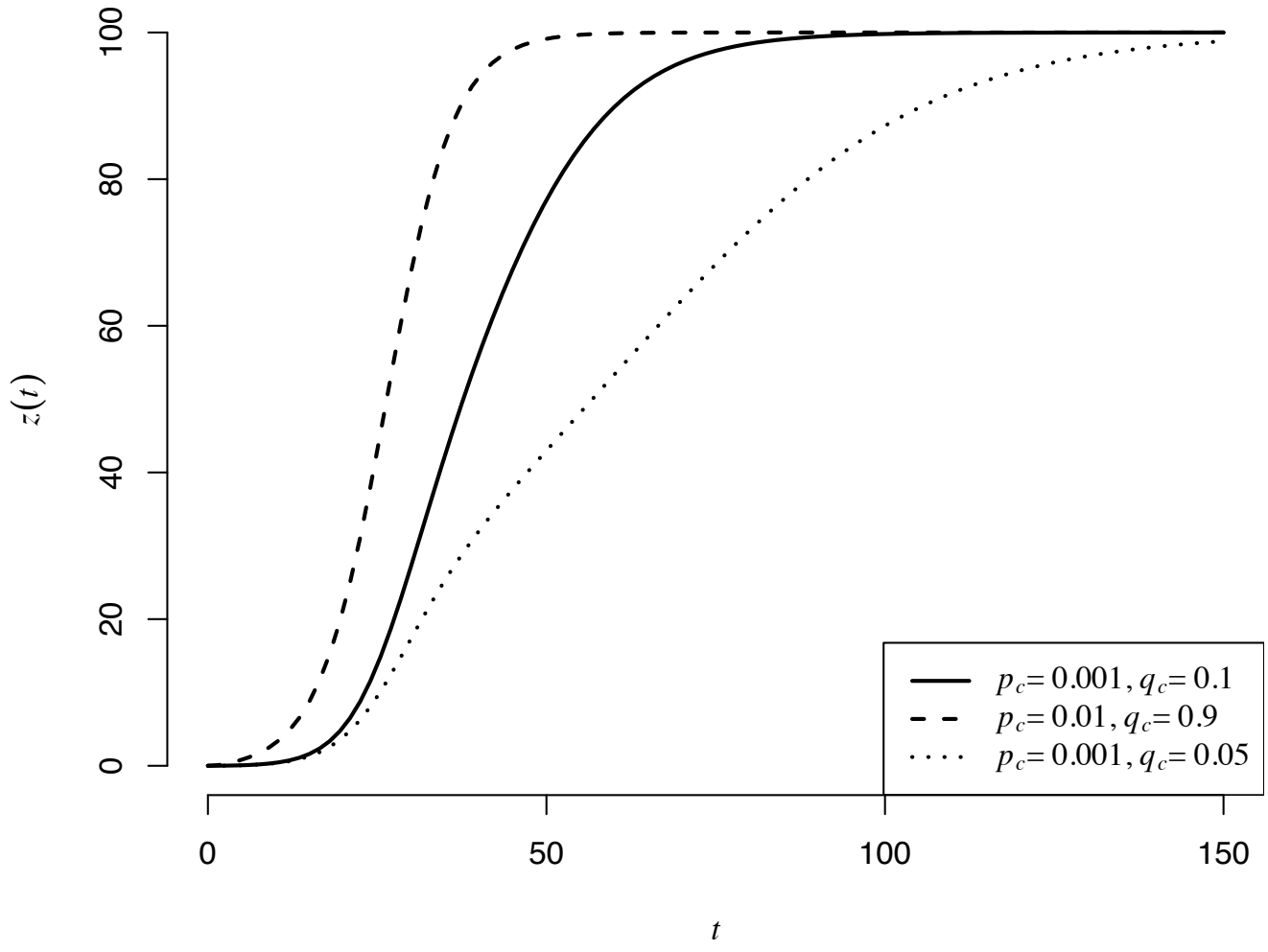
The GGM (Guseo and Guidolin, 2009) is a generalization of the Bass Model, where $m(t)$ is time-dependent

$$z(t) = m(t)F(t) = m(t)\,\frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}}$$

