Nonlinear models for new product growth

New product life cycle: phases

1. Introduction
2. Growth
3. Maturity
4. Decline

What are the variables influencing a product's life cycle?
Marketing strategies play an essential role ...
but the success of a new product ultimately depends on consumers accepting them.

Diffusion of innovations

Diffusion is the process by which an innovation is communicated through certain channels over time among the members of a social system (Rogers, 2003).
Four key elements for describing an innovation diffusion process:

- innovation
- communication channels
- time
- social system

Innovation

An innovation is:

- New product, new service, new technology, new production process, new way of doing things (Schumpeter, 1947).
- Typical distinction: radical vs incremental innovations.
- Radical innovations could be hindered from barriers and social inertia.

New product growth models

General aim: depict the successive increases in the number of adopters and predict the continued development of a diffusion process already in progress (Mahajan and Muller, 1979).

- Fourt and Woodlock model (1960)
- Mansfield model (1961)
- Bass model (1969)
- Generalized Bass model (1994)

Bass Model

The Bass Model is defined by a first order differential equation

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t))$$

Bass Model

innovation

$$z'(t) = \left(p + q\frac{z(t)}{m} \right) (m - z(t))$$

Bass Model

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t))$$

imitation

Bass Model

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t))$$

word-of-mouth

Bass Model

If we pose $\frac{z(t)}{m} = y(t)$ the model becomes

$$y'(t) = (p + qy(t))(1 - y(t))$$

Bass Model: solution

The Bass Model has a closed-form solution

$$y(t) = F(t; p, q) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \qquad t > 0.$$

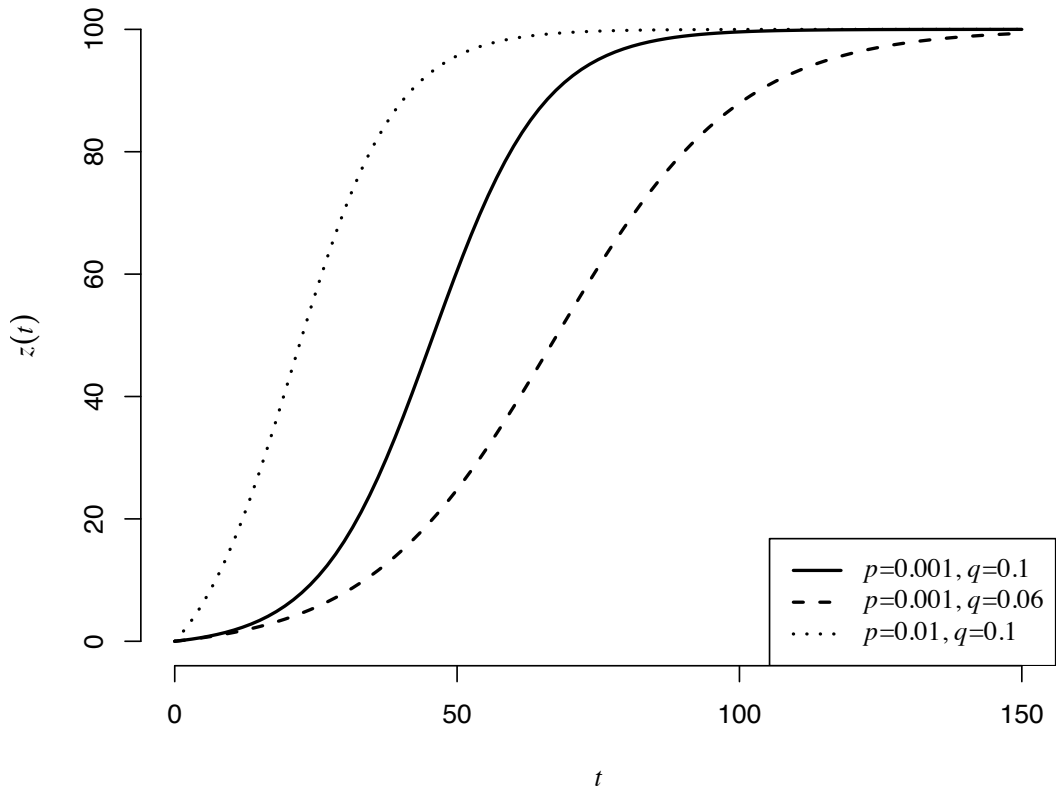
or, by posing $z = ym$

$$z(t) = m\, F(t; p, q) = m\, \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \qquad t > 0.$$

Cumulative sales $z(t)$ 'depend' on parameters p and q.
The market potential m is a scale parameter and is assumed constant.

