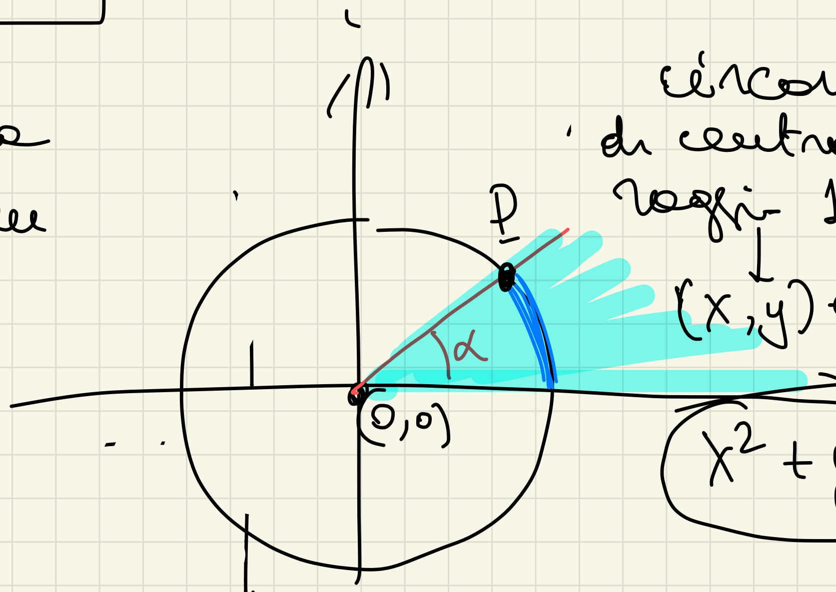
· TUTORATO

· lun 2 ore 10.30 - 12.05
merc 2 ore 10.30 - 12.05
gio 2 ore 10.30 - 12.05
ve 4 ore 10.30 - 13.50 / 13.55

TRIGONOMETRIA

$\alpha \longrightarrow$ identifica nella circonf un pto $\underline{P}$

$$\underline{P} = (x_P, y_P)$$



circonferenza di centro $(0,0)$ e raggio 1

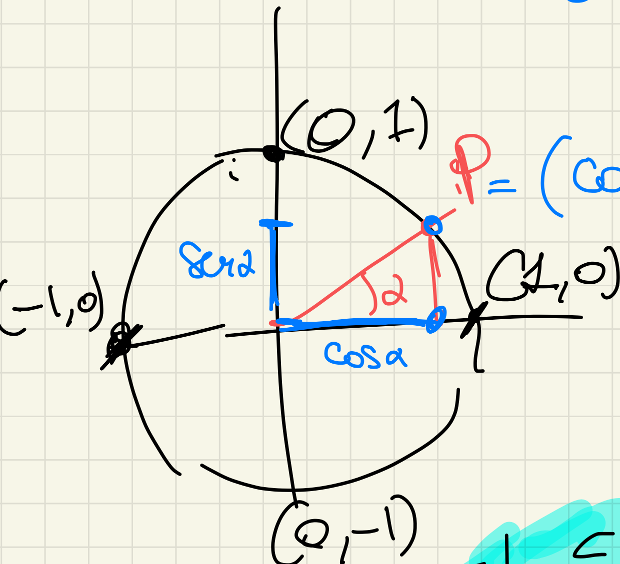
$$\downarrow$$

$$(x, y) \in \mathbb{R}^2$$

$$x^2 + y^2 = 1$$

Chiamo $\cos\alpha = x_P$ coordinata x del
COSENO di α pto P.