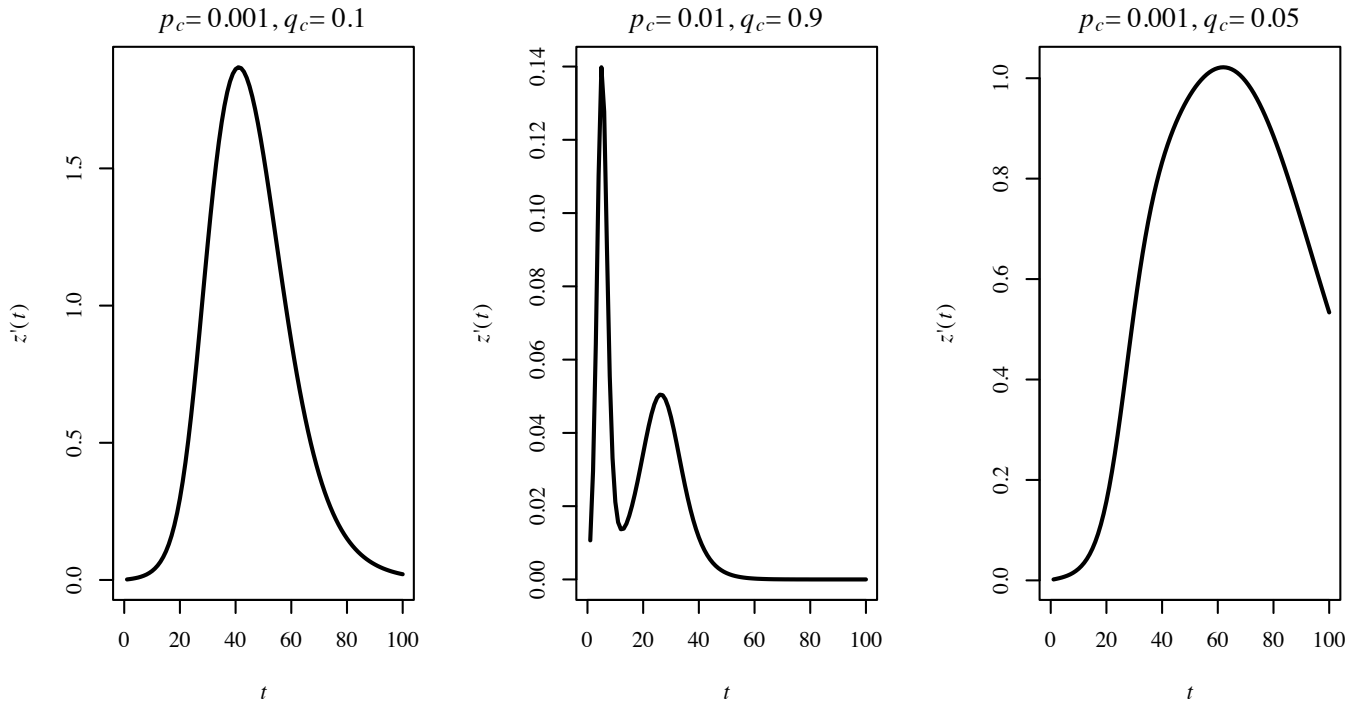
and function of a communication process

$$z(t) = KG(t)F(t) = K\,\sqrt{\frac{1 - e^{-(p_c+q_c)t}}{1 + \frac{q_c}{p_c}e^{-(p_c+q_c)t}}\,\frac{1 - e^{-(p_s+q_s)t}}{1 + \frac{q_s}{p_s}e^{-(p_s+q_s)t}}}$$

