ICEA

Università degli Studi di Padova

Lesson 3:
ESTIMATION OF THE TURBULENT DIFFUSION COEFFICIENTS

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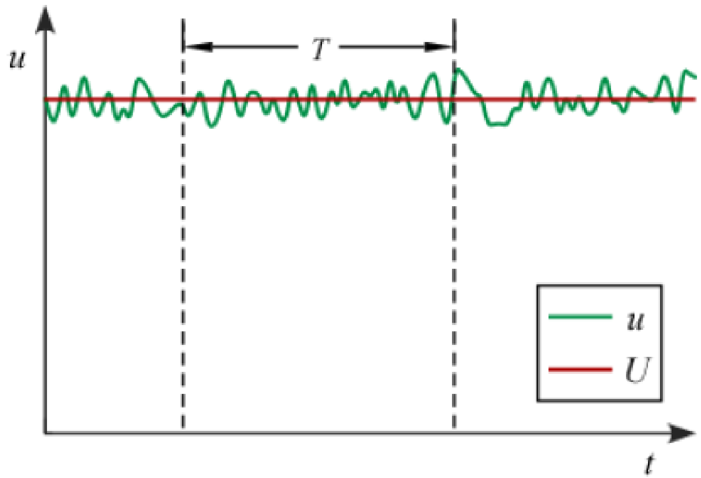
In presence of turbulent velocity field, the velocity $\boldsymbol{u}$ can be expressed as proposed by Reynolds:

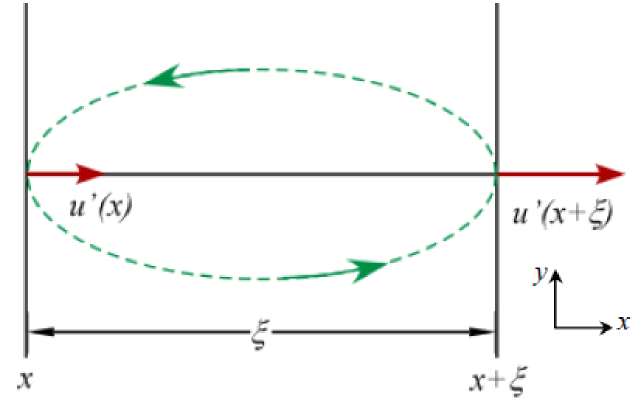
$$\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}'$$



$\boldsymbol{U} =< \boldsymbol{u} >$ mean velocity $\Rightarrow U = \frac{1}{T}\int_0^T u\, dt$ `mean(u);` or `trapz(t,u)/T;`

$\boldsymbol{u}'$ velocity fluctuations $\Rightarrow \boldsymbol{u}' = \boldsymbol{u} -< \boldsymbol{u} >$

The vortex affects the magnitude of the velocity fluctuation



To understand the time evolution of the vortexes, we define the *Eulerian Temporal Correlation Coefficient* as follows:

