Generalized Bass Model

The Bass Model does not account for the effect of exogenous variables, such as marketing mix, public incentives, environmental shocks.
Besides, in some cases the diffusion process does not have a bell shape curve, but a more complex structure.

Generalized Bass Model

The Generalized Bass Model (Bass et al., 1994) adds an intervention function $x(t)$

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t))x(t).$$

where $x(t)$ is an integrable, non negative function.

- The Bass Model is a special case where $x(t) = 1$.
- if $0 < x(t) < 1$ the process slows down,
- if $x(t) > 1$ the process accelerates.

Generalized Bass Model: closed-form solution

The closed-form solution of the model is

$$z(t) = m \, \frac{1 - e^{-(p+q)\int_0^t x(\tau)d\tau}}{1 + \frac{q}{p} e^{-(p+q)\int_0^t x(\tau)d\tau}}, \qquad t > 0.$$

Interesting: function $x(t)$ does not modify the market potential m!
Function $x(t)$ modifies the speed of the process.

Modelling $x(t)$: exponential shock

Function $x(t)$ may take several forms in order to describe various types of shock.
A strong and fast shock may take an exponential form

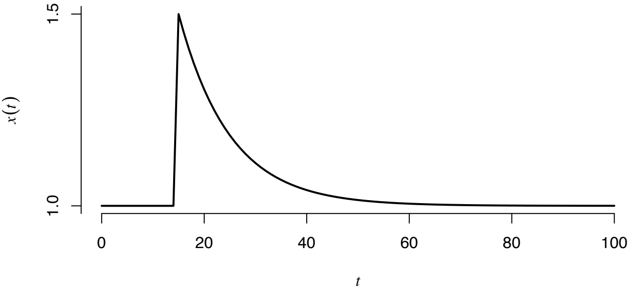
$$x(t) = 1 + c_1 e^{b_1(t-a_1)} I_{t \geq a_1},$$

where parameter c_1 is intensity and sign of the shock, b_1 is the 'memory' of the effect and is typically negative, and a_1 is the starting time of the shock.

Modelling $x(t)$: exponential shock

The use of exponential shock is suitable for identifying the positive effect of marketing strategies or incentive measures, in order to speed up the diffusion process.
Also, a negative shock may represent a fast slowdown in sales due to the entrance of a competitor.

Modelling $x(t)$: exponential shock



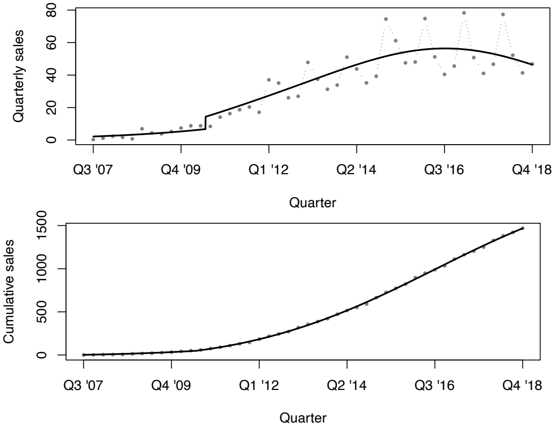
Apple iPhone

GBM for iPhone: estimates and 95% CIs

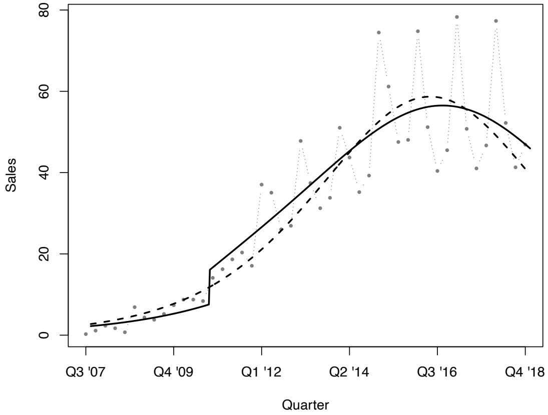
	Estimate	Std.Error	Lower	Upper	p-value
m	2108.9	124.9	1864.1	2353.8	< 0.001
p	0.0009	0.0001	0.0008	0.0011	< 0.001
q	0.10	0.001	0.08	0.12	< 0.001
a_1	12.5	0.99	10.56	14.44	< 0.001
b_1	-0.14	0.06	-0.25	-0.03	0.02
c_1	1.13	0.17	0.78	1.47	< 0.001

$R^2 = 0.9998$

Apple iPhone



Apple iPhone



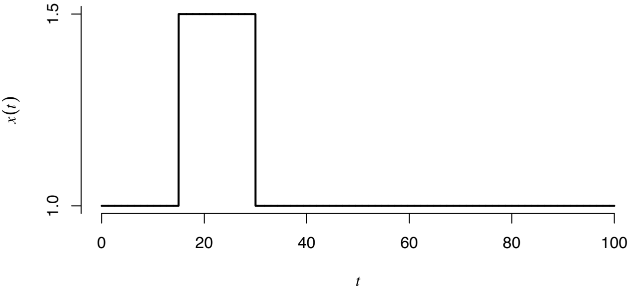
Modelling $x(t)$: rectangular shock

A more stable shock, acting on a longer period of time, may be modeled through a rectangular shock

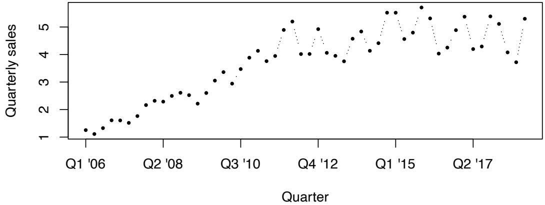
$$x(t) = 1 + c_1 I_{t \geq a_1} I_{t \leq b_1},$$

where parameter c_1 describes intensity of the shock, either positive or negative, parameters a_1 and b_1 define beginning and end of the shock (con $a_1 < b_1$).
The rectangular shock is useful to identify the effect of policies and measures within a limited time interval.

