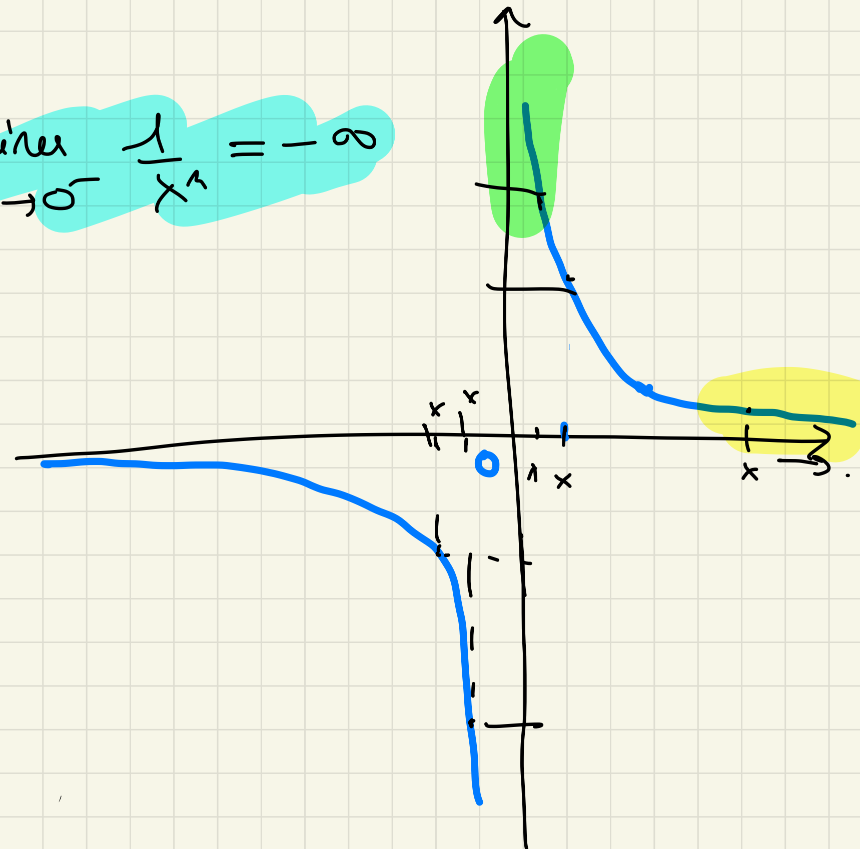
$\frac{1}{x^n}$ n dispari non ha limite a 0

$$\lim_{x \to 0^+} \frac{1}{x^n} = +\infty \qquad \lim_{x \to 0^-} \frac{1}{x^n} = -\infty$$

$$\lim_{x \to +\infty} \frac{1}{x^n} = 0 = \lim_{x \to -\infty} \frac{1}{x^n}$$

$\frac{1}{x^n} > 0$ se $x > 0$

$\frac{1}{x^n} < 0$ $x < 0$



Altre regole di calcolo.

(1) Teorema (no dim) se il limite esiste è UNICO.

(2) LIMITE DI UNA SOMMA È LA SOMMA DEI LIMITI (se tale somma è ben definita)

(a) $\lim_{x \to x_0, +\infty, -\infty} f(x) = L$ $\quad \lim_{x \to x_0, \pm\infty, -\infty} g(x) = M$

$$\lim_{x \to x_0, +\infty, -\infty} f(x) \pm g(x) = L \pm M$$

(b) $\lim_{x \to x_0, +\infty, -\infty} f(x) = +\infty, -\infty$ $\quad \lim_{x \to x_0, +\infty, -\infty} g(x) = M$

$$\lim_{x \to x_0, \pm\infty} f(x) \pm g(x) = \begin{matrix} +\infty + M \\ -\infty \pm M \end{matrix} = \begin{matrix} +\infty \\ -\infty \end{matrix}$$

ⓒ se $\lim_{\substack{x \to x_0 \\ x \to +\infty \\ +\infty}} f(x) = +\infty$ $\lim_{\substack{x \to x_0 \\ x \to \pm\infty}} g(x) = -\infty$

allora $\lim_{\substack{x \to x_0 \\ \pm\infty}} f(x) + g(x)$ $+\infty - \infty$

NON SO DIRE QUANTO VALGA

(FORMA INDETERMINATA)

(non ho una soluzione generale, ma devo vedere di volta in volta come sono fatte le funzioni coinvolte).