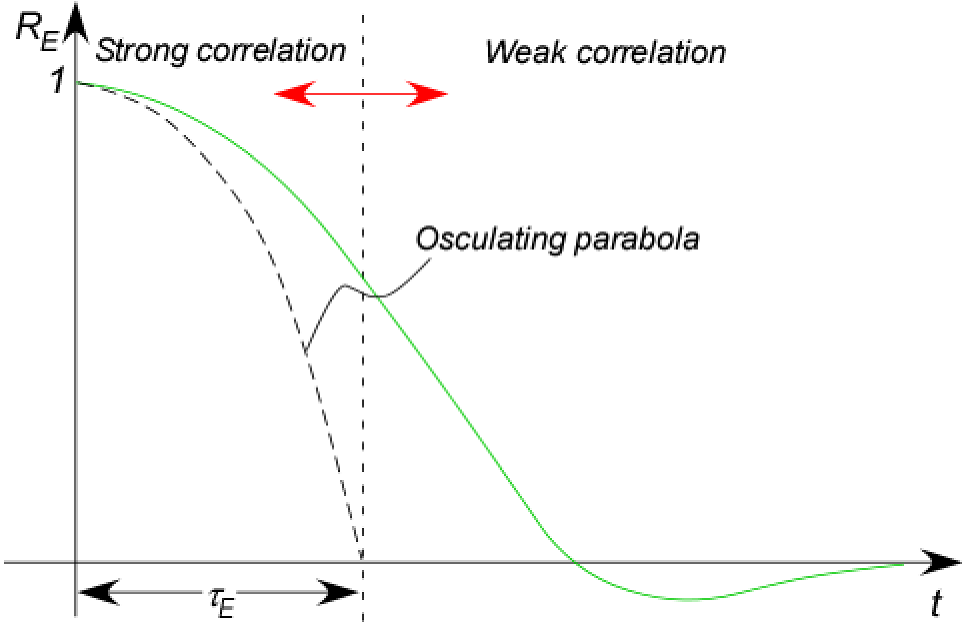
$$R_E(x,t,\tau) = \frac{< u'(x,t)u'(x,t+\tau) >}{\sqrt{< u'^2(x,t) >}\sqrt{< u'^2(x,t+\tau) >}}$$

If the process is *ergodic*:

$$\Rightarrow R_E\ (x,\tau) = \frac{< u'(x)u'(x,\tau) >}{\sqrt{< u'^2(x) >}\sqrt{< u'^2(x,\tau) >}}$$

The correlation function has the following properties:

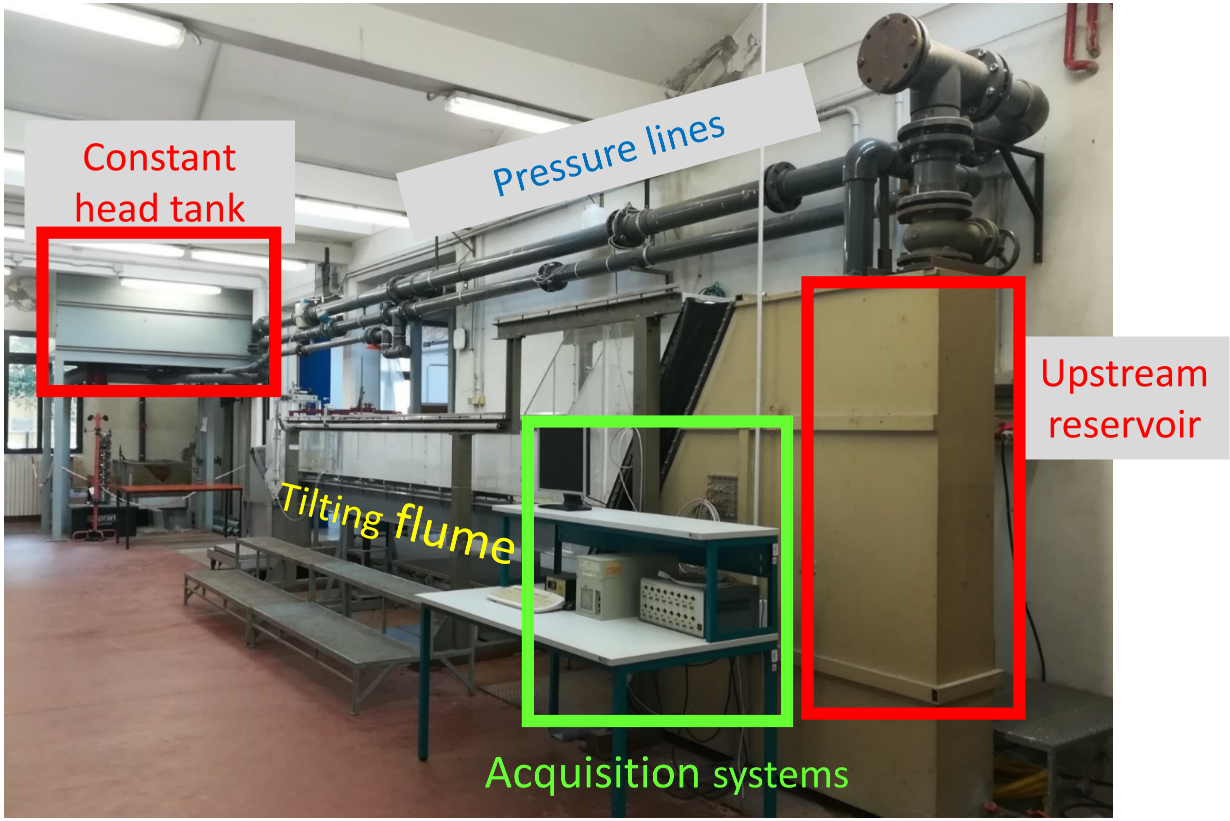
$$\begin{cases} R_E(0) = 1 \\ R_E(t) = R_E(-t) \\ R_E(t) \leq 1 \end{cases}$$