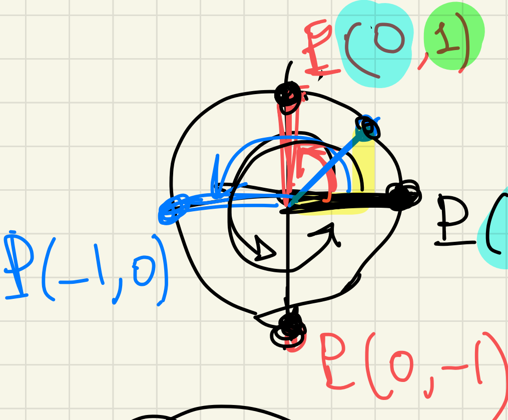
$\sin\alpha = y_P$ coordinata y del
SENO di α punto P.



All'angolo α associo 2 numeri $\cos\alpha$, $\sin\alpha$ (coordinate del punto)

$-1 \leq \cos\alpha \leq +1$ $-1 \leq \sin\alpha \leq 1$

$(\cos\alpha)^2 + (\sin\alpha)^2 = 1$



$\alpha = 0$, $\alpha = 2\pi$

$\cos 0 = \cos 2\pi = 1$

$\sin 0 = \sin 2\pi = 0$

$\alpha = \frac{\pi}{2}$

$\sin \frac{\pi}{2} = 1$ $\cos \frac{\pi}{2} = 0$

$\alpha = \pi$

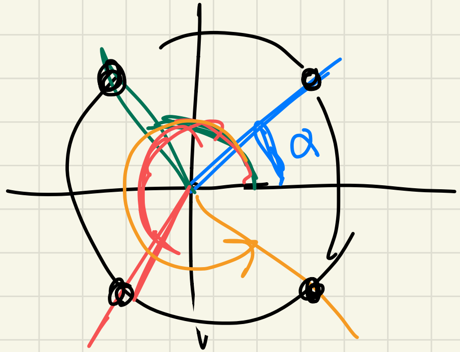
$\sin \pi = 0$ $\cos \pi = -1$

$\alpha = \frac{3}{2}\pi$

$\sin \frac{3}{2}\pi = -1$ $\cos \frac{3}{2}\pi = 0$

$\alpha = \frac{\pi}{4}$ $\cos \frac{\pi}{4} = \sin \frac{\pi}{4}$ $\cos^2 \frac{\pi}{4} = \sin^2 \frac{\pi}{4} = \frac{1}{2}$

$\cos \pi/4 = \sin \pi/4 = 1/\sqrt{2}$



$$0 < \alpha < \frac{\pi}{2} \longrightarrow \sin\alpha, \cos\alpha > 0$$

$$\frac{\pi}{2} < \alpha < \pi \longrightarrow \cos\alpha < 0$$

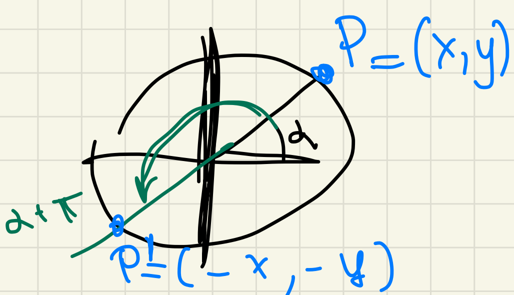
$$\sin\alpha > 0$$

$$\pi < \alpha < \frac{3}{2}\pi \longrightarrow \cos\alpha < 0$$

$$\sin\alpha < 0$$

$$\frac{3}{2}\pi < \alpha < 2\pi \longrightarrow \cos\alpha > 0$$

$$\sin\alpha < 0$$

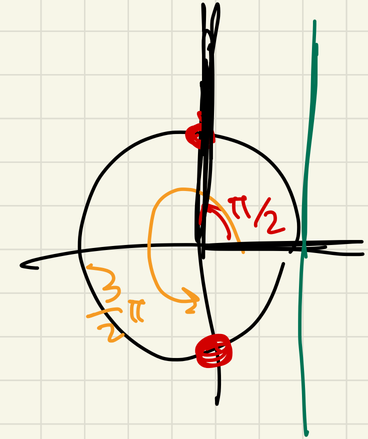


$$\cos(\alpha + \pi) = -\cos\alpha$$

$$\sin(\alpha + \pi) = -\sin\alpha$$

$$\alpha \longrightarrow \begin{array}{l} \cos\alpha \in [-1,1] \\ \sin\alpha \in [-1,1] \end{array}$$

