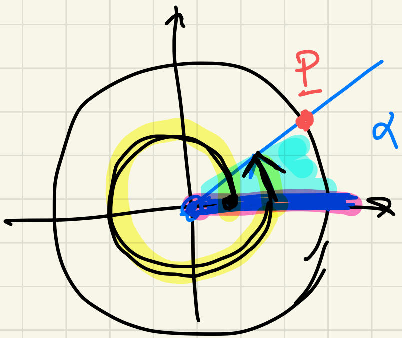
Funzioni trigonometriche

$$\cos(\alpha - 2\pi) = \cos\alpha$$
$$\sin(\alpha - 2\pi) = \sin\alpha$$
$$\operatorname{tg}(\alpha - 2\pi) = \operatorname{tg}\alpha$$



α angolo in radianti

$$\boxed{\alpha \in [0, 2\pi]}$$

$$\boxed{\alpha \in [0, 2\pi]} \rightarrow$$

$\cos\alpha$ = ascissa pto P

$\sin\alpha$ = ordinata pto P

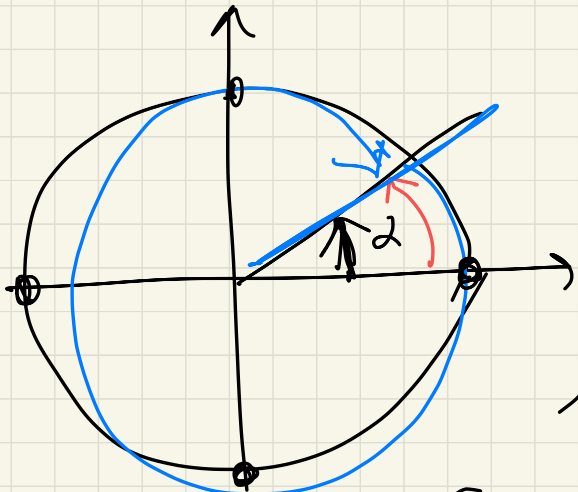
$\operatorname{tg}\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\alpha \in [0, 2\pi]$ $\alpha \neq \frac{\pi}{2}$ $\alpha \neq \frac{3}{2}\pi$

? $x \in \mathbb{R}$

$\cos x$

$$\cos(\alpha + 2\pi) = \cos\alpha$$
$$\sin(\alpha + 2\pi) = \sin\alpha$$
$$\operatorname{tg}(\alpha + 2\pi) = \operatorname{tg}(\alpha)$$