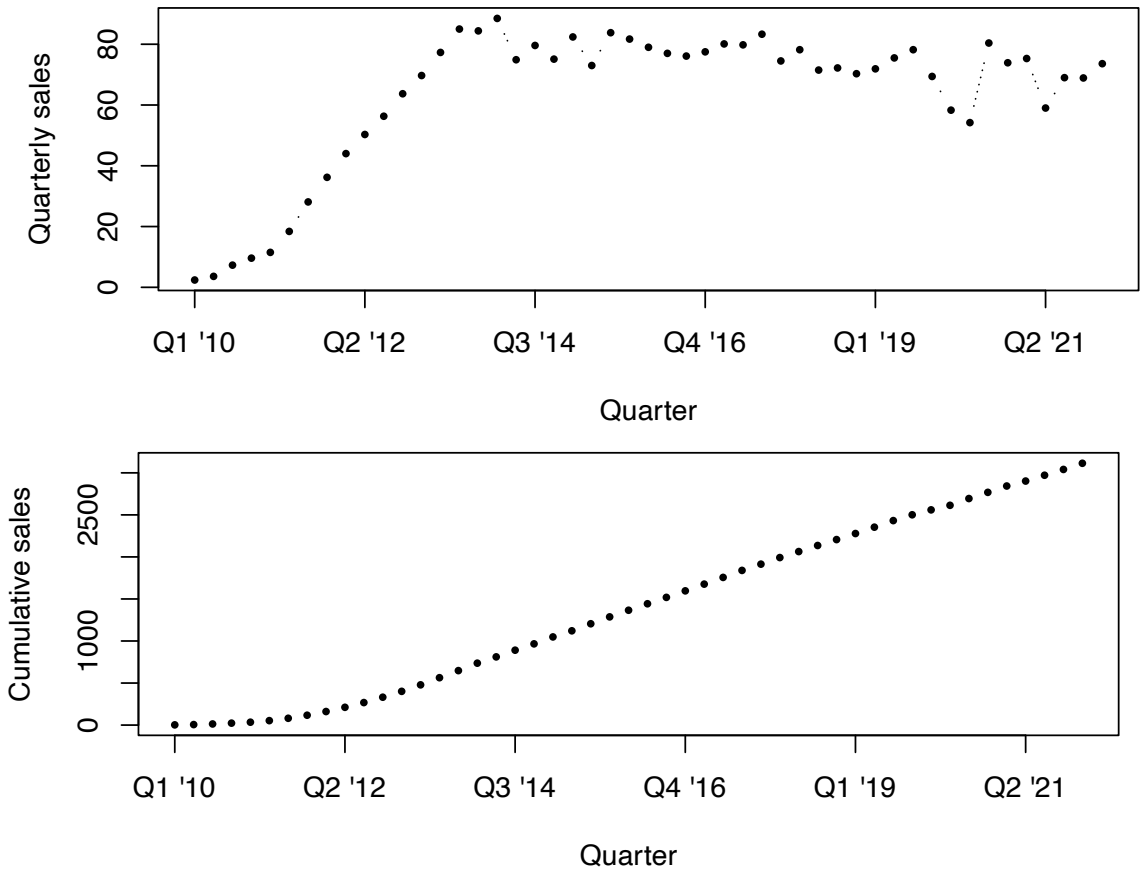
GGM


z(t)
0
20
40
60
80
100
0
50
100
150
t
$p_c = 0.001, q_c = 0.1$
$p_c = 0.01, q_c = 0.9$
$p_c = 0.001, q_c = 0.05$

GGM


$p_c = 0.001, q_c = 0.1$
$z'(t)$
0.0
0.5
1.0
1.5
0
20
40
60
80
100
t
$p_c = 0.01, q_c = 0.9$
$z'(t)$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0
20
40
60
80
100
t
$p_c = 0.001, q_c = 0.05$
$z'(t)$
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
t