Es $\lim_{x \to +\infty} x - x^2$ $\quad$ $+\infty - \infty$ forma indeterminata

$x \to +\infty$, $x^2 \to +\infty$

$f(x) = x \quad g(x) = -x^2$

se ho la somma/differenza di varie potenze della x e ho il limite per $x \to +\infty$ (o $x \to -\infty$)
RACCOLGO a fattor comune la potenza più alta

$$x - x^2 = x^2 \cdot \left[\frac{1}{x} - 1\right]$$

$$\underbrace{x - x^2}_{+\infty - \infty} = \underbrace{x^2}_{\downarrow\, +\infty}\underbrace{\left(\frac{1}{x} - 1\right)}_{\downarrow\, -1}$$

per $x \to +\infty$ $\quad x^2 \to +\infty$

per $x \to +\infty$ $\quad \frac{1}{x} \to 0 \quad \left(\frac{1}{\infty}\right)$

$$\underbrace{\frac{1}{x}}_{\downarrow\, 0} \underbrace{- 1}_{\downarrow\, -1} \longrightarrow 0 - 1 = -1$$

$$\lim_{x \to +\infty} x - x^2 = \lim_{x \to +\infty} x^2\left(\frac{1}{x} - 1\right) = +\infty \cdot (-1) = -\infty$$

③ LIMITE DI UN PRODOTTO È IL PRODOTTO DEI LIMITI (se questo è ben definito)

(a) $\lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} f(x) = \underset{+\infty}{a}$ $\quad \lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} g(x) = \underset{+\infty}{b}$

$$\lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} f(x) \cdot g(x) = a \cdot b$$

$(+\infty) \cdot (+\infty) = +\infty.$

(b) $\lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} f(x) = +\infty$ $\quad -\infty$ $\quad \lim_{\substack{x \to x_0 \\ +\infty \\ +\infty}} g(x) = b \neq 0$

$$\lim_{\substack{x \to x_0 \\ +\infty \\ +\infty}} f(x) g(x) = \begin{cases} +\infty \quad -\infty & \text{se } b > 0 \\ -\infty \quad +\infty & \text{se } b < 0 \end{cases}$$

$+\infty \cdot b$

c) Se $\lim_{x \to x_0, +\infty, -\infty} f(x) = \begin{matrix} +\infty \\ -\infty \end{matrix}$ $\lim_{x \to x_0, +\infty, -\infty} g(x) = 0$

allora $\lim_{x \to x_0, +\infty, -\infty} f(x) \cdot g(x) = (\infty \cdot 0)$

NON SO DIRE QUANTO VALGA

è una $\boxed{\text{forma INDETERMINATA}}$

(dipende da caso a caso).

es. $\lim_{x \to 0^+} \underbrace{x \cdot \log x}$? ($x \to 0$, $\log x \to -\infty$)

$\lim_{x \to -\infty} \underbrace{x e^x}$? ($x \to -\infty$, $e^x \to 0$)

(d) $\lim\limits_{\substack{x \to x_0 \\ +\infty \\ -\infty}} f(x) = 0$ e $\lim\limits_{\substack{x \to x_0 \\ x \to +\infty \\ -\infty}} g(x)$ NON ESISTE

MA $g(x)$ è LIMITATA

$\exists\, C > 0$ $|g(x)| \leq C \quad \forall x \in D$

allora $\lim\limits_{\substack{x \to x_0 \\ +\infty \\ -\infty}} f(x) \cdot g(x) = 0$

$$\lim_{x \to \ldots} \frac{f(x)}{g(x)} = \lim_{x \to \ldots} \underbrace{f(x)} \cdot \underbrace{\frac{1}{g(x)}}$$

$$\lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} f(x) = 0 \quad \pm\infty \qquad \lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} g(x) = 0 \quad \pm\infty$$

$$\lim_{\substack{x \to x_0 \\ +\infty \\ -\infty}} \frac{f(x)}{g(x)} = \lim_{x \to \ldots} f(x) \cdot \frac{1}{g(x)}$$

$f(x) \to 0$, $\frac{1}{g(x)} \to \infty$

F. INDETERM.

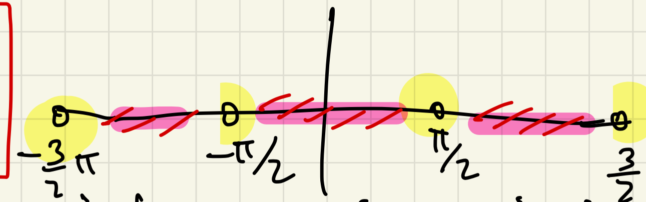
$$\frac{0}{0}$$

$$\lim_{x \to \ldots} \frac{f(x)}{g(x)} = \lim_{x \to \ldots} f(x) \cdot \frac{1}{g(x)}$$