$\alpha - 2\pi$

$\operatorname{sen}\alpha \in [-1, 1]$

$\cos\alpha \in [-1, 1]$

$(\operatorname{sen}\alpha)^2 + (\cos\alpha)^2 = 1$

Definisco

$\operatorname{sen} x : \mathbb{R} \to [-1, 1]$

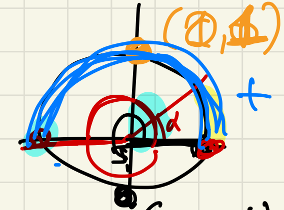
$\cos x : \mathbb{R} \to [-1, 1]$

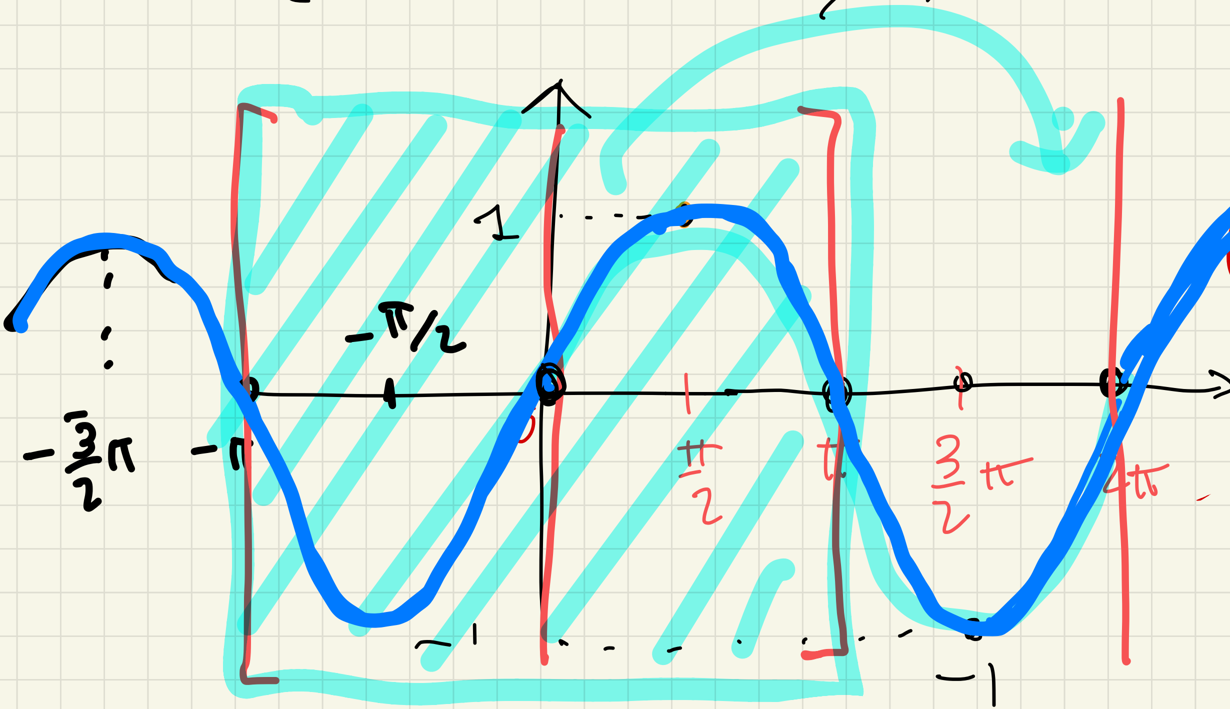
Sono funzioni che si ripetono sempre uguali a se stesse su ogni intervallo di lunghezza 2π

$\sin(x + 2\pi) = \sin x \quad \forall x \qquad \cos(x + 2\pi) = \cos x$

FUNZIONI PERIODICHE

$\text{sen}(3\pi) = \text{sen}(\pi + \cancel{2\pi}) = \text{sen}\,\pi$





ORDINATA

$\text{sen}\,0 = 0$

$0 = \text{sen}\,\pi = \text{sen}\,2\pi$

angolo $\alpha = 0$

$\alpha = 2\pi$, $\alpha = \pi$

corrispondono al pto

$(1,0)$ $(-1,0)$

$\text{sen}\,\frac{\pi}{2} = 1$

$\text{sen}\,\frac{3}{2}\pi = -1$

$\text{sen}\,x = \text{sen}(x + 2k\pi) \quad k \in \mathbb{Z}$

periodico di periodo 2π

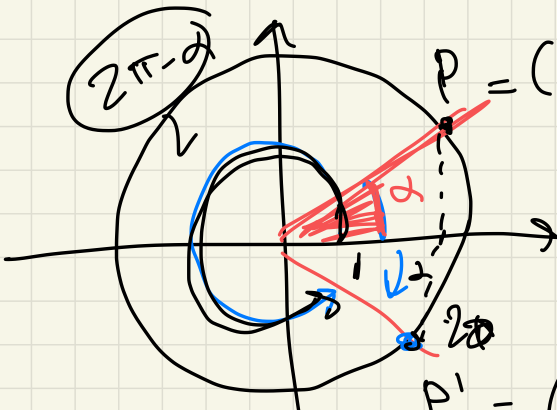
LIMITATA $-1 \le \sin x \le 1 \quad \forall x \in \mathbb{R}$

$\operatorname{sen} x : \mathbb{R} \to \mathbb{R}$ è DOMINIO $= \mathbb{R}$

- periodica di periodo 2π $\sin(x+2\pi) = \sin x \quad \forall x \in \mathbb{R}$

- LIMITATA $-1 \le \operatorname{sen} x \le 1 \quad \forall x \in \mathbb{R}$

- DISPARI $\operatorname{sen}(-x) = -\operatorname{sen} x$



$P = (\cos\alpha, \sin\alpha)$

$P' = (\cos(2\pi-\alpha), \operatorname{sen}(2\pi-\alpha))$

$\cos(-\alpha)$
$\parallel$

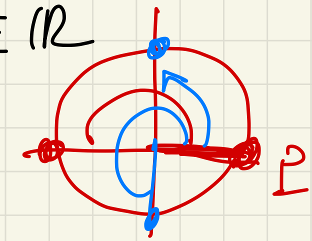
$\boxed{\begin{aligned} \cos(2\pi-\alpha) &= \cos\alpha \\ \sin(2\pi-\alpha) &= -\sin\alpha \end{aligned}}$