$T_{E_i} = \int_0^\infty R_E(\tau)d\tau$ is the time scale of macrovortex in a selected direction i

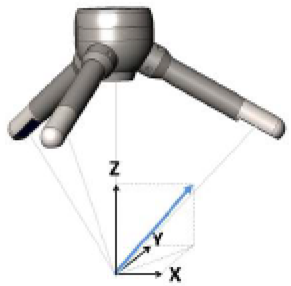
$e_i = < u_i'^2 > T_{E_i}$ is the diffusion coefficient for $t \gg T_{E_i}$

Experiment: Set-up

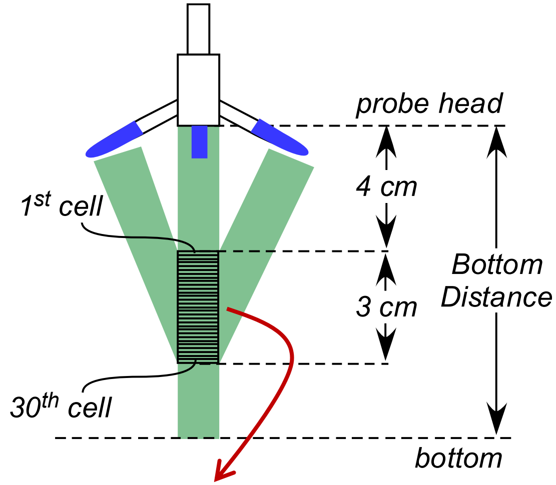


Q [L/s]	Water depth [cm]	Slope [%]
3.0	12	0.1
5.0	13	0.1

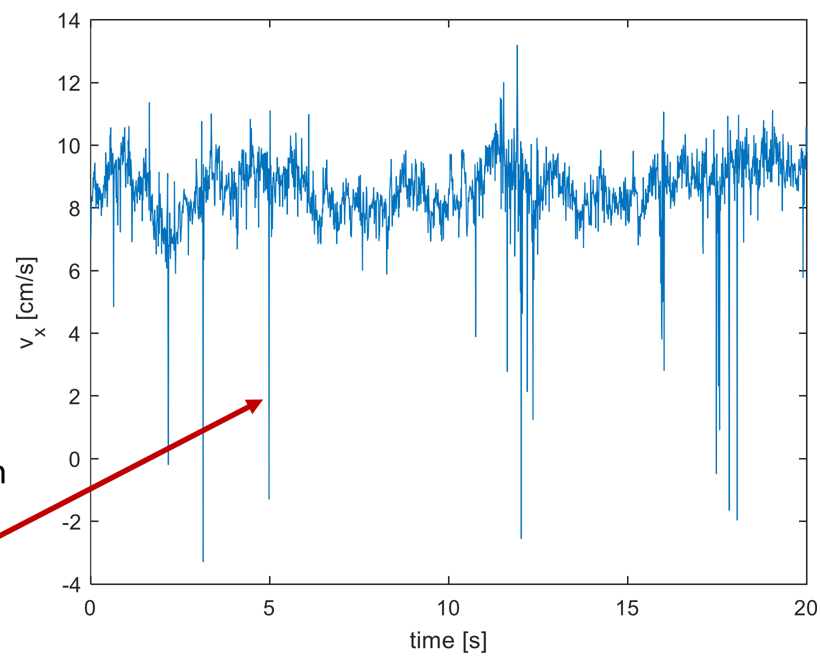
Experiment: ADV



The blue arrow indicates a positive velocity



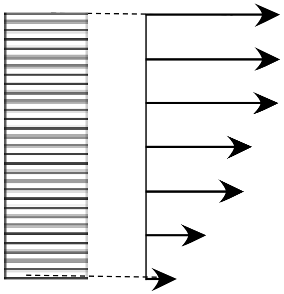
30 cells in the sampling volume



Spikes neglected in the post-process thanks to the statistical analysis

Post-processing

Velocity profile



Goal #1:

1. Compute the mean velocities, the fluctuations, and the standard deviation in the 3 directions
2. Compute the mean velocity profile along the vertical direction

Goal #2:

3. Compute the coefficient of temporal autocorrelation ($R_E(x, \tau)$) in at least one cell
4. Compute the time scale of the macrovortex (T_{E_i})

Goal #3:

5. Compute the diffusion coefficient (e_i)
6. Compare e_i with the value provided by the literature, e.g., Reynolds analogy theory

MATLAB goal #1

1. Load the velocity data
2. Extract the velocities (`Vx = Data.Profiles_VelX, Vy = Data.Profiles_VelY, Vz = Data.Profiles_VelZ`) (in the file the velocities are in [m/s], attention to the units!)
3. Extract the distance between the bottom and the head of the sensor (`Data.BottomCheck_BottomDistance`) (consider the mean value, in the file it is in [m])
