INTERPOLAZIONE DI FOURIER

$f(x)$ è periodica con periodo L $\quad f(x+L)=f(x) \quad \forall x$

NB $\quad f: \mathbb{R} \to \mathbb{C}$

Serie di Fourier

$$f(x) = \sum_{m=-\infty,+\infty} c_m e^{i\frac{2\pi}{L}m \cdot x}$$

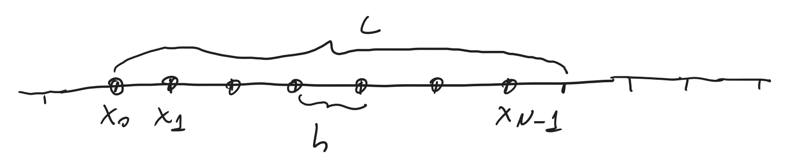
$$c_m = \frac{1}{L}\int_0^L e^{-i\frac{2\pi}{L}mx} f(x)\,dx$$

Infatti:

$$\frac{1}{L}\int_0^L e^{-i\frac{2\pi}{L}mx}\left(\sum_{m'=-\infty,+\infty} c_{m'} e^{i\frac{2\pi}{L}m'x}\right)dx$$

$$= \frac{1}{L}\sum_{m'=-\infty,+\infty} c_{m'} \underbrace{\int_0^L e^{i\frac{2\pi}{L}(m'-m)x}\,dx}_{L\,\delta_{m,m'}} = c_m$$

SERIE DI FOURIER DISCRETA



$$h = \frac{L}{N}$$

$$f_i = \frac{1}{N}\sum_{j=0,N-1} g_j e^{i\frac{2\pi}{L}jx_i}$$

$$g_j = \sum_{i=0,N-1} e^{-i\frac{2\pi}{L}jx_i} f_i$$

$L = hN \qquad x_i = h x_i$

$$\boxed{\begin{aligned} f_i &= \frac{1}{N}\sum_{j=0,N-1} g_j e^{i\frac{2\pi}{N}ji} \\ g_j &= \sum_{i=0,N-1} f_i e^{-i\frac{2\pi}{N}ji} \end{aligned}}$$

TRASFORMATA DI FOURIER DISCRETA

$$g_j \stackrel{?}{=} \sum_{i=0,N-1}\left(\frac{1}{N}\sum_{k=0,N-1} g_k e^{i\frac{2\pi}{N}ki}\right)e^{-i\frac{2\pi}{N}ji}$$

$$\stackrel{?}{=} \sum_{k=0,N-1}\frac{1}{N} g_k \underbrace{\sum_{i=0,N-1} e^{i\frac{2\pi}{N}(k-j)i}}_{N\,\delta_{k,j}} = g_j \quad \text{ok!}$$

In teoria $\propto N^2$ operazioni $\quad O(N^2)$

algoritmo FAST FOURIER TRANSFORM (FFT)

$O(N\log(N))$

Se ho i $\{g_j\}$

allora posso interpolare

$$f(x) = \frac{1}{N}\sum_{j=0,N-1} g_j e^{i\frac{2\pi}{L}jx} \qquad (I)$$

Calcolo di $\frac{d}{dx}f(x)$ per $f(x)$ scritta come (I)

$$\frac{d}{dx}f(x) = \frac{1}{N}\sum_{j=0,N-1} g_j\left(\frac{i2\pi}{L}j\right)e^{i\frac{2\pi}{L}jx}$$

Quindi l'operazione d/dx corrisponde a:

$$g_j \to g_j \frac{i2\pi}{L}j$$

(i: unità immaginaria)

Se $f(x) \in \mathbb{R} \quad \forall x$

allora $\quad f(x) = f^*(x)$

$$\frac{1}{N}\sum_{j=0,N-1} g_j e^{i\frac{2\pi}{L}jx} = \left(\frac{1}{N}\sum_{j'=0,N-1} g_{j'} e^{i\frac{2\pi}{L}j'x}\right)^*$$

$$= \frac{1}{N}\sum_{j'=0,N-1} g^*_{j'} e^{-i\frac{2\pi}{L}j'x}$$

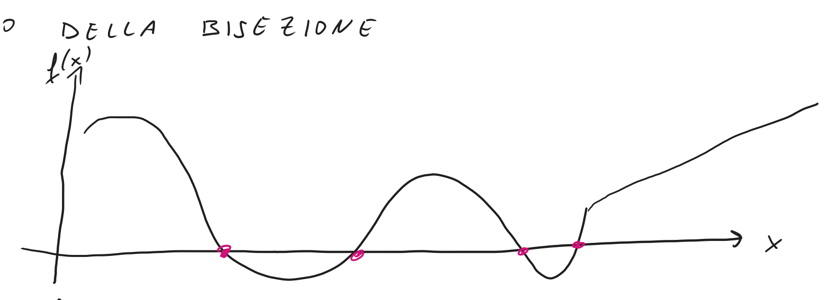
$$= \frac{1}{N}\sum_{k=0,N-1} g^*_{-k} e^{i\frac{2\pi}{L}kx}$$