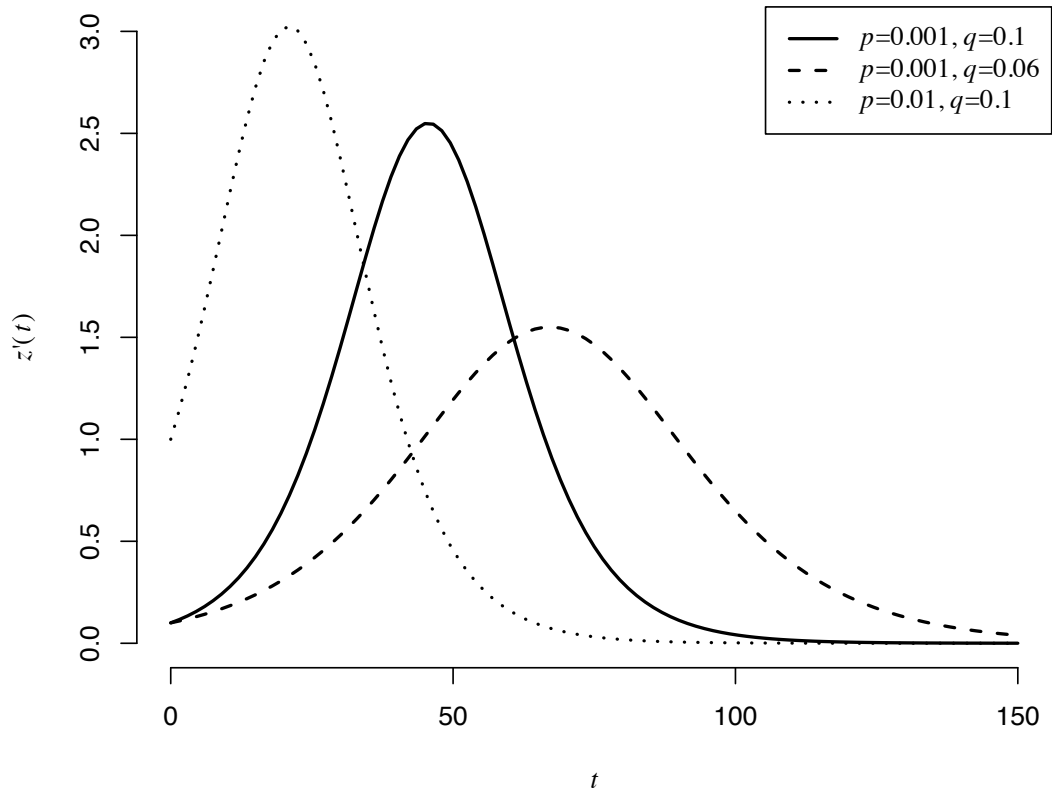
Bass Model

Cumulative process



Bass Model

Instantaneous process



Bass Model: estimation

The Bass Model is a nonlinear model

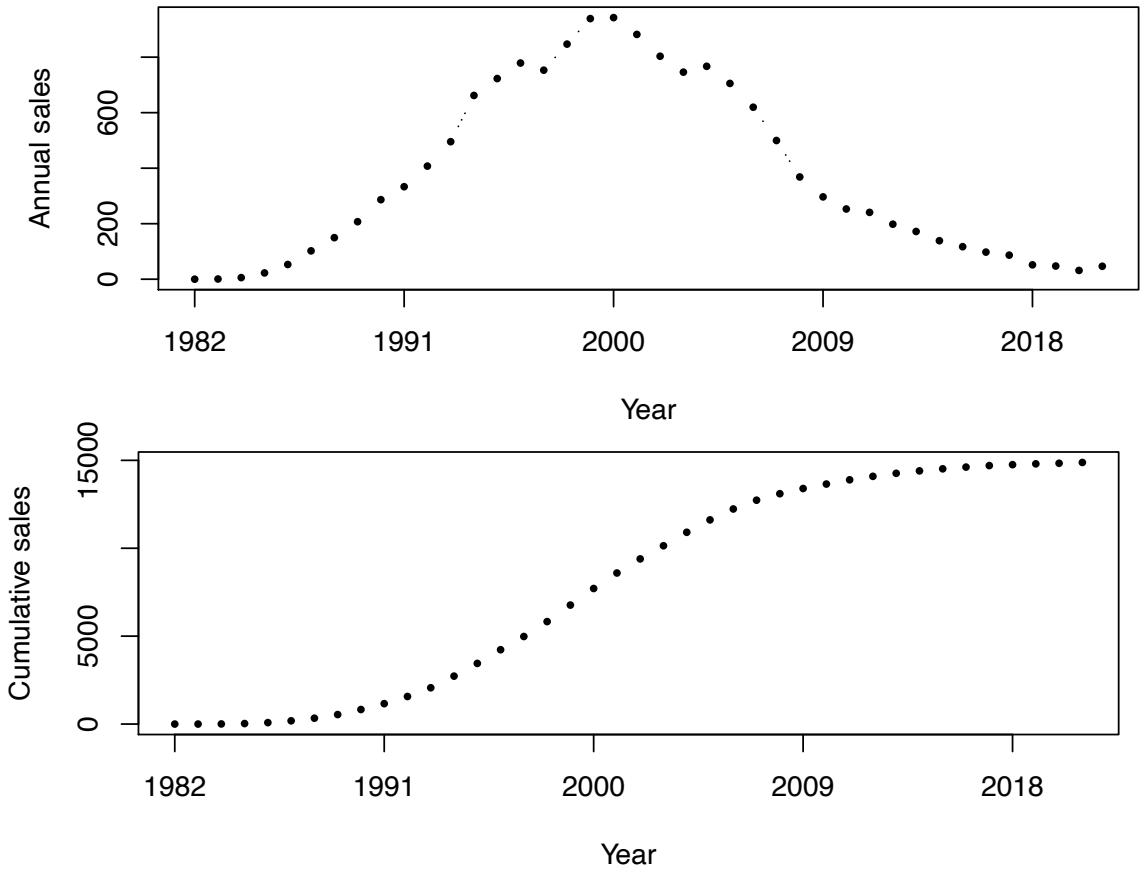
$$Z(t) = f(\beta, t) + \varepsilon(t)$$

where $Z(t)$ is the dependent variable, $f(\beta, t)$ is the deterministic term, function of $\beta \in R^k$ and of time t.
The second term, $\varepsilon(t)$, is the error term, for which usual assumptions hold, namely $M(\varepsilon(t)) = 0$,$Var(\varepsilon(t)) = \sigma^2$, $Cov(\varepsilon(t), \varepsilon(t')) = 0, t \neq t'$.

Bass Model: estimation

- Typical starting values for p and q are 0.01 and 0.1.
- Estimating m is the most difficult task.
- Parameter estimates are very sensitive to the number of available data.
- Reliable estimates are obtained after the maximum peak, but . . . *"By the time sufficient observations have been developed for realiable estimation, it is too late to use the estimates for forecasting purposes"* (Mahajan, Muller, Bass, 1990).