$\parallel$

$\sin(-\alpha+2\pi) = \sin(-\alpha)$

$\cos x : \mathbb{R} \to \mathbb{R}$ $\qquad D = \mathbb{R}$

periodica di periodo 2π $\qquad \cos x = \cos(x + 2\pi)$ $\quad \forall x \in \mathbb{R}$



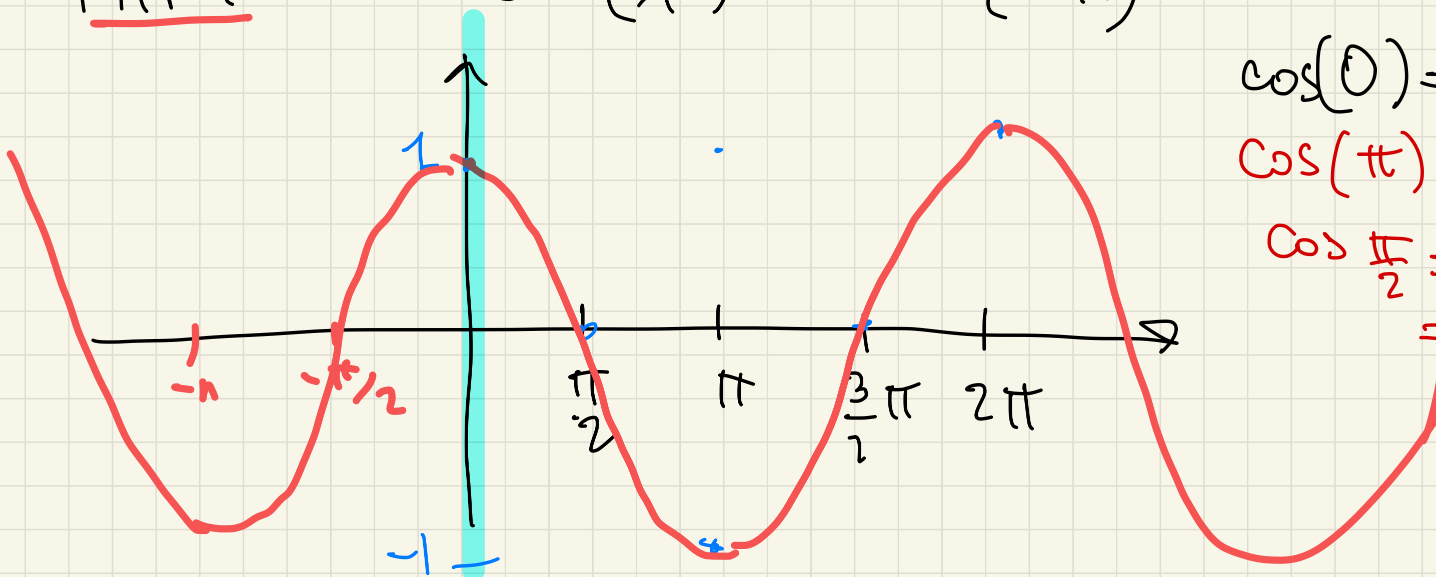
LIMITATA $-1 \le \cos x \le 1$ $\quad \forall x \in \mathbb{R}$

