Limiti di composizioni di funzioni

$$\lim_{x \to x_0} f(\underbrace{g(x)}) = \lim_{x \to x_0} f \circ g(x)$$

$$x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$$

$$e^{-\frac{1}{x^2}}$$

$$x \longrightarrow -\frac{1}{x^2} \longrightarrow e^{-\frac{1}{x^2}}$$

$$\lim_{x \to x_0} f(g(x))$$

abbiamo di saper calcolare

$$\lim_{x \to x_0} g(x) = L$$

$$+\infty$$

$$-\infty$$

$$\lim_{x \to x_0} f(g(x)) = \lim_{y \to L} f(y)$$

$$y \to +\infty$$

$$y \to -\infty$$

Es

$$\lim_{x\to 0} e^{-\frac{1}{x^2}} = \lim_{y\to -\infty} e^{y} = 0$$

$$\lim_{x\to 0} -\frac{1}{x^2} = -(+\infty) = -\infty$$

$x \to 0 \Rightarrow x^2 \to 0$ ma $x^2 > 0$ & $x \neq 0$

$\Rightarrow \frac{1}{x^2} \to +\infty$ $\left(\frac{1}{0} \quad x^2 > 0\right)$

$$\lim_{x\to+\infty} e^{-\frac{1}{x^2}} = \lim_{y\to 0} e^{y} = e^{0} = 1$$

$$x \to \begin{matrix} +\infty \\ -\infty \end{matrix} \qquad -\frac{1}{x^2} \to 0 \qquad \cancel{o}\ 1$$

$$\lim_{x\to-\infty} e^{-\frac{1}{x^2}} = \lim_{y\to 0} e^{y} = e^{0} = 1$$

$$e^{-\frac{1}{x^2}} = f(x)$$

$$D = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

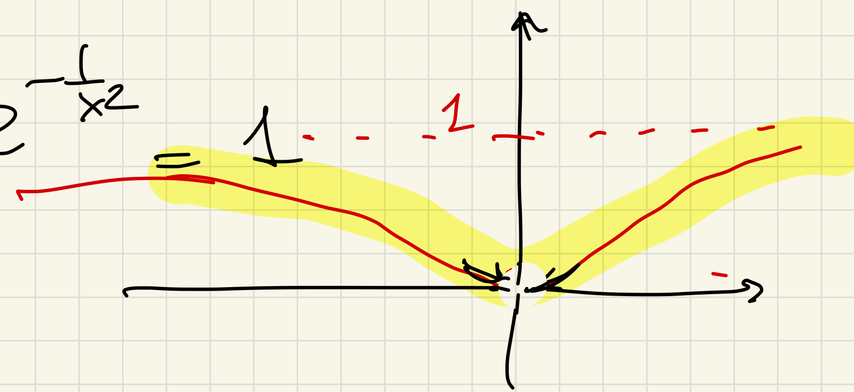
$$f(x) > 0 \quad \forall x \in D$$

$$f(-x) = f(x) \qquad f(-x) = e^{-\frac{1}{(-x)^2}} = e^{-\frac{1}{x^2}} = f(x)$$

pari

$$\boxed{\lim_{x \to 0} e^{-\frac{1}{x^2}} = 0}$$

$$\lim_{\substack{x \to +\infty \\ -\infty}} e^{-\frac{1}{x^2}} = 1$$



ES

$$\lim_{x\to+\infty} \underbrace{\boxed{\sqrt{x^2+1}} - x}_{+\infty \; - \; \infty} = +\infty - \infty$$

F. INDETERMINATA

$x \to +\infty \qquad x^2+1 \to +\infty \qquad \lim_{x\to+\infty} \sqrt{x^2+1} = \lim_{y\to+\infty} \sqrt{y} = +\infty$

$$\underbrace{(A-B)}(A+B) = A^2 - B^2$$

$A = \sqrt{x^2+1} \qquad B = x$

$A^2 = x^2+1 \qquad B^2 = x^2$

$$\frac{\left(\sqrt{x^2+1} - x\right)}{1} \cdot \frac{\left(\sqrt{x^2+1} + x\right)}{\left(\sqrt{x^2+1} + x\right)} = \frac{\cancel{x^2}+1 - \cancel{x^2}}{\sqrt{x^2+1} + x} = \frac{1}{\sqrt{x^2+1} + x}$$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+1}-x\right)=\lim_{x\to+\infty}\frac{1}{\sqrt{x^2+1}+x}=0$$

$$+\infty+\infty=+\infty$$

ES $\lim_{x \to +\infty} \sqrt{x+2} - x$

$+\infty - \infty$ f. ind.