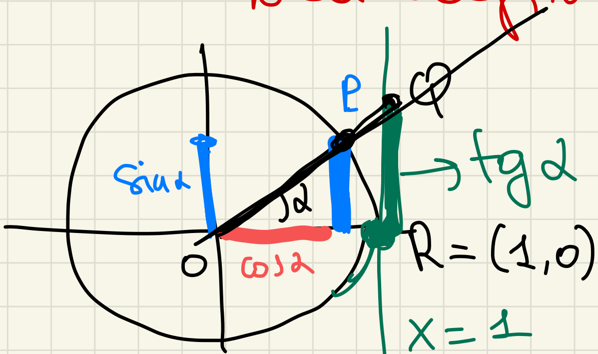
$$(\sin\alpha)^2 + (\cos\alpha)^2 = 1$$



$$\operatorname{tg}\alpha = \text{tangente di } \alpha = \frac{\sin\alpha}{\cos\alpha}$$

ben definito solo se $\alpha \neq \frac{\pi}{2}$

$\alpha \neq \frac{3}{2}\pi$



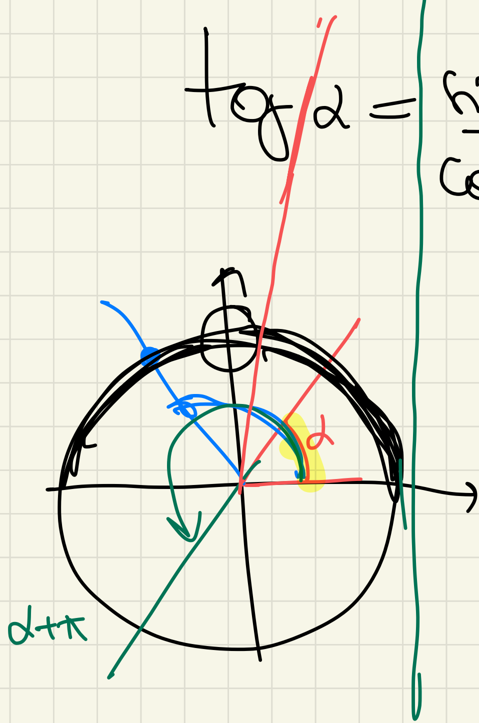
$\operatorname{tg}\alpha = |RQ|$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$\alpha \neq \frac{\pi}{2}$

$\alpha \neq \frac{3}{2}\pi$

$\operatorname{tg}\alpha \in \mathbb{R}$



$0 < \alpha < \frac{\pi}{2}$ $\quad \operatorname{tg}\alpha > 0$

$\frac{\pi}{2} < \alpha < \pi$ $\quad \operatorname{tg}\alpha < 0$

$$\operatorname{tg}0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0 = \operatorname{tg}2\pi$$

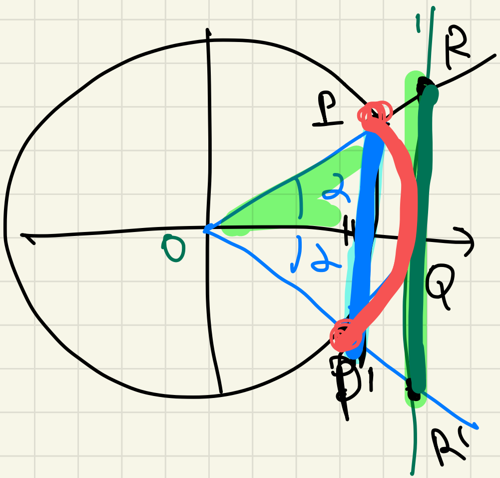
$$\operatorname{tg}\pi = \frac{\sin\pi}{\cos\pi} = \frac{0}{-1} = 0$$

$$\operatorname{tg}(\alpha+\pi) = \frac{\sin(\alpha+\pi)}{\cos(\alpha+\pi)} = \frac{\cancel{-}\sin\alpha}{\cancel{-}\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha$$

due disuguaglianze importanti

$0 < \alpha < \frac{\pi}{2}$ $\quad (\sin\alpha > 0, \; \cos\alpha > 0, \; \operatorname{tg}\alpha > 0)$