Samsung smartphones



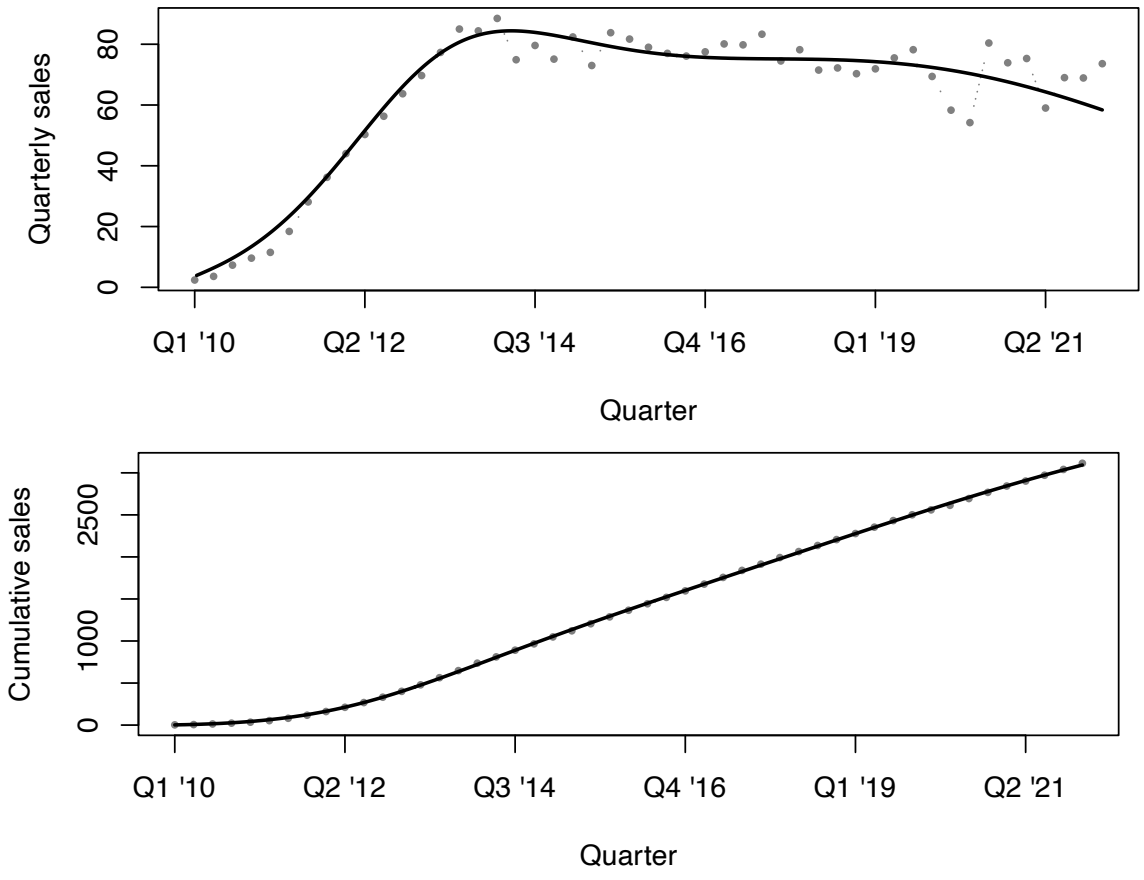
Esempio: Samsung smartphones

GGM for Samsung: estimates and 95% CIs

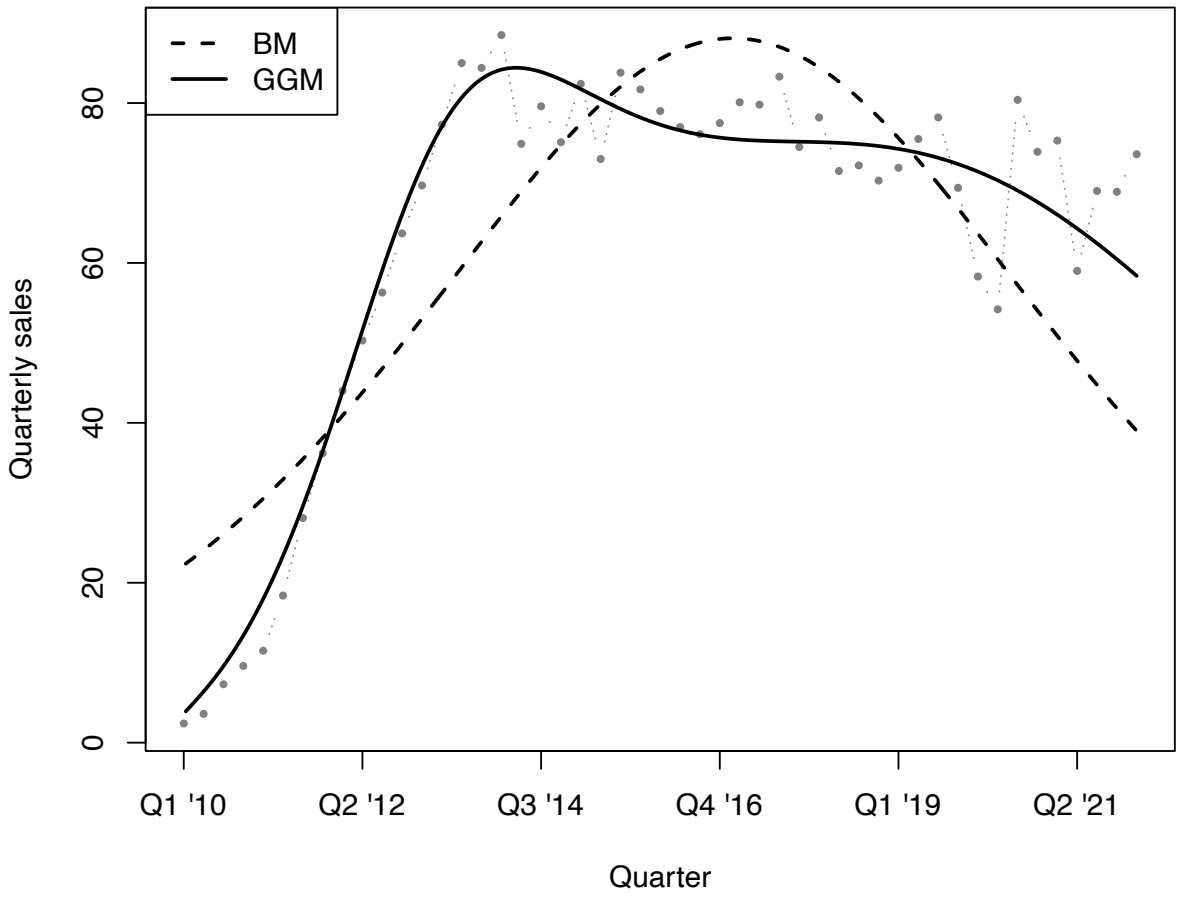
	Estimate	Std.Error	Lower	Upper	P-value
K	4030.7	75.47	3882.8	4178.6	< 0.0001
p_c	0.0015	0.00001	0.0014	0.0016	< 0.0001
q_c	0.08	0.0026	0.08	0.09	< 0.0001
p_s	0.012	0.0006	0.011	0.014	< 0.0001
q_s	0.21	0.008	0.20	0.23	< 0.0001