DOMINIO di

$f(x) = \sqrt{x+2} - x$

è $x+2 \geq 0$

$[-2, +\infty)$

dominio superiormente ILLIMITATO

$A^2 = x+2 \qquad B = x^2$

$A = \sqrt{x+2} \qquad B = x$

$$\left(\sqrt{x+2} - x\right) \cdot \frac{\left(\sqrt{x+2} + x\right)}{\sqrt{x+2} + x} =$$

$$= \frac{x+2 - x^2}{\sqrt{x+2} + x}$$

$$\lim_{x\to+\infty} \sqrt{x+2}-x = \lim_{x\to+\infty} \frac{x+2-x^2}{\sqrt{x+2}+x} =$$

RACCOLGO termine di GRADO MASSIMO

$$= \lim_{x\to+\infty} \frac{\overset{+\infty}{x^2}\left[\frac{1}{x}+\frac{2}{x^2}-1\right]}{\sqrt{x+2}+x} \quad \begin{matrix} \to -\infty \\ \to +\infty \end{matrix}$$

$$f.i. \ \frac{-\infty}{+\infty}$$

$$\underbrace{x^2}_{+\infty}\cdot\left[\underbrace{\frac{1}{x}+\frac{2}{x^2}}-1\right] \longrightarrow +\infty\cdot[0+0-1] =$$

$$\frac{1}{\infty}=0 \qquad \frac{2}{\infty}=0$$

$$= +\infty\cdot(-1) = -\infty$$

a denominatore raccolgo x di grado massimo

$$\sqrt{\underbrace{x+2}} + x = \sqrt{x \cdot \left(1+\frac{2}{x}\right)} + x =$$

$$\sqrt{x} = x^{1/2}$$

$$= \sqrt{x} \cdot \sqrt{1+\frac{2}{x}} + x$$

$$= \underset{+\infty}{x} \left[\frac{1}{\sqrt{x}} \cdot \sqrt{1+\frac{2}{x}} + 1 \right] = +\infty \cdot [0 \cdot 1 + 1]$$

$$\frac{1}{\infty} = 0$$

$$\sqrt{1+0} = 1$$

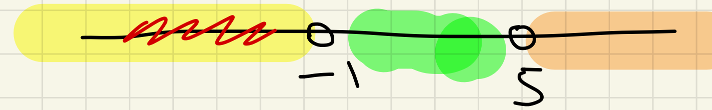
$$\lim_{x\to+\infty} \frac{\cancel{x^2}\left[\frac{1}{x}+\frac{2}{x^2}-1\right]}{\cancel{x}\left[\frac{1}{\sqrt{x}}\sqrt{1+\frac{2}{x}}+1\right]} =$$

$$= \lim_{x\to+\infty} \frac{\overset{\to+\infty}{x}\cdot \underbrace{\left[\frac{1}{x}+\frac{2}{x^2}-1\right]}_{\to [0+0-1]=-1}}{\underbrace{\left[\frac{1}{\sqrt{x}}\sqrt{1+\frac{2}{x}}+1\right]}_{0+1=1}}$$

$$= \frac{+\infty\cdot(-1)}{1}$$

$$= -\infty.$$

es $f(x) = \dfrac{1}{|x-2|-3}$



det dominio, poss. simmetrie, segno, limiti agli estremi del dominio (nei pti di accumulazione e a $\pm\infty$ se possibile)