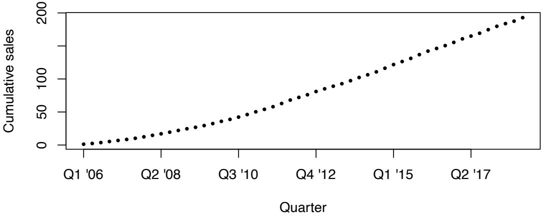
Modelling $x(t)$: rectangular shock



Apple iMac



Quarterly sales
1
2
3
4
5
Q1 '06
Q2 '08
Q3 '10
Q4 '12
Q1 '15
Q2 '17
Quarter



Cumulative sales
0
50
100
200
Q1 '06
Q2 '08
Q3 '10
Q4 '12
Q1 '15
Q2 '17
Quarter

Apple iMac

GBM for iMac: estimates and 95% CIs

	Estimate	Std.Error	Lower	Upper	p-value
m	281.66	3.58	274.65	288.68	< 0.0001
p	0.0047	0.0042	0.0047	0.0048	< 0.0001
q	0.061	0.001	0.059	0.063	< 0.0001

$R^2 = 0.9999088$

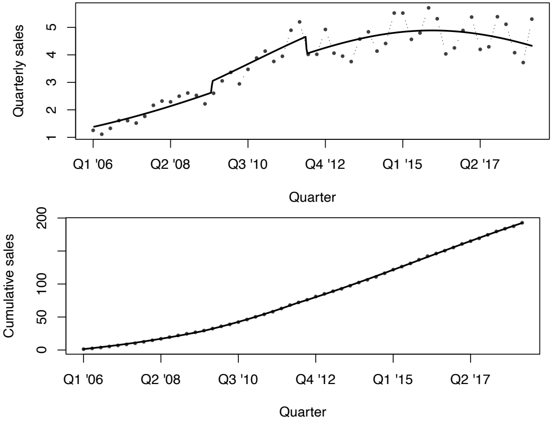
Apple iMac

GBM for iMac: estimates and 95% CIs

	Estimate	Std.Error	Lower	Upper	P-value
m	304.1	3.67	296.9	311.3	< 0.0001
p	0.0043	0.00001	0.0042	0.0044	< 0.0001
q	0.055	0.00	0.053	0.056	< 0.0001
a_1	14.67	0.96	12.79	16.54	< 0.0001
b_1	25.95	0.71	24.55	27.35	< 0.0001
c_1	0.16	0.02	0.13	0.20	< 0.0001

$R^2 = 0.9999$

Apple iMac



Quarterly sales
1
2
3
4
5
Q1 '06
Q2 '08
Q3 '10
Q4 '12
Q1 '15
Q2 '17
Quarter
Cumulative sales
0
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200
Q1 '06
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Q1 '15
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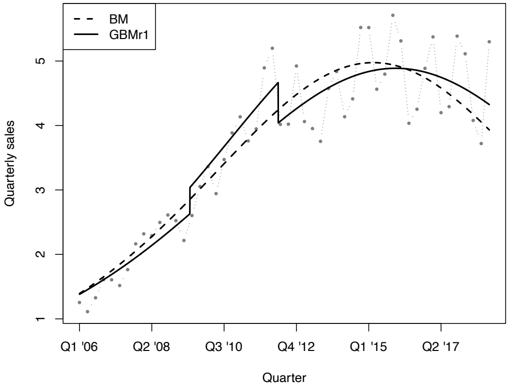
Quarter

Apple iMac



Model comparison . . .

Modelling $x(t)$: mixed shock

It may be useful to have more than one shock of different nature. A simple case is made of a couple of shocks, rectangular and exponential,

$$x(t) = 1 + c_1 I_{t \geq a_1} I_{t \leq b_1} + c_2 e^{b_2(t-a_2)} I_{t \geq a_2}$$

Other combinations are possible.

Model performance and selection

The usual performance indicator is the R^2

$$R^2 = \frac{\mathsf{SST} - \mathsf{SSE}}{\mathsf{SST}} = \frac{\sum(y_i - \bar{y})^2 - \sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

where y_i, $i = 1, 2, ..., n$ are calculated with the selected model. Further evaluations are performed through analysis of residuals (e.g. residual plots, Durbin-Watson statistic).

Model selection: $\tilde{R}^2$

In order to select between two 'nested' models, a suitable tool is the $\tilde{R}^2$

$$\tilde{R}^2 = \frac{\mathsf{SSE}_{m_1} - \mathsf{SSE}_{m_2}}{\mathsf{SSE}_{m_1}} = (R^2_{m_2} - R^2_{m_1})/(1 - R^2_{m_1}),$$

where $R^2_{m_i},\ i = 1, 2$ is the R^2 of model m_i.
If $\tilde{R}^2 > 0.3$ then the more complex model is significant.