4. Define the vector of cells' position to be used to plot the mean velocity (remember the definition of the ADV and how it measures the distances: 30 cells in 3 cm of sampling volume)

For each cell:

5. Compute the 3 mean velocity components (`mean()`)
6. Compute the fluctuations for each direction (remember the formula!)
7. Compute the standard deviation for the 3 velocities ($SD = \sqrt{< {u'_i}^2 >} = \sqrt{\frac{\sum_{i=1}^{N} u'_i \cdot u'_i}{N}}$, hint: use the matrix operation or for cycle)

For the longitudinal direction:

8. Plot the mean velocity

For 1 or more cells:

1. Load the velocity data (v_x, v_y, v_z)
2. Define the time of measurements: $f = 100\ Hz,\ T_{end} = \Delta t \cdot N$
3. Create the time vector (`linspace(…)`)
4. Divide the time vector in two: the first half is used for the analysis (we will see later why we need to divide the data in two vectors!)

For each velocity vector:

5. Compute the mean velocity (`mean()` or `trapz(t,v)/T`)
6. Compute the fluctuations (or extract from the vectors computed for *goal #1*)
7. Compute and plot $R_E(x,\tau) = \frac{<u'(x)u'(x,\tau)>}{\sqrt{<u'^2(t)>}\sqrt{<u'^2(x,\tau)>}}$
8. Compute $T_E = \int_0^\infty R_E(\tau)d\tau$

MATLAB goal #2

7. Compute $\boxed{R_E(x,t,\tau) = \frac{<u'(x,t)u'(x,t+\tau)>}{\sqrt{<u'^2(x,t)>}\sqrt{<u'^2(x,t+\tau)>}}}$

$< \boxed{\boldsymbol{u'(x)}}\boxed{\boldsymbol{u'(x,\tau)}} >$ *Ergodic process* (remember that usually **x** is fixed!)