$f(x) \to \begin{matrix} +\infty \\ -\infty \end{matrix}$, $\frac{1}{g(x)} \to 0$

$$\frac{\infty}{\infty}$$

Limiti di $\boxed{\operatorname{tg} x = \dfrac{\sin x}{\cos x}}$



$\operatorname{tg} x$ è una funzione periodica di periodo π

$$\operatorname{tg}(x+\pi) = \operatorname{tg} x \quad \forall x \in D$$

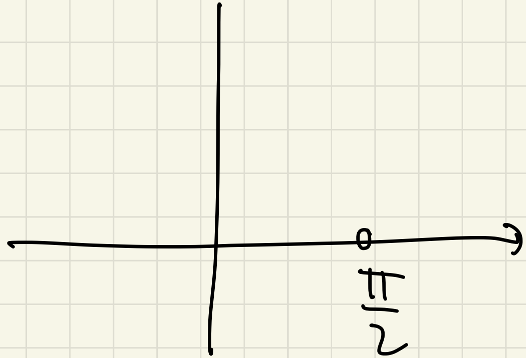
quindi $\lim_{x\to+\infty} \operatorname{tg} x$, $\lim_{x\to-\infty} \operatorname{tg} x$ NON ESISTONO

$\operatorname{tg} x$ è ben definita $x \neq \frac{\pi}{2} + k\pi$

$$D = \{ x \in \mathbb{R} \quad x \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z} \} =$$

$$= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3}{2}\pi\right) \cup \left(-\frac{3}{2}\pi, -\frac{\pi}{2}\right) \cup \dots$$

$$\forall x_0 \in D \quad \lim_{x\to x_0} \operatorname{tg} x = \operatorname{tg} x_0$$



$$\lim_{x \to \frac{\pi}{2}} \operatorname{tg} x = $$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \sin x \cdot \frac{1}{\cos x}$$

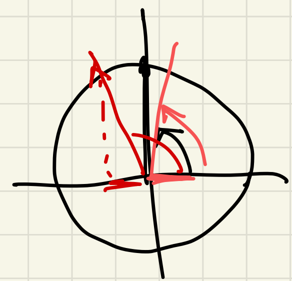
$$\sin \frac{\pi}{2} = 1$$

$$\text{“} \frac{1}{0} \text{”}$$

per $x \to \frac{\pi}{2}$ $\quad \sin x \to \sin \frac{\pi}{2} = 1$

$$\lim_{x \to \frac{\pi}{2}} \sin x = \sin \frac{\pi}{2} = 1$$

$$\lim_{x \to \frac{\pi}{2}} \cos x = \cos \frac{\pi}{2} = 0$$



$0 < \alpha < \frac{\pi}{2}$

$\cos \alpha > 0$

$\frac{\pi}{2} < \alpha < \pi$

$\cos \alpha < 0$

$$\lim_{x \to \left(\frac{\pi}{2}\right)^+} tg\, x = \lim_{x \to \left(\frac{\pi}{2}\right)^+} \sin x \cdot \frac{1}{\cos x} = 1 \cdot (-\infty) = -\infty$$

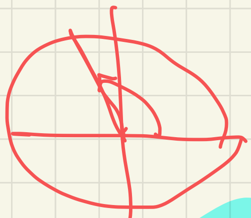
$\sin x \to 1$, $\frac{1}{\cos x} \to -\infty$