$R^2 = 0.9999$

Samsung smartphones

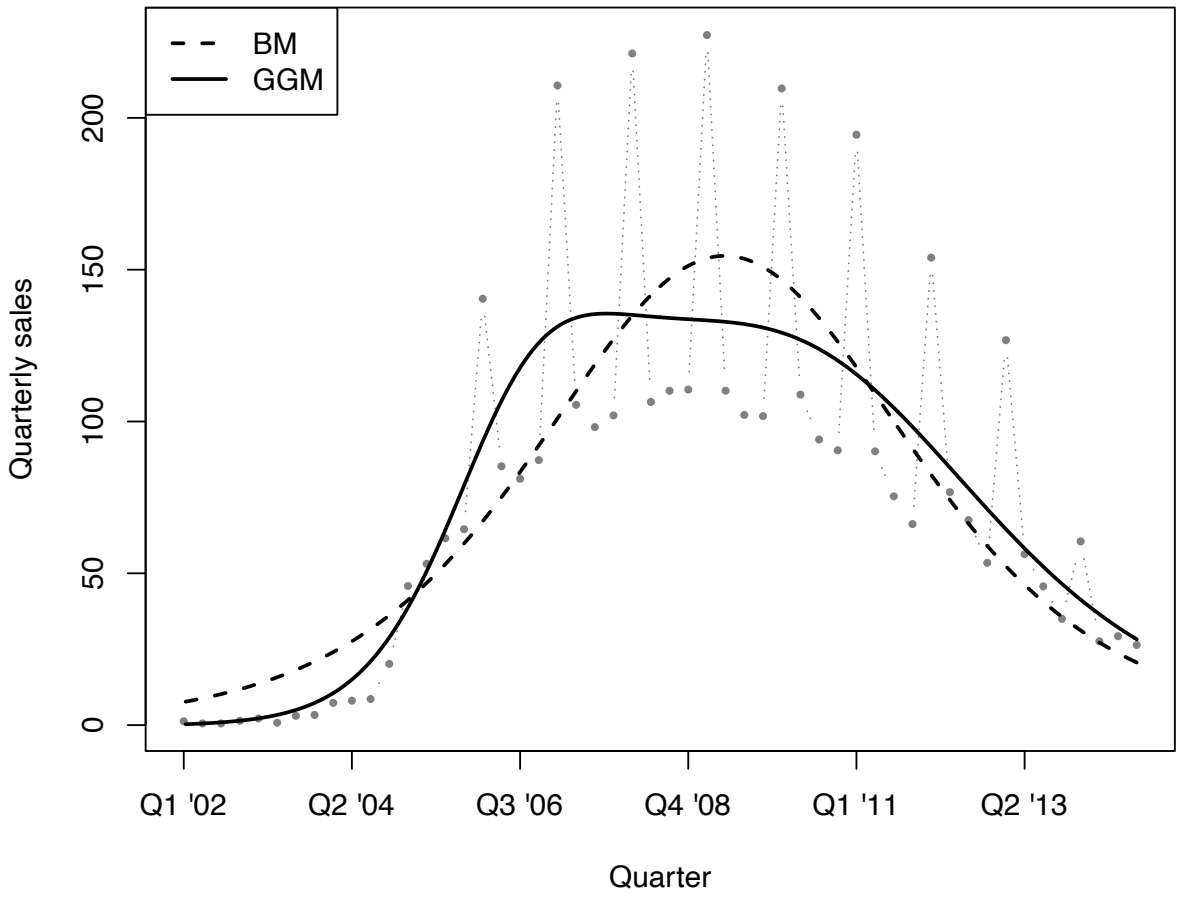


Samsung smartphones



Model comparison . . .

Apple iPod



Model comparison . . .

Competition between two products

Unbalanced competition and regime change diachronic model

$$
\begin{aligned}
z_1'(t) &= m\Bigg\{ \left[p_{1a} + q_{1a}\frac{z(t)}{m}\right](1 - I_{t>c_2}) \\
&\quad + \left[p_{1c} + (q_{1c} + \delta)\frac{z_1(t)}{m} + q_{1c}\frac{z_2(t)}{m}\right] I_{t>c_2} \Bigg\} \left[1 - \frac{z(t)}{m}\right], \\
z_2'(t) &= m\left[p_2 + (q_2 - \gamma)\frac{z_1(t)}{m} + q_2\frac{z_2(t)}{m}\right]\left[1 - \frac{z(t)}{m}\right] I_{t>c_2},
\end{aligned}
$$