Compact Discs in USA



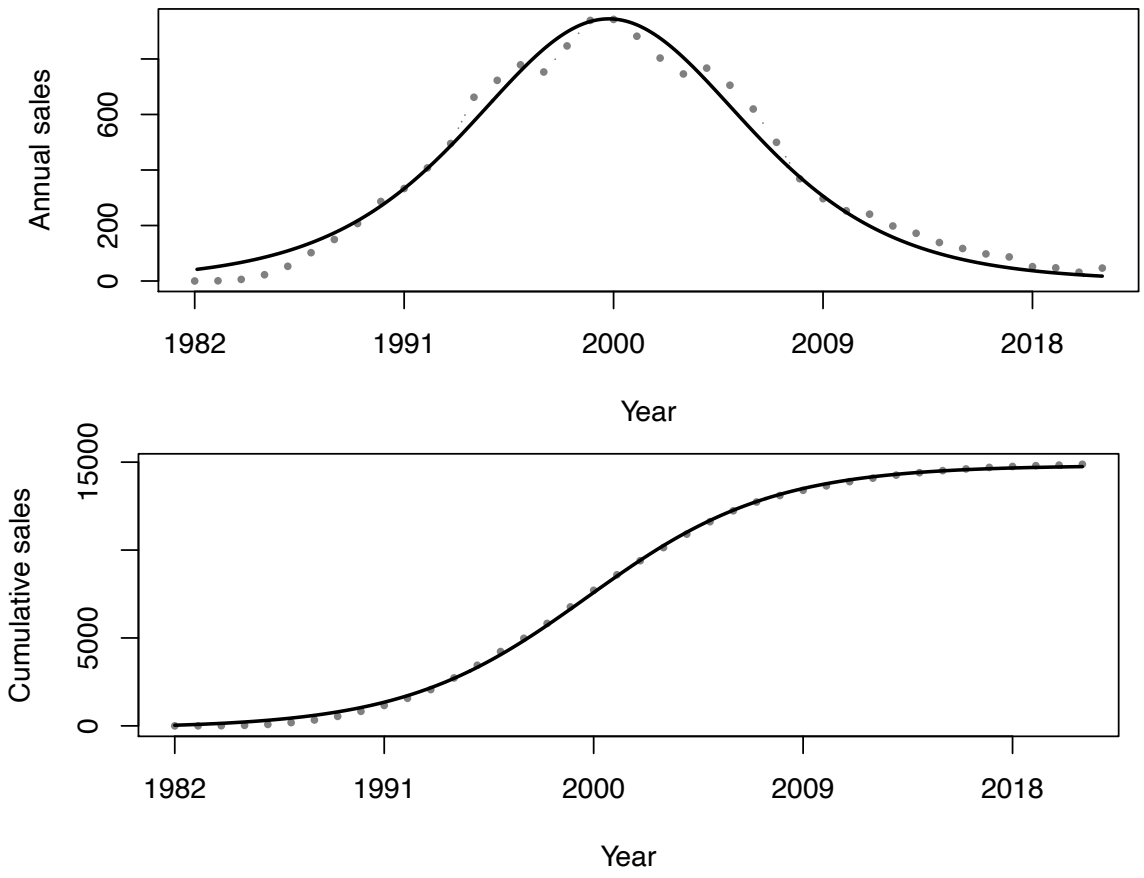
Compact Discs in USA

Bass Model for CD: estimates and 95% CIs

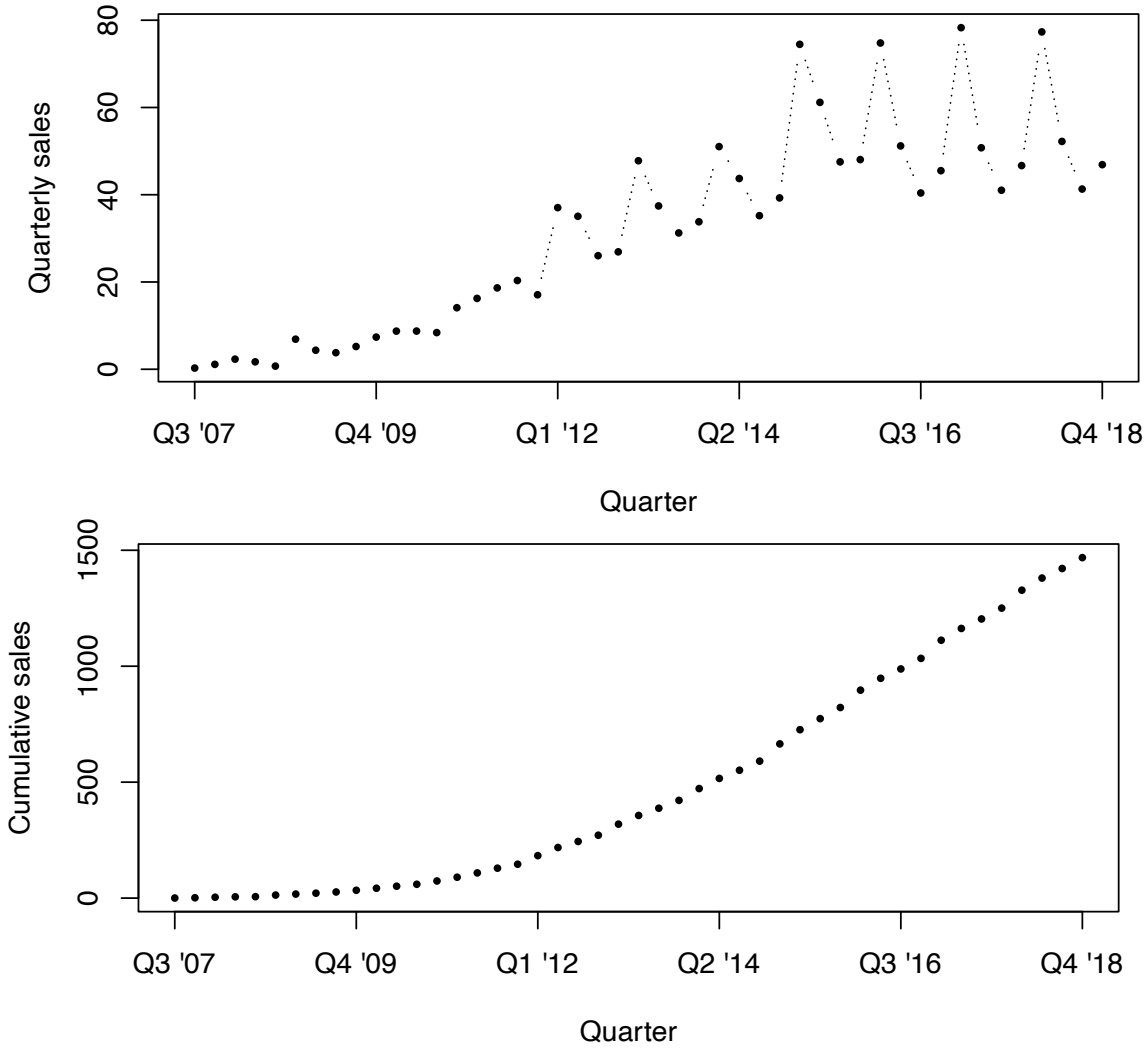
	Estimate	Std.Error	Lower	Upper	p-value
m	14814	49	14716	14911	< 0.0001
p	0.0022	0.0001	0.0020	0.0024	< 0.0001
q	0.25	0.0035	0.24	0.26	< 0.0001