calcolo limite per $x > \frac{\pi}{2}$, $x \to \frac{\pi}{2}$

$\cos x < 0$

$x > \frac{\pi}{2}$

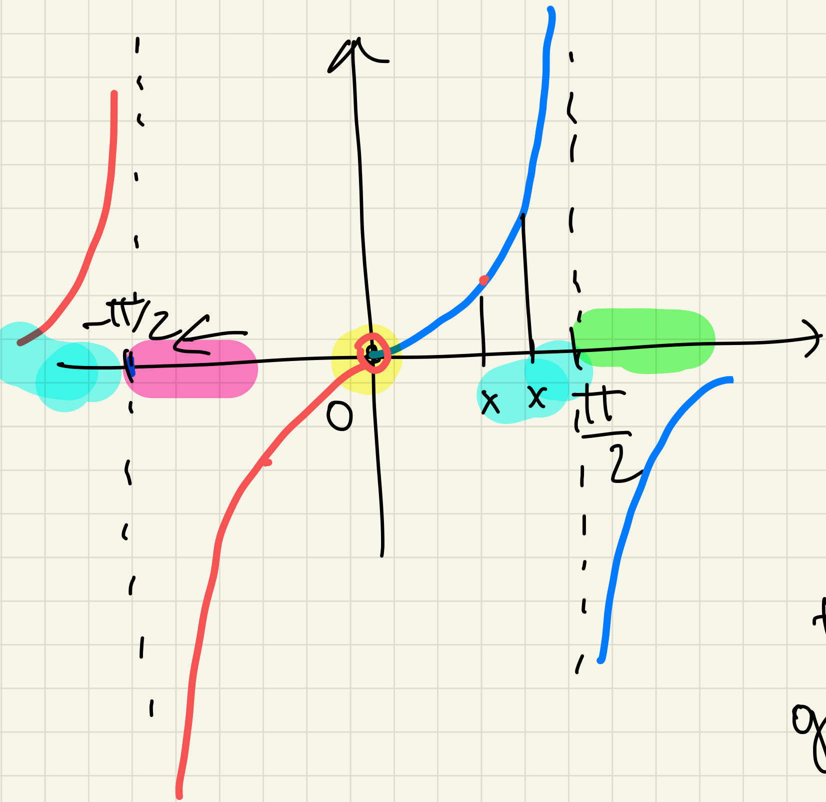
$x \to \frac{\pi}{2}$



$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} tg\, x = \lim_{x \to \left(\frac{\pi}{2}\right)^-} \sin x \cdot \frac{1}{\cos x} = 1 \cdot (+\infty) = +\infty$$

$\sin x \to 1$, $\frac{1}{\cos x} \to +\infty$

$x < \frac{\pi}{2}$ $x \to \frac{\pi}{2}$ $\cos x > 0$



$$\boxed{\lim_{x \to \left(\frac{\pi}{2}\right)^-} tg\, x = +\infty}$$

$$\lim_{x \to \left(\frac{\pi}{2}\right)^+} tg\, x = -\infty$$

$tg\, x$ è DISPARI

$tg(-x) = -tg\, x$

grafico è simmetrico rispetto a $(0,0)$.

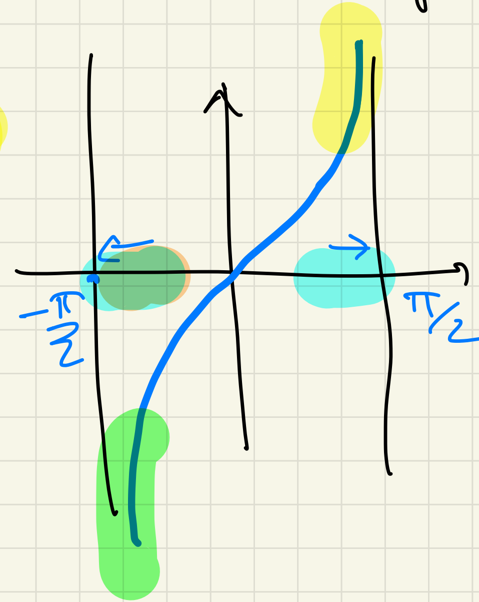
$$\lim_{x \to \left(-\frac{\pi}{2}\right)^+} tg\, x = -\infty$$

$$\lim_{x \to \left(-\frac{\pi}{2}\right)^-} tg\, x = +\infty$$

arctg x è inversa della tangente ristretta a $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} \text{tg}\, x = +\infty$$

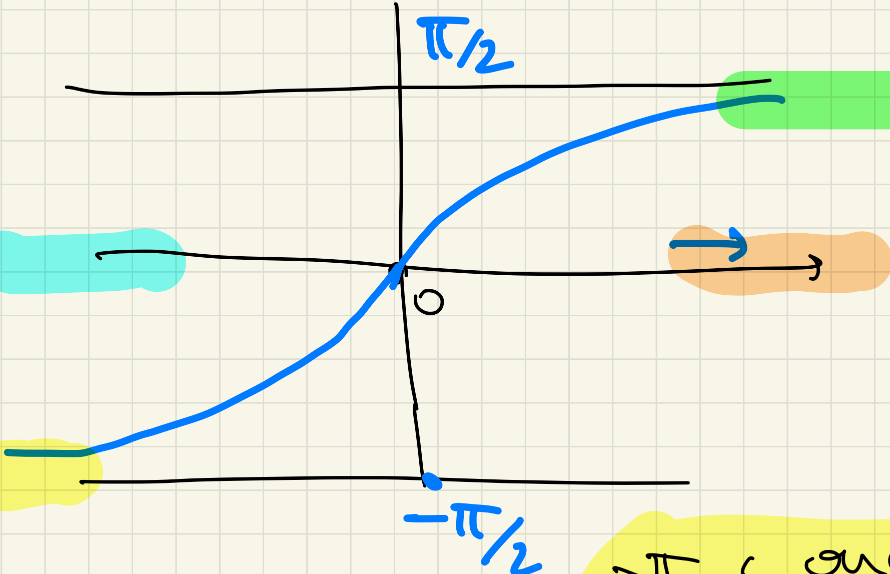
$$\lim_{x \to \left(-\frac{\pi}{2}\right)^+} \text{tg}\, x = -\infty$$