$P = (\cos\alpha, \sin\alpha)$



$|\overline{QR}| = \operatorname{tg}\alpha$

$O\hat{P}H \cong O\hat{P'}H$

$O\hat{Q}R \cong O\hat{Q}R'$

$|PP'| = 2\sin\alpha$

$\boxed{|RR'| = 2\operatorname{tg}\alpha}$

l'arco di circonferenza tra P e P' è lungo 2α

$\cancel{2}\sin\alpha \le \cancel{2}\alpha$

$$\boxed{\frac{\sin\alpha}{\alpha} \le 1 \quad 0 < \alpha < \frac{\pi}{2}}$$

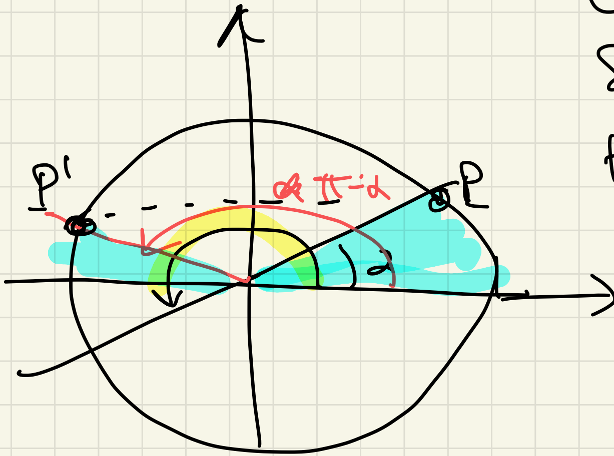
$$\underbrace{\cancel{2}\alpha}_{\text{arco tra } P \text{ e } P'} \le \cancel{2}\underbrace{\operatorname{tg}\alpha}_{\text{segmento } |RR'|}$$

$$\boxed{0<\alpha<\frac{\pi}{2} \qquad \cos\alpha \le \frac{\sin\alpha}{\alpha} \le 1}$$

$$\alpha \le \frac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha$$

$$\frac{\cos\alpha}{\cancel{\alpha}}\,\cancel{\alpha} \le \frac{\sin\alpha}{\cancel{\cos\alpha}} \cdot \frac{\cancel{\cos\alpha}}{\alpha}$$

$$\cos\alpha \le \frac{\sin\alpha}{\alpha} \qquad 0<\alpha<\frac{\pi}{2}$$



$$\cos(\alpha+\pi) = -\cos\alpha$$
$$\sin(\alpha+\pi) = -\sin\alpha$$
$$\operatorname{tg}(\alpha+\pi) = \operatorname{tg}\alpha$$

$$\cos(\pi-\alpha) = -\cos\alpha$$
$$\sin(\pi-\alpha) = \sin\alpha$$
$$\operatorname{tg}(\pi-\alpha) = -\operatorname{tg}\alpha$$

$$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\operatorname{tg}\left(\frac{\pi}{4}\right) = \frac{\sin \pi/4}{\cos \pi/4} = 1$$