PARI $\qquad \cos(x) = \cos(-x)$

$\cos(0) = \cos 2\pi = 1$

$\cos(\pi) = -1$

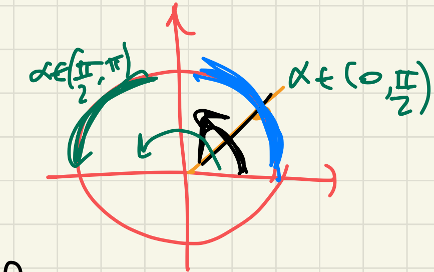
$\cos \frac{\pi}{2} = 0 = \cos \frac{3}{2}\pi$



$$\operatorname{tg} x = \frac{\sin x}{\cos x} \rightarrow \cos x \neq 0 \,!$$

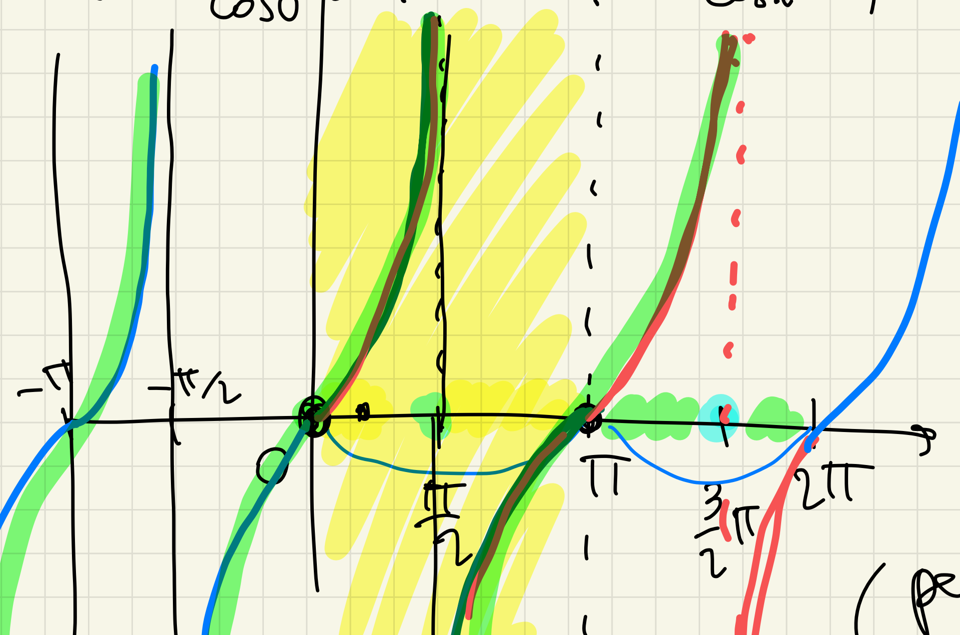
$\alpha \in [0, 2\pi]$

$\operatorname{tg} \alpha$



1) esiste solo per $\alpha \neq \frac{\pi}{2}$, $\alpha \neq \frac{3}{2}\pi$

$$\operatorname{tg} 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = \operatorname{tg} \pi = \frac{\sin \pi}{\cos \pi} = \frac{0}{-1}$$



2) $\boxed{\underbrace{\operatorname{tg}(\alpha) = \operatorname{tg}(\alpha + \pi)}}$ → basta conoscere la funzione (per $\alpha \in [0, \pi]$ $\alpha \neq \frac{\pi}{2}$

$\operatorname{tg}(x+\pi) = \operatorname{tg} x$ PERIODICA di periodo π

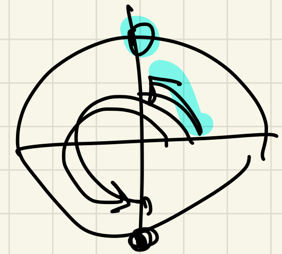
- $\operatorname{tg} x = \operatorname{tg}(x+\pi)$ periodica di periodo π
- ILLIMITATA $\operatorname{tg} x$ può assumere tutti i valori positivi negativi o nulli
- $\operatorname{tg}(-x) = \dfrac{\operatorname{sen}(-x)}{\cos(-x)} = \dfrac{-\operatorname{sen} x}{\cos(x)} = -\dfrac{\operatorname{sen} x}{\cos x} = -\operatorname{tg} x$

seno è DISPARI
coseno è PARI

tangente è funzione DISPARI

$$\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}$$
$$\forall x \neq \frac{\pi}{2} + k\pi$$
$$k \in \mathbb{Z}$$