arctg x ha dominio $\mathbb{R}$ ed è continua

$$\lim_{x \to x_0} \text{arctg}\, x = \text{arctg}\, x_0 \quad \forall\, x_0 \in \mathbb{R}$$

$$\lim_{x \to +\infty} \text{arctg}\, x = \frac{\pi}{2} \qquad \lim_{x \to -\infty} \text{arctg}\, x = -\frac{\pi}{2}$$



$\operatorname{arctg} x$

$$\lim_{x \to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \operatorname{arctg} x = -\frac{\pi}{2}$$

$$-\frac{\pi}{2} < \operatorname{arctg} x < \frac{\pi}{2}$$

$\operatorname{arctg} 0 = 0$ $\quad$ $\operatorname{arctg} x$ è dispari

Es

$$\lim_{x\to+\infty} \cosh x = \lim_{x\to+\infty} \frac{e^{x}+e^{-x}}{2} =$$

$$= \lim_{x\to+\infty} \frac{e^{x}}{2} + \frac{e^{-x}}{2} = +\infty + 0 = +\infty$$

$$\frac{+\infty}{2} = +\infty \qquad \frac{0}{2} = 0$$

$x \to +\infty$

$e^{x} \to +\infty$

$$e^{-x} = \frac{1}{e^{x}} = \text{"}\frac{1}{+\infty}\text{"} = 0$$

$$\lim_{x \to -\infty} \frac{e^x}{2} + \frac{e^{-x}}{2} = 0 + +\infty = +\infty$$

$\frac{e^x}{2} \to \frac{0}{2} = 0$

$\frac{e^{-x}}{2} \to \frac{+\infty}{2} = +\infty$

$x \to -\infty$

$e^x \to 0$

$$e^{-x} = \frac{1}{e^x} = \text{"}\frac{1}{0}\text{"} = +\infty$$

$e^x > 0$!

---

es. $\lim_{x \to +\infty} \frac{e^x - e^{-x}}{2} = +\infty$ $\quad$ $\lim_{x \to -\infty} \frac{e^x - e^{-x}}{2} = -\infty$

$$\lim_{x \to +\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \frac{+\infty}{+\infty} \qquad \boxed{\text{forma indeterminata}}$$

$$e^x \to +\infty \qquad e^{-x} = \frac{1}{e^x} \to 0$$

$$e^x - e^{-x} \to +\infty - 0 = +\infty$$

$$e^x + e^{-x} \to +\infty + 0 = +\infty$$

raccolgo a fattor comune il termine "più grande"

$$e^x \mp e^{-x} = e^x \cdot (1 \mp e^{-2x})$$

$$\lim_{x\to+\infty} \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} = \lim_{x\to+\infty} \frac{\cancel{e^{x}}\left(1-e^{-2x}\right)}{\cancel{e^{x}}\left(1+e^{-2x}\right)} = \frac{1-0}{1+0} = 1$$

$$e^{-2x} \to 0$$

$$e^{-2x} = \frac{1}{e^{2x}} = \frac{1}{(e^{x})^{2}} = \frac{1}{e^{x}}\cdot\frac{1}{e^{x}} = 0\cdot 0$$

$$e^{x} \to +\infty$$

$$\lim_{x\to-\infty} \frac{e^{+x}-e^{-x}}{e^{x}+e^{-x}} = \lim_{x\to-\infty} \frac{\cancel{e^{-x}}\left(e^{2x}-1\right)}{\cancel{e^{-x}}\left(e^{2x}+1\right)} = \frac{0-1}{0+1} = \frac{-1}{1} = -1$$

$$e^{2x} \to 0$$

$$e^{2x} = \underbrace{e^{x}}\cdot\underbrace{e^{x}} \to 0\cdot 0$$

$$x \to -\infty \qquad e^{x} \to 0$$