Competition between two products

Unbalanced competition and regime change diachronic model

$$
\begin{aligned}
z_1'(t) &= m\Bigg\{ \left[p_{1a} + q_{1a}\frac{z(t)}{m}\right](1 - I_{t>c_2}) \\
&\quad + \left[p_{1c} + (q_{1c} + \delta)\frac{z_1(t)}{m} + q_{1c}\frac{z_2(t)}{m}\right] I_{t>c_2} \Bigg\} \left[1 - \frac{z(t)}{m}\right], \\
z_2'(t) &= m\left[p_2 + (q_2 - \gamma)\frac{z_1(t)}{m} + q_2\frac{z_2(t)}{m}\right]\left[1 - \frac{z(t)}{m}\right] I_{t>c_2},
\end{aligned}
$$

within imitation

Competition between two products

Unbalanced competition and regime change diachronic model

$$
\begin{aligned}
z_1'(t) &= m\Bigg\{ \left[p_{1a} + q_{1a}\frac{z(t)}{m}\right](1 - I_{t>c_2}) \\
&\quad + \left[p_{1c} + (q_{1c} + \delta)\frac{z_1(t)}{m} + q_{1c}\frac{z_2(t)}{m}\right] I_{t>c_2} \Bigg\} \left[1 - \frac{z(t)}{m}\right], \\
z_2'(t) &= m\left[p_2 + (q_2 - \gamma)\frac{z_1(t)}{m} + q_2\frac{z_2(t)}{m}\right]\left[1 - \frac{z(t)}{m}\right] I_{t>c_2},
\end{aligned}
$$