$$(\operatorname{sen} x)^2 + (\cos x)^2 = 1 \quad \forall x$$



Dominio tangente $= \{ x \in \mathbb{R}$

$$x \neq \frac{\pi}{2} + 2k\pi$$

$$x \neq \frac{3}{2}\pi + 2k\pi$$

Funzioni iperboliche

$\sinh x$ = seno iperbolico di x =

$$= \frac{e^x - e^{-x}}{2}$$

$\cosh x$ = coseno iperbolico $= \dfrac{e^x + e^{-x}}{2}$

$\text{th}\, x$ = tangente iperbolica $= \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$(\cosh x)^2 - (\sinh x)^2 = 1$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad D = \mathbb{R}$$

segno $\quad \sinh x \geq 0 \qquad \frac{e^x - e^{-x}}{2} \geq 0$

$\downarrow$

$$e^x - e^{-x} \geq 0 \qquad 2^{-3} = \left(\frac{1}{2}\right)^3 = = \frac{1}{2^3}$$

$$e^x - \frac{1}{e^x} \geq 0$$

N: $e^{2x} - 1 \geq 0 \qquad e^{2x} \geq 1 = e^0$

D $e^x > 0 \ \forall x \qquad \Downarrow$

$$\frac{e^x \cdot e^x - 1}{e^x} = \frac{e^{2x} - 1}{e^{\cancel{x}}} \geq 0 \qquad 2x \geq 0 \quad x \geq 0$$

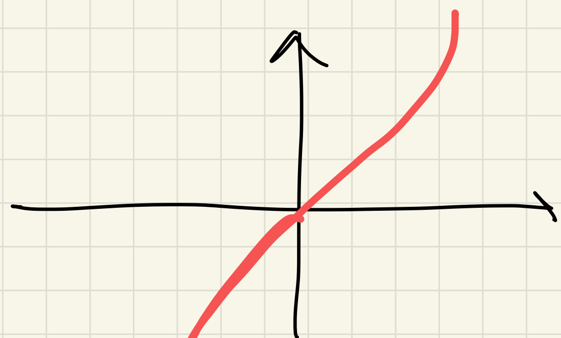
$\sinh x \geq 0 \iff x \geq 0 \qquad \sinh 0 = 0.$

$$\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} =$$

$$= \frac{e^{-x} - e^{x}}{2} = \frac{-e^{x} + e^{-x}}{2} =$$

$$= -\frac{(e^{x} - e^{-x})}{2} = -\sinh x$$

DISPARI



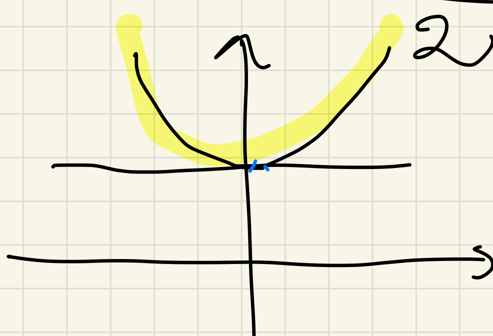
$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad D = \mathbb{R}$$

$$\cosh x \geq 0 \implies \frac{\overset{>0}{e^x} + \overset{>0}{e^{-x}}}{2} \geq 0$$

$\cosh x > 0 \quad \forall x \in \mathbb{R}$ LIMITATA INFERIORM

$$\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^{x}}{2} = \cosh x$$

PARI



$$\cosh 0 = 1$$

OSS $\boxed{\dfrac{e^x+e^{-x}}{2} \geq 1}$ $\forall x \in \mathbb{R}$

$\cosh x \geq 1 \quad \forall x \in \mathbb{R}$

$$\frac{e^x+e^{-x}}{2} - 1 \geq 0 \qquad \frac{e^x+e^{-x}-2}{\cancel{2}} \geq 0$$

$$e^x+e^{-x}-2 \geq 0 \; ?$$

$$e^x + \frac{1}{e^x} - 2 \geq 0 \qquad \frac{e^{2x}+1-2e^x}{e^x} \geq 0$$

$e^x > 0$ $\qquad$ > 0

$e^x > 0$

$$e^{2x}+1-2e^{x}\geq 0 \qquad (e^{x}-1)^{2}\geq 0$$

$$e^{2x}=(e^{x})^{2}$$

$$y=e^{x} \qquad \forall x\in\mathbb{R}$$

$$y^{2}+1-2y\geq 0 \qquad y^{2}-2y+1\geq 0 \qquad (y-1)^{2}\geq 0$$

$$y^{2}-2y+1=0 \qquad y_{1,2}=1 \qquad \forall y\in\mathbb{R}$$

$$(y-1)^{2}$$

$$\text{th}\,x = \frac{e^x - e^{-x}}{\underbrace{e^x + e^{-x}}} = \frac{\sinh x}{\cosh x}$$