$R^2 = 0.9998$

Compact Discs in USA



Apple iPhone



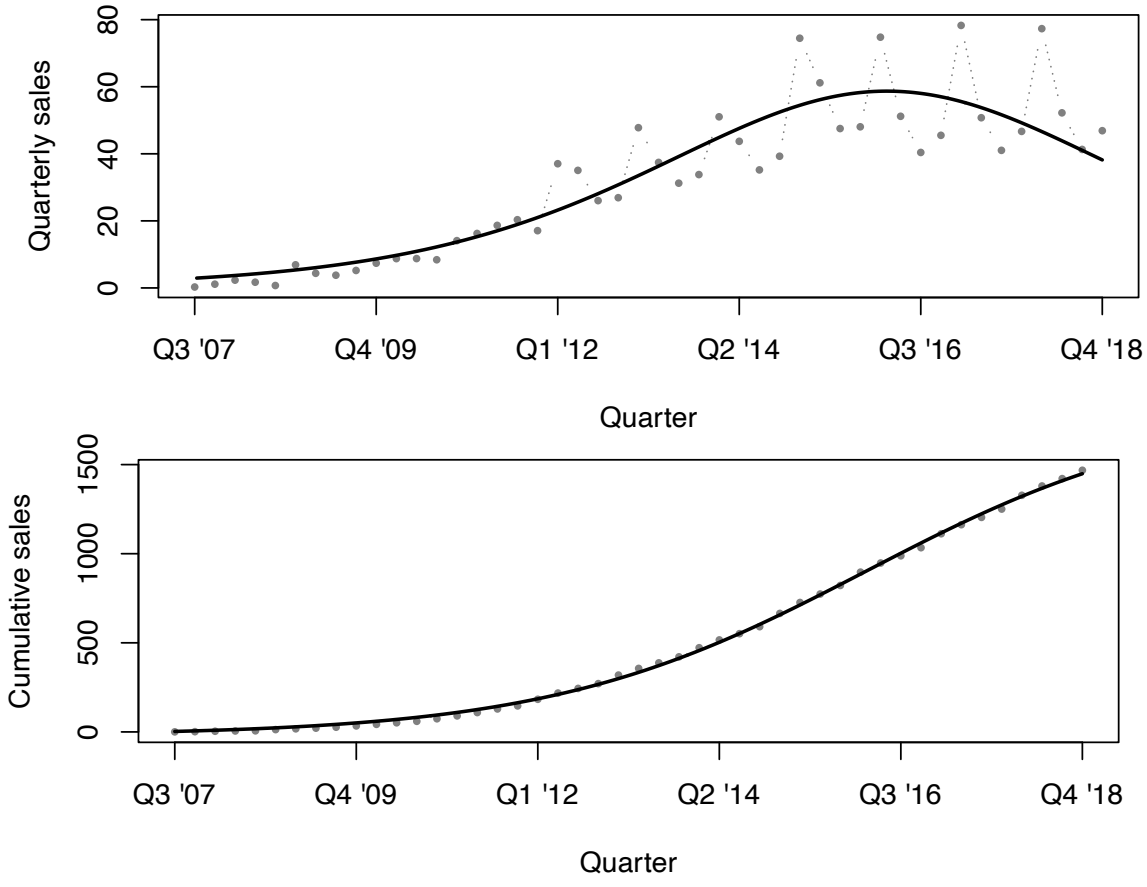
Apple iPhone

Bass Model for iPhone: estimates and 95% CIs

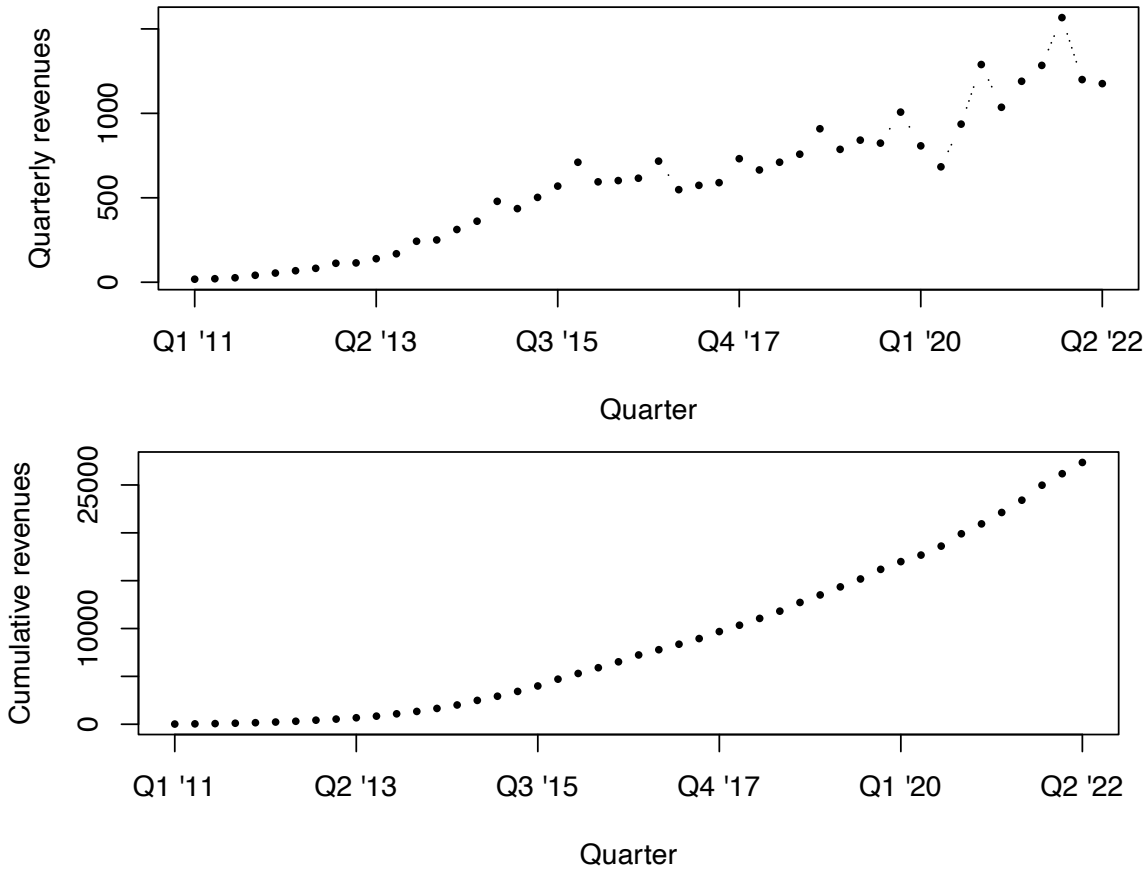
	Estimate	Std.Error	Lower	Upper	p-value
m	1823.7	34.12	1756.8	1890.6	< 0.0001
p	0.0014	0.0001	0.0013	0.0015	< 0.0001
q	0.1259	0.0027	0.1206	0.1311	< 0.0001