Dominio $|x-2|-3 \neq 0$ $\quad |x-2| \neq 3$

$$x-2 \neq 3 \rightarrow x \neq 5$$
$$x-2 \neq -3 \rightarrow x \neq -1$$

$$D = \{x \neq 5,\ x \neq -1\} = (-\infty, -1) \cup (-1, 5) \cup (5, +\infty)$$

non è simmetrica perché $1 \in D$

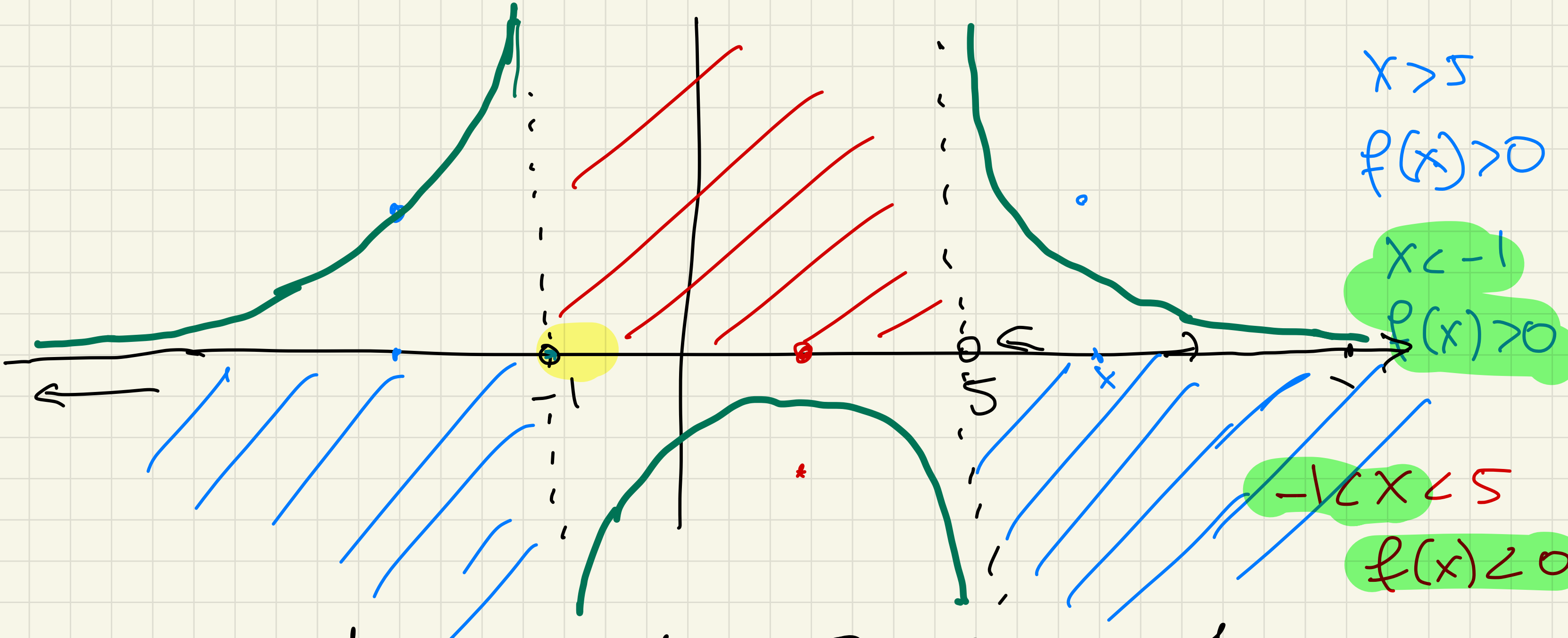
$-1 \notin D$

segno $f(x) > 0$ $\qquad \dfrac{1}{|x-2|-3} \geq 0$

$|x-2|-3 > 0 \qquad |x-2| > 3 \quad \begin{cases} x-2 > 3 & \Rightarrow x > 5 \\ x-2 < -3 & \Rightarrow x < -1 \end{cases}$

$|A| > B$ con $B > 0$

$A > B$ oppure $A < -B$

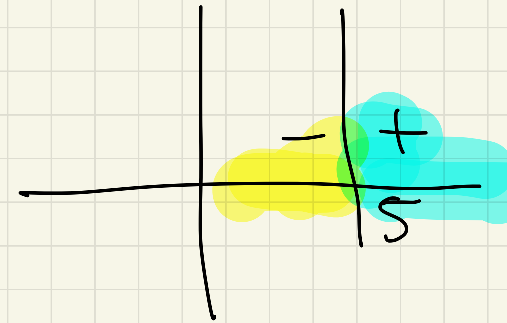


$$\lim_{x\to+\infty}\frac{1}{|x-2|-3}=\frac{1}{+\infty}=0=\lim_{x\to-\infty}\frac{1}{|x-2|-3}$$