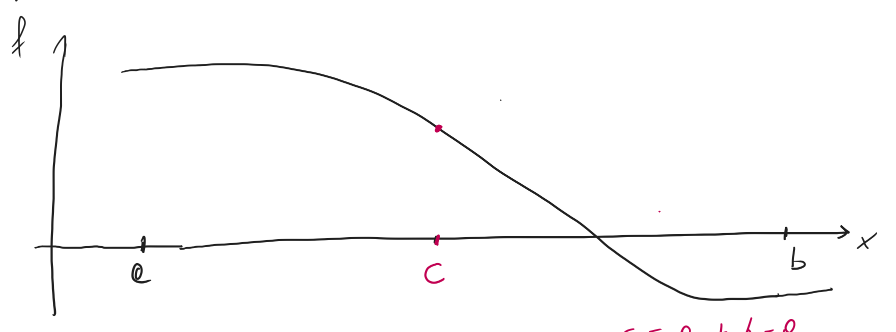
$$\Rightarrow \boxed{g_j = g^*_{-j}}$$

METODI PER LA SOLUZIONE NUMERICA DI

$f(x) = 0 \qquad f: \mathbb{R} \to \mathbb{R}$

METODO DELLA BISEZIONE





$c = e + \frac{b-e}{2}$

Se $f(x)$ è continua

Se $f(e)f(b) < 0$ allora ho almeno una soluzione $\in\,]e,b[$

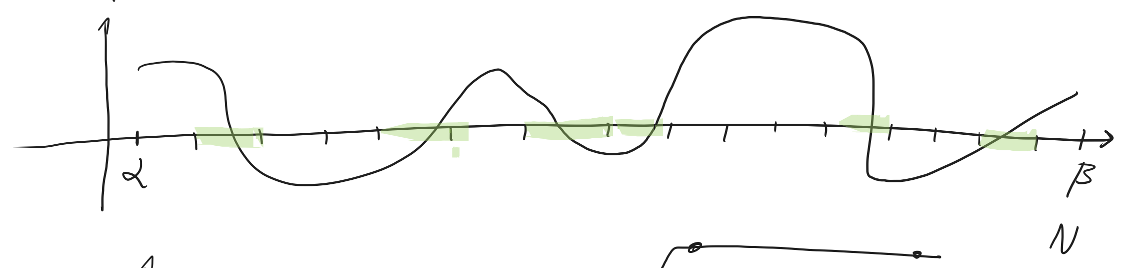
pongo $c = e + \frac{b-e}{2}$ e valuto $f(c)$

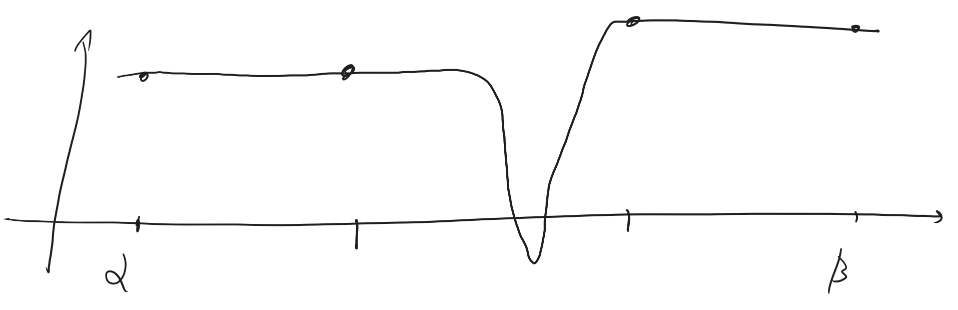
se $f(e)f(c) > 0$ allora pongo $e = c$ e ricomincio

altrimenti pongo $b = c$ e ricomincio

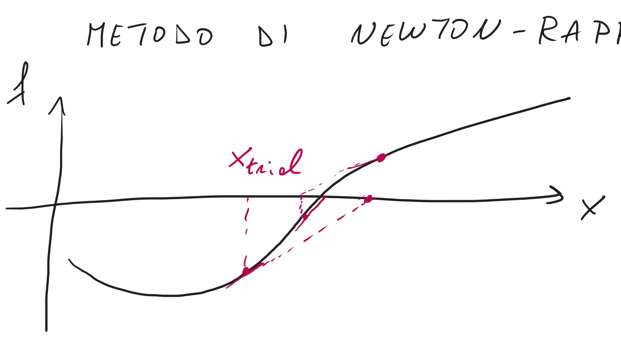
mi fermo quando $(b-e) < \varepsilon$ ← soglia di convergenza

In pratica





METODO DI NEWTON-RAPHSON



Consideriamo x_{trial}

$$\tilde{f}(x) = f(x_{trial}) + f^{(1)}(x_{trial})(x - x_{trial})$$

risolviamo $\tilde{f}(x) = 0$ e

$$x = \frac{-f(x_{trial}) + f^{(1)}(x_{trial})x_{trial})}{f^{(1)}(x_{trial})}$$

e poniamo il nuovo

$$x_{trial} = x$$