$e^x + e^{-x} > 0$

D? $e^x + e^{-x} > 0 \ \forall x \Rightarrow \boxed{D = \mathbb{R}}$

segno

$$\text{th}\,x \geq 0 \iff \frac{e^x - e^{-x}}{e^x + e^{-x}} \geq 0 \qquad e^x - e^{-x} \geq 0$$

$e^x + e^{-x} > 0 \ \forall x$

$x \geq 0$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} \leq \frac{\cancel{e^x + e^{-x}}}{\cancel{e^x + e^{-x}}} = 1 \qquad \text{th}\,x \leq 1 \ \forall x \in \mathbb{R}$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} \geq -e^x - e^{-x} = \frac{-(\cancel{e^x + e^{-x}})}{\cancel{e^x + e^{-x}}}$$

$$\text{th}\, x \geq -1 \quad \forall x$$

$$-1 \leq \text{th}\, x \leq 1 \quad \forall x \in \mathbb{R}$$

è una funzione limitata

es $f(x) = \sqrt[4]{\dfrac{1}{tg\,x}}$

D, segno, simmetrie

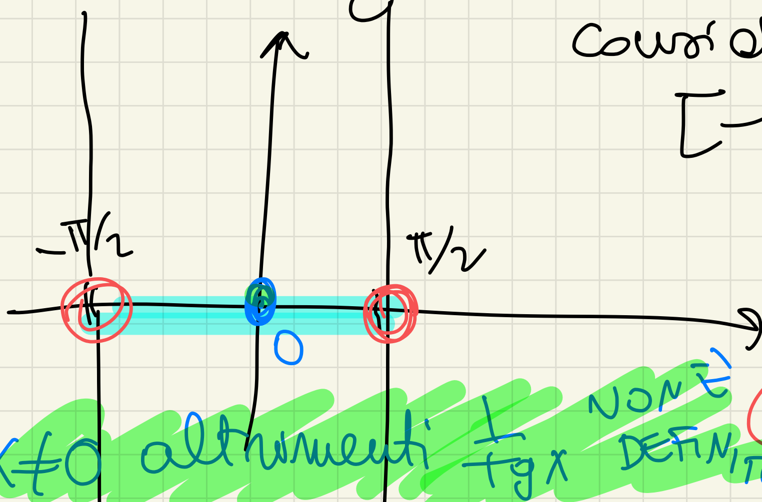
$\dfrac{1}{tg\,x} > 0$ altrimenti radice non è def

$tg\,x \neq 0$ $\dfrac{1}{tg\,x}$ non è def.

$tg\,x$ $x \neq \dfrac{\pi}{2} + k\pi$ altrimenti tangente non è definita

$$f(x+\pi) = \sqrt[4]{\frac{1}{tg(x+\pi)}} = \sqrt[4]{\frac{1}{tg\,x}} = f(x)$$

f è PERIODICA di periodo π come la tangente



consideriamo solo intervallo $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ e studiamo