`u'x,y,z(i)` where *i* defines the time

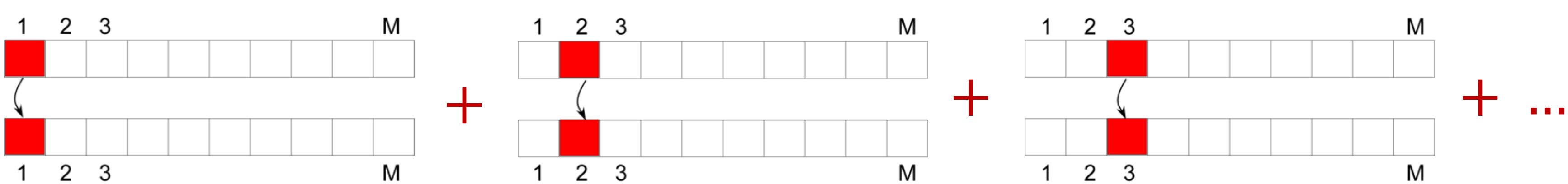
`u'x,y,z(i+j)` where *j* resembles $\tau = 0, \tau = \Delta t, \tau = 2\Delta t, \ldots., \tau = \boxed{\mathrm{M}}\Delta t$

Remember that you're using half of the data!

$< u'(x)u'(x,\tau) > \Rightarrow < \cdots >$ remember that it means the time averaged value!

Where:

$\boldsymbol{\tau = 0 \Rightarrow < u'(x)u'(x) > =}$



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M

7. Compute $\boxed{R_E(x,t,\tau) = \frac{<u'(x,t)u'(x,t+\tau)>}{\sqrt{<u'^2(x,t)>}\sqrt{<u'^2(x,t+\tau)>}}}$

$< \boxed{\boldsymbol{u'(x)}}\boxed{\boldsymbol{u'(x,\tau)}} >$ *Ergodic process* (remember that usually **x** is fixed!)

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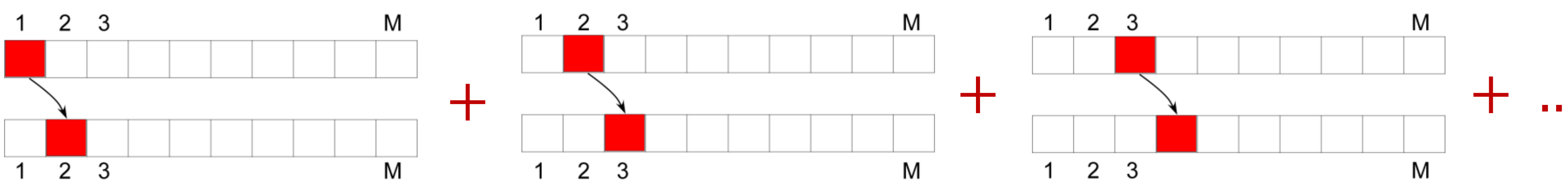
Remember that you're using half of the data!

$< u'(x)u'(x,\tau) > \Rightarrow < \cdots >$ remember that it means the time averaged value!

Where:

$\boldsymbol{\tau = \Delta t \Rightarrow < u'(x)u'(x+\Delta t) > =}$



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7. Compute $\boxed{R_E(x,t,\tau) = \frac{<u'(x,t)u'(x,t+\tau)>}{\sqrt{<u'^2(x,t)>}\sqrt{<u'^2(x,t+\tau)>}}}$

$$< \boxed{\boldsymbol{u'(x)}}\boxed{\boldsymbol{u'(x,\tau)}} >$$

Ergodic process (remember that usually **x** is fixed!)