$x\to+\infty \quad |x-2|\to+\infty \quad |x-2|-3\to+\infty$

$x\to-\infty \quad |x-2|\to+\infty \quad |x-2|-3\to+\infty$

$$\lim_{x\to 5} \frac{1}{|x-2|-3}$$ NON ESISTE



$$|x-2|-3 \to |5-2|-3=0$$

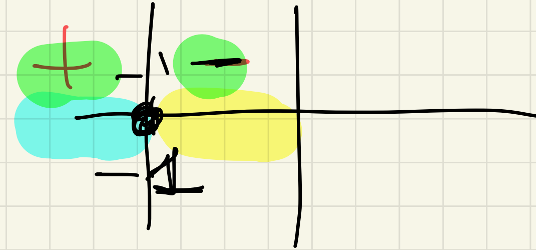
Se mi avvicino a 5 da destra $|x-2|-3>0$

se mi avvicino a 5 da sinistra $|x-2|-3<0$

$$\lim_{x\to 5^+} \frac{1}{|x-2|-3} = +\infty$$

$$\lim_{x\to 5^-} \frac{1}{|x-2|-3} = -\infty$$

$$\lim_{x \to -1} \frac{1}{|x-2|-3}$$ NON ESISTE



$$x \to -1 \qquad |x-2|-3 \to |-1-2|-3 = |-3|-3 = 0$$

$$\lim_{x \to (-1)^-} \frac{1}{|x-2|-3} = +\infty$$

$$\lim_{x \to (-1)^+} \frac{1}{|x-2|-3} = -\infty$$

Es $f(x) = \log\left(e^{2x} - 3e^{x} + 2\right)$

dominio, segno, simmetrie, limiti

D: $e^{2x} - 3e^{x} + 2 > 0$

$$e^{2x} = (e^{x})^2 = e^{x} \cdot e^{x}$$

$$(e^{x})^2 - 3e^{x} + 2 > 0$$

$y = e^{x}$

$$\boxed{y^2 - 3y + 2 \geq 0}$$

$y_{12} = \begin{cases} 1 \\ 2 \end{cases}$

$y > 2 \quad \text{o} \quad y < 1$

$$y > 2 \Rightarrow e^x > 2 = e^{\lg 2} \Rightarrow x > \lg 2$$

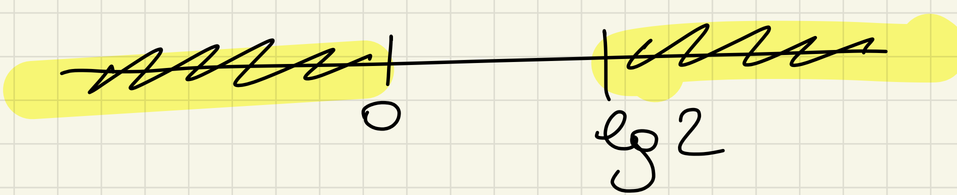
$$y < 1 \Rightarrow e^x < 1 = e^{\lg 1} = e^0 \qquad x < 0 \Rightarrow$$

$$\boxed{a = e^{\lg a} \quad \forall a > 0}$$

$$D = \{ x > \lg 2, \quad x < 0 \}$$

$$(\lg 1 = 0 < \lg 2 < 1 = \lg e$$

$$D = (-\infty, 0) \cup (\lg 2, +\infty)$$



f non è simmetrica

segno

$f(x) \geq 0$

$\lg(e^{2x} - 3e^x + 2) \geq 0 = \lg 1$

$\Downarrow$

$e^{2x} - 3e^x + 2 \geq 1$

$e^{2x} - 3e^x + 1 \geq 0$

$b = \lg e^b$

$\forall b \in \mathbb{R}$

$y = e^x$

$y^2 - 3y + 1 \geq 0$

$y_{12} = \dfrac{3 \pm \sqrt{9-4}}{2}$ — $\dfrac{3+\sqrt{5}}{2}$, $\dfrac{3-\sqrt{5}}{2}$

$y \geq \dfrac{3+\sqrt{5}}{2}$

$y \leq \dfrac{3-\sqrt{5}}{2}$

$$y \geq \frac{3+\sqrt{5}}{2} \Rightarrow e^{x} \geq \frac{3+\sqrt{5}}{2} = e^{\lg\left(\frac{3+\sqrt{5}}{2}\right)}$$