Es trovare gli angoli che soddisfano

$$\boxed{\sqrt{1-(\sin\alpha)^2} \geq \frac{1}{\sqrt{2}}}$$

$$x^2 - 1 \overset{?}{\leq} 0$$

$$\Updownarrow$$

$$-1 \leq x \leq 1$$

$$+1 - (\sin\alpha)^2 \geq 0 \;?$$

$$+(\sin\alpha)^2 - 1 \leq 0$$

$$-1 \leq \sin\alpha \leq 1$$

sempre vero

$$\left(\sqrt{1-(\sin\alpha)^2}\right)^2 \geq \left(\frac{1}{\sqrt{2}}\right)^2$$

$$1-(\sin\alpha)^2 \geq \frac{1}{2}$$

$$1-(\sin\alpha)^2-\frac{1}{2} \geq 0$$

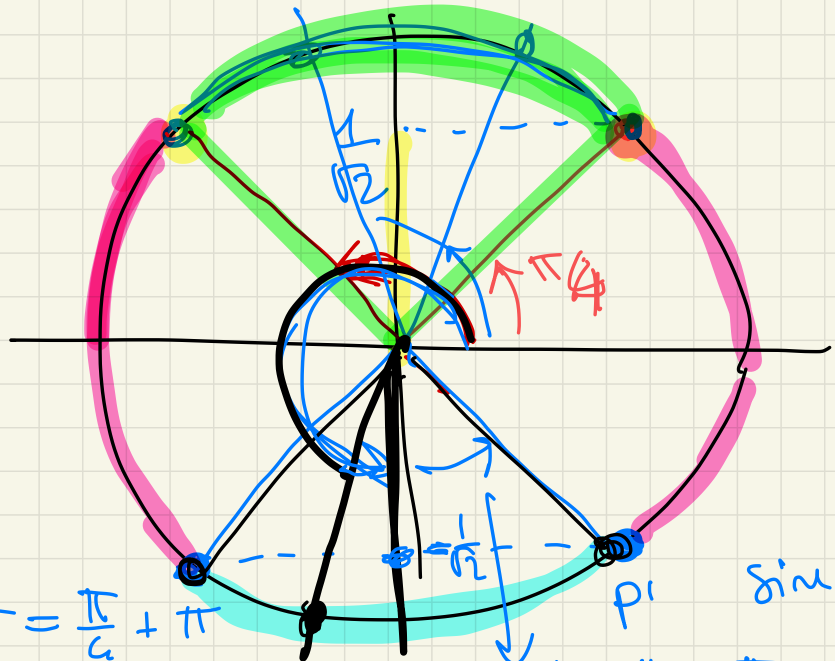
$$-\frac{1}{2}-(\sin\alpha)^2 \geq 0$$

$$+(\sin\alpha)^2-\frac{1}{2} \leq 0$$

$$x^2-\frac{1}{2} \leq 0$$

$$-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$-\frac{1}{\sqrt{2}} \leq \sin\alpha \leq \frac{1}{\sqrt{2}}$$



$$-\frac{1}{\sqrt{2}} \le \sin\alpha \le \frac{1}{\sqrt{2}}$$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sin\frac{3}{4}\pi = \frac{1}{\sqrt{2}}$$

$$\frac{5}{4}\pi = \frac{\pi}{4} + \pi$$

$$\sin\left(\frac{5}{4}\pi\right) = -\frac{1}{\sqrt{2}} = -\sin\frac{\pi}{4} = \sin\frac{7}{4}\pi$$

$$\pi + \frac{\pi}{2} + \frac{\pi}{4} = \frac{7}{4}\pi$$

$$\frac{3}{4}\pi = \frac{\pi}{2} + \frac{\pi}{4}$$

$\frac{\pi}{4} < \alpha < \frac{3}{4}\pi \rightarrow$ $\sin\alpha > \frac{1}{\sqrt{2}}$

$\frac{5}{4}\pi < \alpha < \frac{7}{4}\pi \rightarrow$ $\sin\alpha < -\frac{1}{\sqrt{2}}$

NON LI VOGLIO

Sol. senō

$$0 \leq \alpha \leq \frac{\pi}{4}$$

$$\frac{3}{4}\pi \leq \alpha \leq \frac{5}{4}\pi$$

$$\frac{7}{4}\pi \leq \alpha \leq 2\pi \ .$$

FUNZIONE reale di variabile reale

$$f : \boxed{A \subseteq \mathbb{R}} \longrightarrow B$$

$\boxed{A \subseteq \mathbb{R}}$ $\quad B \subseteq \mathbb{R}$

f si chiama funzione se è una CORRISPONDENZA che associa a ogni elemento di A UNO E UN SOLO ELEMENTO di B.