$R^2 = 0.9995$

Apple iPhone



Twitter revenues



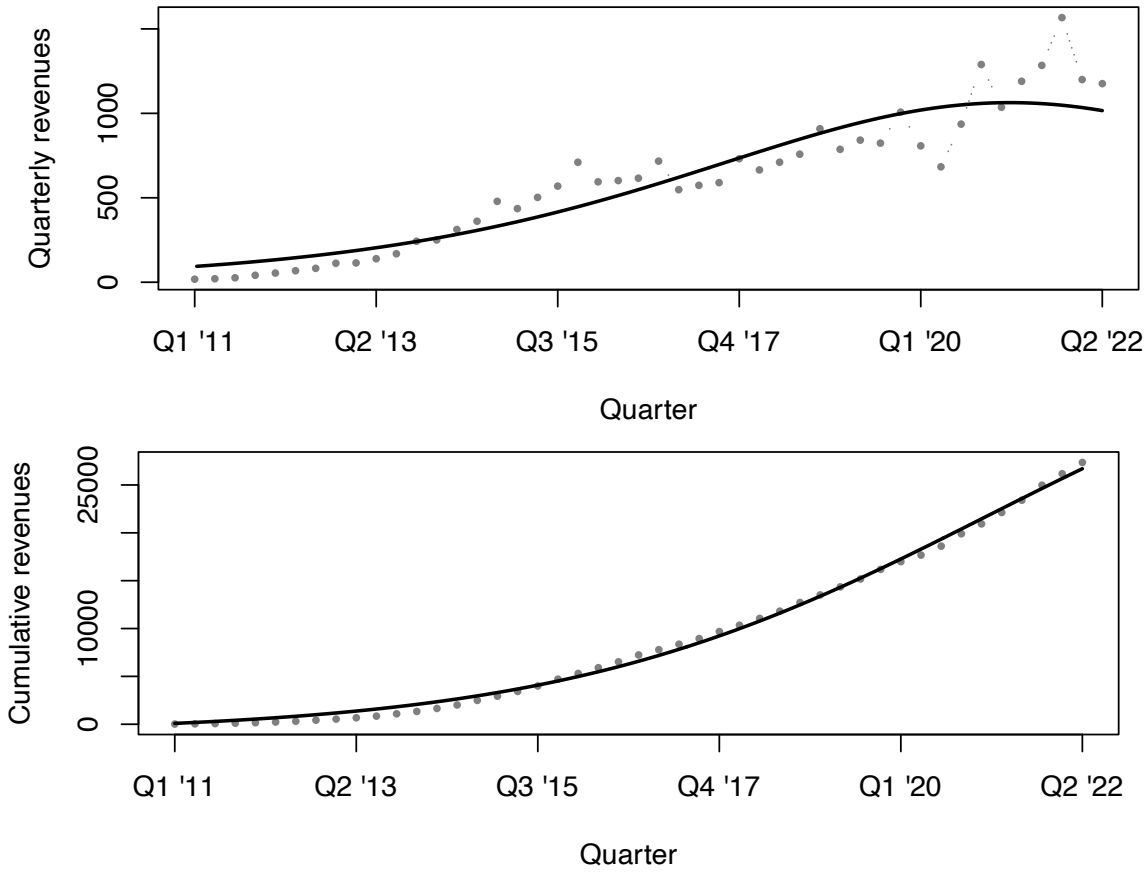
Twitter revenues

Bass Model for Twitter: estimates and 95% CIs

	Estimate	Std.Error	Lower	Upper	p-value
m	44633.7	3557.9	37660.3	51607.0	< 0.0001
p	0.0019	0.0001	0.0018	0.0021	< 0.0001
q	0.09	0.004	0.08	0.10	< 0.0001

$R^2 = 0.9995$

Twitter revenues



Bass Model: interesting properties

- Parsimonious model with just three parameters m, p, q.
- Only needs aggregate sales data.
- Easy to interpret.

Bass Model: limitations

- The market potential m is constant along the whole life cycle.
- The Bass Model does not account for marketing mix strategies.
- It is a model for products with a limited life cycle: needs a hypothesis.