`u'x,y,z(i)`
where *i* defines the time

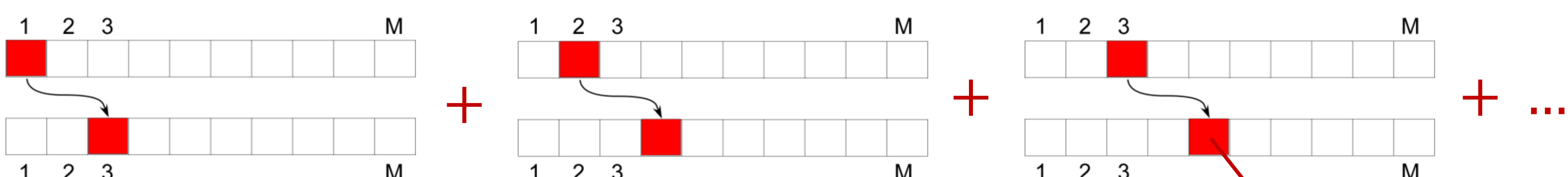
`u'x,y,z(i+j)`
where *j* resembles $\tau = 0, \tau = \Delta t, \tau = 2\Delta t, \ldots., \tau = \boxed{M}\Delta t$

Remember that you're using half of the data!

$< u'(x)u'(x,\tau) > \Rightarrow < \cdots >$ remember that it means the time averaged value!

Where:

$\boldsymbol{\tau = 2\Delta t \Rightarrow < u'(x)u'(x + 2\Delta t) > =}$



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That's why we need to divide the data in two vectors!

7. Compute $\boxed{R_E(x,t,\tau) = \frac{<u'(x,t)u'(x,t+\tau)>}{\sqrt{<u'^2(x,t)>}\sqrt{<u'^2(x,t+\tau)>}}}$

$< \boxed{\boldsymbol{u}'(\boldsymbol{x})}\boxed{\boldsymbol{u}'(\boldsymbol{x},\boldsymbol{\tau})} >$ *Ergodic process* (remember that usually **x** is fixed!)

`u'`$_{\texttt{x,y,z}}$`(i)`
where *i* defines the time

`u'`$_{\texttt{x,y,z}}$`(i+j)`
where *j* resembles $\tau = 0, \tau = \Delta t, \tau = 2\Delta t, \ldots., \tau = \boxed{\mathrm{M}}\Delta t$

Remember that you're using half of the data!

$< u'(x)u'(x,\tau) > \Rightarrow < \cdots >$ remember that it means the time averaged value!

Thus, the numerator is computed as:

$$\Rightarrow < \boldsymbol{u}'(\boldsymbol{i})\boldsymbol{u}'(\boldsymbol{i}+\boldsymbol{j}) > = \frac{\mathbf{1}}{\boldsymbol{M}}\sum_{\boldsymbol{j}}\sum_{\boldsymbol{i}}\boldsymbol{u}'(\boldsymbol{i})\cdot\boldsymbol{u}'(\boldsymbol{i}+\boldsymbol{j})$$

For the denominator, the same methodology has to be applied! (Pay attention that there is the square root and the square of $\boldsymbol{u}'$ in the formula!)

7. Compute $\boxed{R_E(x,t,\tau) = \frac{<u'(x,t)u'(x,t+\tau)>}{\sqrt{<u'^2(x,t)>}\sqrt{<u'^2(x,t+\tau)>}}}$

Hint:

1. $< \mathbf{u'(i)u'(i+j)} > = \frac{\mathbf{1}}{\mathbf{M}}\sum_{\mathbf{j}} \sum_{\mathbf{i}} \mathbf{u'(i)} \cdot \mathbf{u'(i+j)}$: use two for loops

```
For j= …
    for i= …

    end
End
```

2. $\frac{\boxed{<\mathbf{u'(x,t)u'(x,t+\tau)}>}}{\boxed{\sqrt{<\mathbf{u'^2(x,t)}>}}\boxed{\sqrt{<\mathbf{u'^2(x,t+\tau)}>}}}$: create three temporary variables (1, 2, and 3)

8. Compute $\boxed{T_E = \int_0^{\infty} R_E(\tau)d\tau}$

Hint:

1. Once you computed R_E, T_E is simply the time integral (pay attention to use the correct time vector! Where is define R_E?)

MATLAB goal #3

For 1 or more cells:

1. You already know how to compute $< {u_i'}^2 >$
2. $e_i =< {u_i'}^2 > T_{E_i}$ (T_{E_i} computed in *goal #2*)
3. Compare e_i with the value provided by the literature, e.g., Reynolds analogy theory