$x \in \mathbb{R}$ positivo $\longrightarrow \sqrt[2]{x} = y$

(per def. y è quel numero POSITIVO che elevato al quadrato dà x)

$4 \longrightarrow 2$

$x \in \mathbb{R} \longrightarrow \sqrt[3]{x}$

$f : A \subseteq \mathbb{R} \to \mathbb{R}$

Chiamiamo DOMINIO NATURALE di f,

il più grande (*tolto*) insieme di $\mathbb{R}$ più grande dove sia

possibile definire la nostra funzione

DOMINIO NATURALE

$[0, +\infty) = \{x \in \mathbb{R} \ \ x \geq 0\}$

$$\underbrace{x \longmapsto \sqrt[2]{x}}$$

insieme di numeri reali x maggiore possibile di cui abbia senso definire RADICE QUADRATA? $\longrightarrow$ NUMERI POSITIVI $x \geq 0$

$$x \longrightarrow \sqrt[3]{x}$$

DOMINIO NATURALE è $\mathbb{R} = (-\infty, +\infty)$

$\forall x \in \mathbb{R} \ \exists \sqrt[3]{x}$

D dominio naturale della funzione f

$D \subseteq \mathbb{R}$

Immagine di $f = \{ y \in \mathbb{R}$ tali che esiste $x \in D$ con $f(x) = y \}$

Sol.

$$\boxed{|x-1| < |2x-3|}$$

$|A| < B$

$A > B$ oppure $|A| > B$

$$\begin{cases} x-1 < |2x-3| \\ x-1 > -|2x-3| \\ -x+1 < |2x-3| \end{cases}$$

$\longrightarrow$ $|2x-3| > x-1$

$\longrightarrow$ $|2x-3| > -x+1$

$$\begin{cases} 2x-3 > x-1 \text{ oppure } 2x-3 < -(x-1) \\ 2x-3 > -x+1 \text{ oppure } 2x-3 < -(-x+1) \end{cases}$$

$$2x - 3 > x - 1$$

$$2x - \cancel{x} > 3 - 1$$

$$x > 2$$

oppure

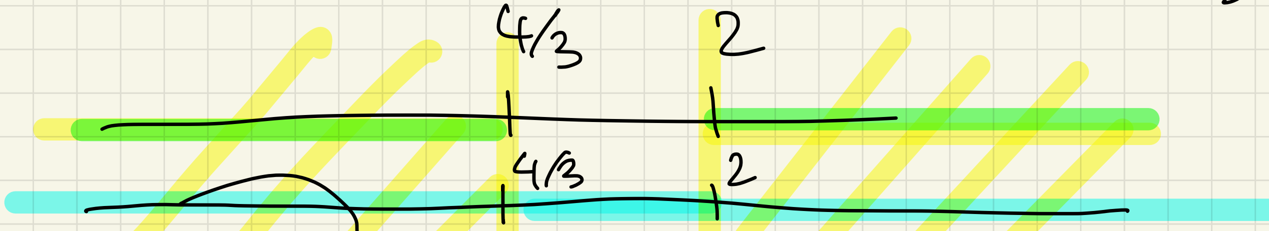
$$2x - 3 < -(x - 1)$$

$$2x - 3 < -x + 1$$

$$2x + x < 3 + 1$$

$$3x < 4$$

$$x < \frac{4}{3}$$



$$2x - 3 > -x + 1$$

$$2x + x > 3 + 1$$

$$3x > 4$$

$$x > 4/3$$

oppure

oppure

$$2x - 3 < -(-x + 1)$$

$$2x - 3 < x - 1$$

$$2x - x < 3 - 1$$

$$x < 2$$