Cantor set in $\mathbb{R}$.

$$C = \bigcap C_n$$

$$C_0 = [0,1] \qquad C_1 = \left[0,\tfrac{1}{3}\right] \cup \left[\tfrac{1}{3}, 1\right]$$

$$C_{n+1} = \frac{1}{3} C_n \cup \left(\frac{1}{3} C_n + \frac{2}{3}\right)$$

C_n is a closed set which contains 2^n intervals of length $\frac{1}{3^n}$. (distance between intervals at most $\frac{1}{3^n}$)

take $\delta = \frac{1}{3^n}$ $\quad s>0 \Rightarrow C_n$ is a cover of C

$$\mathcal{H}^s_\delta(C) \leq \frac{\omega_s}{2^s} \sum_i \left(\operatorname{diam} C_n^i\right)^s = \frac{\omega_s}{2^s} \cdot \underbrace{\left(\frac{1}{3^n}\right)^s}_{\text{length}} \cdot \overbrace{2^n}^{\text{number}}$$

$$\mathcal{H}^s_{\left(\frac{1}{3}\right)^n}(C) \leq \frac{\omega_s}{2^s}\left(\frac{2}{3^s}\right)^n$$

letting $\delta \to 0$ $\left(\delta = \frac{1}{3^n} \to 0 \text{ as } n \to +\infty\right)$

$$\boxed{\text{if } \frac{2}{3^s} < 1 \Rightarrow \left(\frac{2}{3^s}\right)^n \to 0 \quad \mathcal{H}^s(C) = 0 \quad s > \log_3 2}$$

$$\boxed{\text{So } \mathcal{H}\text{-dim}(C) \leq \log_3 2}$$

$$\mathcal{H}^{\log_3 2}(C) \leq \frac{\omega_{\log_3 2}}{\cancel{2}\,3}$$

take a cover E_i $\quad$ $\text{diam } E_i \leq \delta$ $\quad$ $\cup E_i \supseteq C$

$\boxed{\text{take } s = \log_3 2}$ $\quad$ $2 = 3^s$

we can take E_i open and since C is compact and $C \subseteq \cup_i E_i$, we may pass to a finite cover

assume $\frac{1}{3^{n+1}} \leq \text{diam } E_i \leq \frac{1}{3^n}$ for some n

① $\# E_i \geq 2^n$ (each E_i intersects at most 1 ~~set~~ interval in C_n)

② $k > n$ $\quad$ each E_i intersects at most 2^{k-n} intervals in C_k

$$2^{k-n} = 2^k 3^{-sn} \leq 2^k 3^s (\text{diam } E_i)^s$$

Sum over all E_i $\Longrightarrow$ $2^k \leq 2^k 3^s \sum_i (\text{diam } E_i)^s$

$\Rightarrow \sum_i (\operatorname{diam} E_i)^s \geq 3^{-s}$ $\quad$ E_i cover such that

$C \subseteq \bigcup_i E_i$

$s = \log_3 2$

$\frac{1}{3^{n+1}} \leq \operatorname{diam} E_i \leq \frac{1}{3^n}$

$\Downarrow$

So $\quad \mathcal{H}^s(C) \geq \frac{3^{-s}\,\omega_s}{2^s}$