$$\Rightarrow x \geq \lg\left(\frac{3+\sqrt{5}}{2}\right) > \lg 2$$

$$\frac{3+\sqrt{5}}{2} \geq \frac{3+2}{2} = \frac{5}{2}$$

$$y \leq \underbrace{\frac{3-\sqrt{5}}{2}}_{\vee \atop 0}$$

$$\Downarrow$$

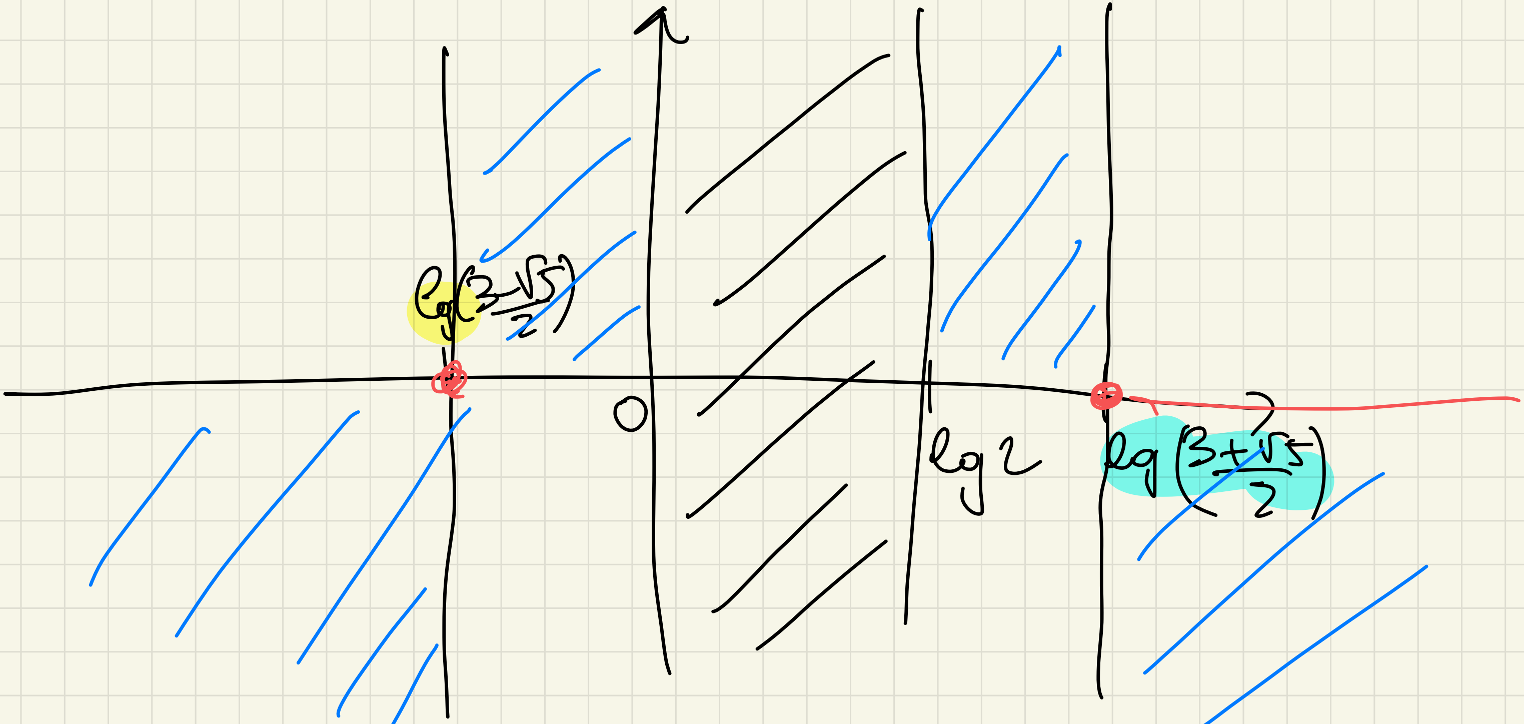
$$e^{x} \leq \frac{3-\sqrt{5}}{2} = e^{\lg\left(\frac{3-\sqrt{5}}{2}\right)}$$

$$x \leq \lg\left(\frac{3-\sqrt{5}}{2}\right) < 0$$

$$0 \leq \frac{3-\sqrt{5}}{2} \leq \frac{3-2}{2} = \frac{1}{2} < 1$$

$$2 \leq \sqrt{5} \leq 3$$

$$\sqrt{4} \leq \sqrt{5} \leq \sqrt{9}$$



$f(x) > 0$ $\quad x > \log\left(\frac{3+\sqrt{5}}{2}\right)$

$\quad\quad\quad\quad x < \log\left(\frac{3-\sqrt{5}}{2}\right)$

$f(x) = 0$

$x = \log\left(\frac{3+\sqrt{5}}{2}\right)$

$x = \log\left(\frac{3-\sqrt{5}}{2}\right)$