within imitation

cross imitation

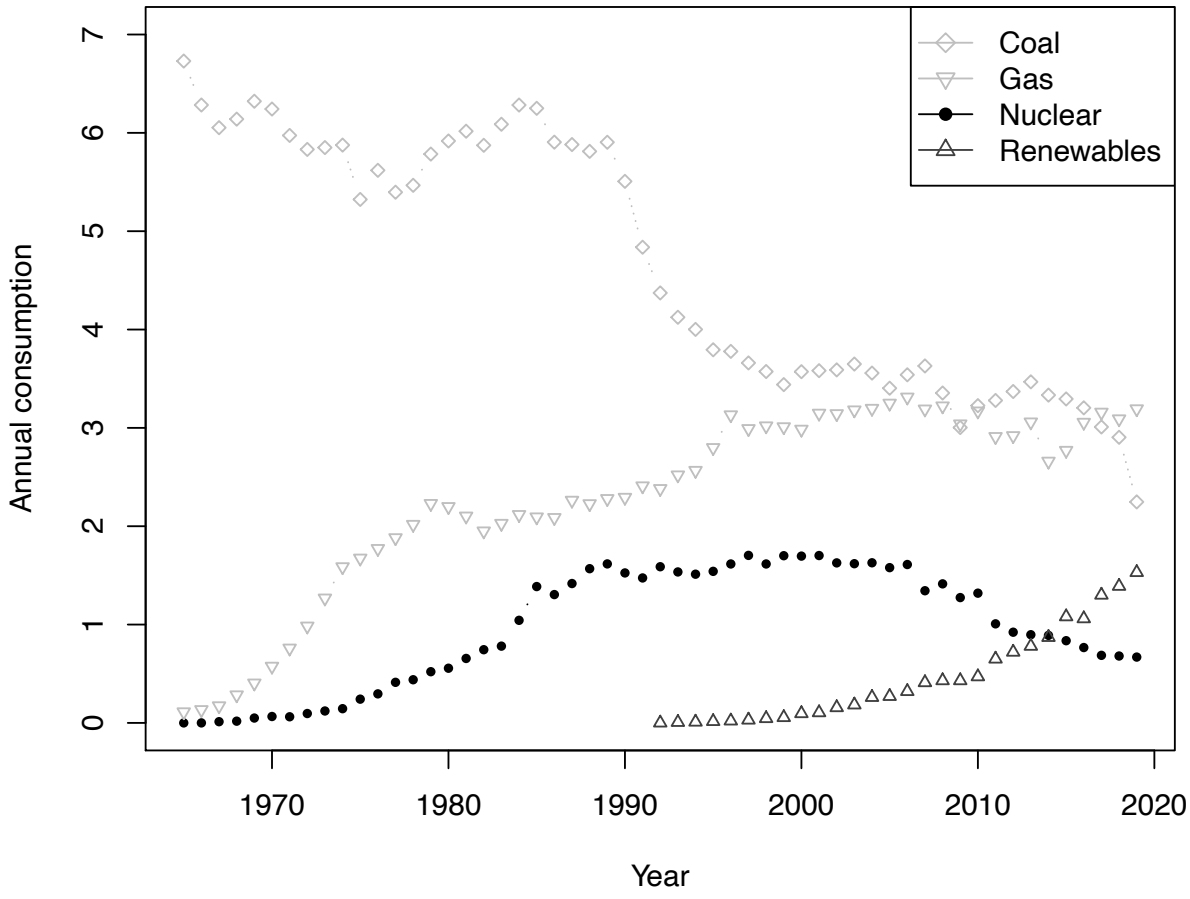
Model

Sign of cross-imitation coefficients: competition-collaboration

q_{1c}	$q_2 - \gamma$	interpretation
negative	negative	full competition
negative	positive	2 competes with 1, 1 collaborates with 2
positive	negative	2 collaborates with 1, 1 competes with 2
positive	positive	full collaboration

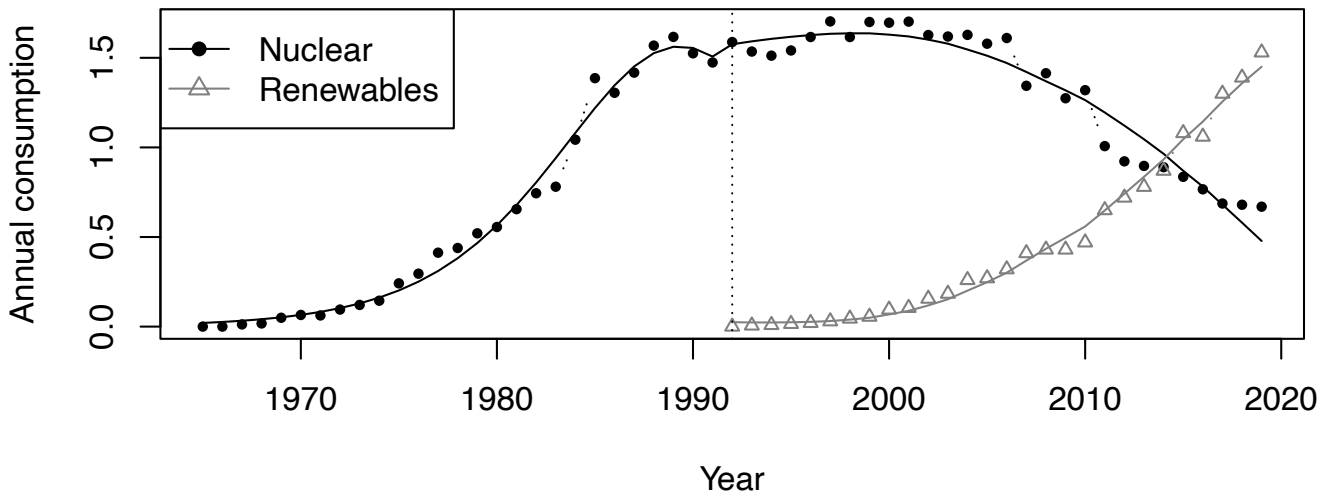
Germany energy transition

Energy consumption in Germany



Germany energy transition

Competition between renewables and nuclear in Germany



Germany energy transition

UCRCD for renewables vs nuclear: estimates and 95% CIs

	Estimate	Std.Error	Lower	Upper
m_a	26.6	0.73	25.1	28.0
p_{1a}	0.0007	0.0000	0.0006	0.0007
q_{1a}	0.23	0.004	0.22	0.24
m_c	99.9	9.87	80.5	119.2
p_{1c}	0.012	0.0012	0.010	0.014
p_2	0.001	0.0015	-0.002	0.003
q_{1c}	-0.145	0.015	-0.176	-0.114
q_2	0.342	0.0683	0.208	0.475
δ	0.183	0.0186	0.146	0.219
γ	0.343	0.0730	0.200	0.487

$R^2 = 0.9915$

- nuclear $q_{1c} = -0.145$,
- renewables $q_2 - \gamma = -0.002$