DOMINIO

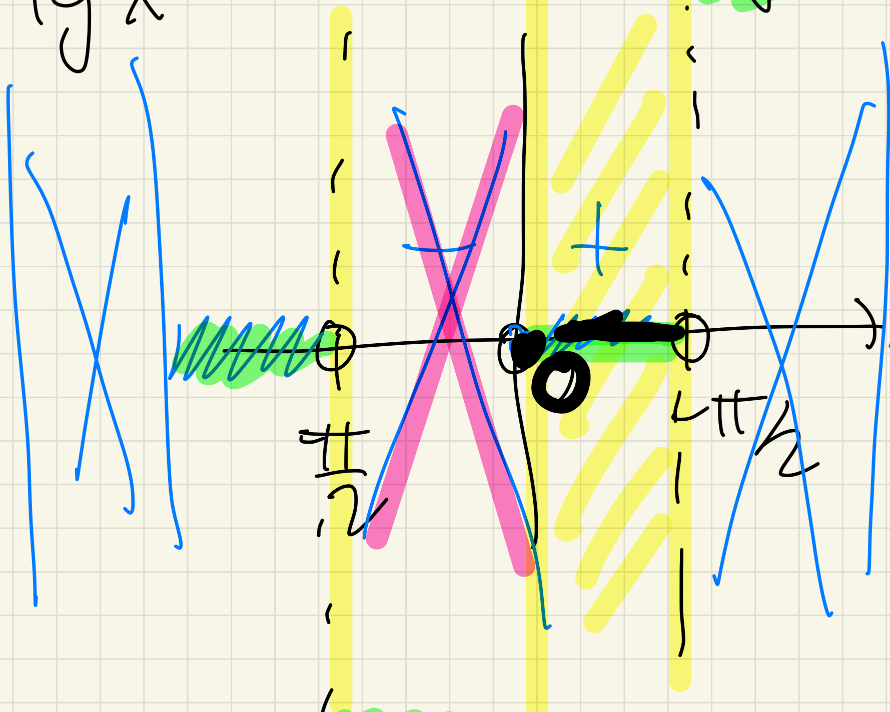
~~$tg \geq 0$~~

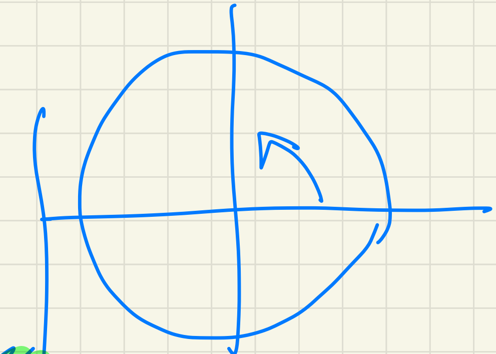
$x \neq \frac{\pi}{2}$, $x \neq -\frac{\pi}{2}$

(altrimenti tangente NON È DEFINITA!

$x \neq 0$ altrimenti $\frac{1}{tg\,x}$ NON È DEFINITO

$$\frac{1}{\operatorname{tg} x} > 0 \implies \operatorname{tg} x > 0$$





$\operatorname{tg} x > 0$

$x \in \left(0, \frac{\pi}{2}\right)$

$\operatorname{tg} x < 0$

$x \in \left(-\frac{\pi}{2}, 0\right)$

DOMINIO $\left(0, \frac{\pi}{2}\right) + k\pi$

NON È SIMM.

$f(x) > 0 \ \forall x \in D.$

Es $f(x) = \log\left(\underline{x - \sqrt{x} + 2}\right)$

DOMINIO

$$\boxed{\underline{x - \sqrt{x} + 2 > 0}}$$ altrimenti logaritmo non è def.

$\sqrt{x}$ è ben definito $\Longleftrightarrow$ $\boxed{x \geq 0}$

(sicuram. f NON HA SIMMETRIE)

$x \geq 0$

$$\boxed{x - \sqrt{x} + 2 > 0}$$

$$x = (\sqrt{x})^2$$

$$y = \sqrt{x}$$

$$y^2 - y + 2 > 0$$

$$\forall y \in \mathbb{R}$$

$$y^2 - y + 2 = 0$$

$$y_{12} = \frac{1 \pm \sqrt{1-8}}{2}$$

$$\Rightarrow D = \{x \geq 0\} = [0, +\infty)$$

segno

$$f(x) \geq 0$$

$$\lg\left(x-\sqrt{x}+2\right) \geq 0 = \lg 1$$

$$x-\sqrt{x}+2 \geq 1$$

$$x-\sqrt{x}+1 \geq 0$$

$y=\sqrt{x}$

$y^2-y+1 \geq 0$

$y = \sqrt{x}$

$y^2 - y + 1 \geq 0$

$$y = \frac{1 \pm \sqrt{1-4}}{2}$$

$\Delta < 0$

$\downarrow$

$\forall y$

$f(x) \geq 0 \quad \forall x \in D$