ICEA

UNIVERSITÀ DEGLI STUDI DI PADOVA

# LESSON 16: EXERCISES

Through the models that take into account the transport and the reaction of pollutants we can determine the concentration of $DO$ along watercourses. The aim is to ensure the respect of the limits prescribed by the regulations.

Just to remind:
- $\bar{O} > 5$ mg/l
- $O > 4$ mg/l

Usually the maximum $O_c$ occurs in summertime at sunrise because the water is warmer, hence the saturated oxigen concentration $O_s$ is the minimum, and the minimum difference $P - R$ persists for the whole night. However problem of concentration of $DO$ occurs also in winter if the freesurface freezes, i.e. $r_a \rightarrow 0$.

Of course the deficit of $DO$ depends also on the hydrodynamic conditions of ther river. To assess the river condition we have to use the hydraulic regime in which $\mathrm{Q}_{min}$ has return period of 10 years and duration of 1 week.

Generally the river paths is rather complex; we can find:

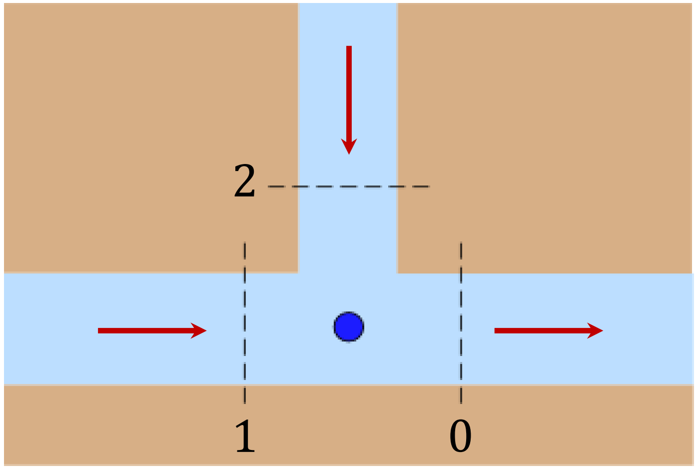
- Tributaries
- Emissaries
- Hydraulic structures regulating the flow rate and the channel level
- Geometric variation of the riverbed
- Change of slope or, in general all the elements that affect the hydraulic regime
- Variation of Biomass
- Composition of sediments

They change the parameters $R$, $P$ and $S$

To correctly apply the analitical solutions provided we need to divide the river path in several segments within the assumption of homogeneity (and stationarity) is valid!
For instance, we can apply the S-P model for each reach. The final condition of the upstream tract is used to determined the initial condition for the downstream segment.

Let's see some examples.

# EXERCISE 1: CASE OF TRIBUTARY




| transect | 1 | 2 | 0 |
|---|---|---|---|
| $T\ (°C)$ | 26.4 | 24.8 | ? |
| $O_s\ (mg/l)$ | 8.0 | 8.3 | ? |
| $O\ (mg/l)$ | 7.3 | 6.8 | ? |
| $B\ (mg/l)$ | 3.0 | 6.0 | ? |
| $Q\ (m^3/s)$ | 55 | 14 | ? |

To determine the value of tha parameters in section 0 we have to use the mass balance. First, the continuity:

$$Q_0 = Q_1 + Q_2 = 55 + 14 = 69 \text{ m}^3/\text{s}$$

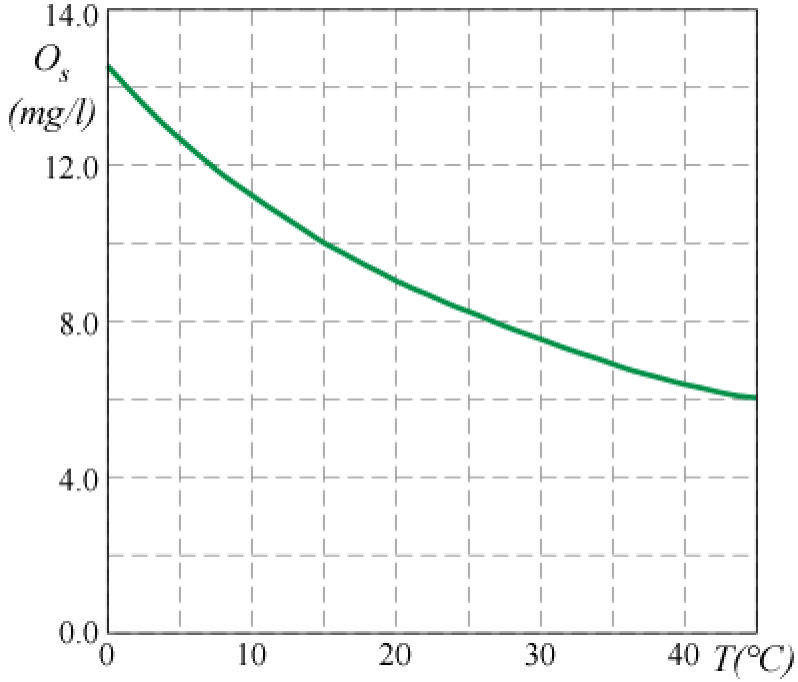
The concentration $B_0$ and $O_0$ is given by the weighted average of $B_i$ and $O_i$:

$$B_0 = \frac{B_1 Q_1 + B_2 Q_2}{Q_0} = \frac{55 \cdot 3.0 + 14 \cdot 6.0}{69} = 3.6 \quad \text{mg/l}$$

$$O_0 = \frac{O_1 Q_1 + O_2 Q_2}{Q_0} = \frac{7.3 \cdot 55 + 6.8 \cdot 14}{69} = 7.2 \quad \text{mg/l}$$

Similarly, the temperature $T_0$ (°C):

$$T_0 = \frac{T_1 Q_1 + T_2 Q_2}{Q_0} = \frac{26.4 \cdot 55 + 24.8 \cdot 14}{69} = 26.1$$




By knowing $T_0$ we can determine $O_{s,0}$ by the table:

$$O_{s,0}(26.1\ °\mathrm{C}) = 8.1 \quad \text{mg/l}$$

Finally the deficit of Oxygen $\tilde{O}_0$ is:

$$\tilde{O}_0 = O_{s,0} - O_0 = 8.1 - 7.2 = 0.9 \quad \text{mg/l}$$

| T (°C) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Os (mg/l) | 14.60 | 14.19 | 13.81 | 13.44 | 13.09 | 12.75 | 12.43 | 12.12 | 11.83 | 11.55 | 11.27 | 11.01 | 10.76 | 10.52 | 10.29 |
| T (°C) | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| Os (mg/l) | 10.07 | 9.85 | 9.65 | 9.45 | 9.26 | 9.07 | 8.90 | 8.72 | 8.56 | 8.40 | 8.24 | 8.09 | 7.95 | 7.81 | 7.67 |
| T (°C) | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 |
| Os (mg/l) | 7.54 | 7.41 | 7.28 | 7.16 | 7.06 | 6.93 | 6.82 | 6.71 | 6.61 | 6.51 | 6.41 | 6.31 | 6.22 | 6.13 | 6.04 |

# EXERCISE 2: ESTIMATION OF $r_a$



Let's determine the aeration coefficient $r_a$ for rectangular channel having the following characteristics:

- $i = 0.1\%$
- $B = 10$ m
- $Q = 30$ m³/s
- $K_s = 30$ m$^{1/3}$/s

$\left.\begin{array}{l} B = 10 \text{ m} \\ Q = 30 \text{ m}^3/\text{s} \end{array}\right\} q = 3.0 \text{ m}^3/\text{s·m}$ *Flow rate per unit of width*

The coefficient $r_a$ can be estimated by the formula of *O'Connor & Dobbins*. To apply the formula, we have to calculate the uniform flow condition, i.e. $U$ and $z_0$.
The Strickler formula reads:

$$U = K_s\, {R_H}^{2/3} i^{1/2} \longrightarrow q = z_0 K_s\, {R_H}^{2/3} i^{1/2}$$

$$= z_0 K_s \left(\frac{B z_0}{B + 2z_0}\right)^{2/3} i^{1/2}$$

$$= {z_0}^{5/3} K_s i^{1/2} \left(\frac{B}{B + 2z_0}\right)^{2/3} \xrightarrow{B \gg z_0} q = {z_0}^{5/3} K_s\, i^{1/2}$$

If $B \gg z_0$, we can determine $z_0$ as: $z_0 = \left(\dfrac{q}{K_s\, i^{1/2}}\right)^{3/5}$

# EXERCISE 2: ESTIMATION OF $r_a$



When $R_H \neq z_0$, the uniform depth is determined iteratively, in according to the following:

$$z_{0,n+1} = \left(\frac{q}{K_s\, i^{1/2}}\right)^{3/5} \left(\frac{B + 2z_{0,n}}{B}\right)^{2/5}$$

In this case the formulas becomes: $z_{0,n+1} = 2.00 \cdot \left(1 + \frac{z_{0,n}}{5.0}\right)^{2/5}$

The value of first attempt is the aproximated solution assuming $B \gg z_0$, i.e. $z_0 = 2.00$ m.

We need of few iteations to determine $z_0 = 2.32$ m

Thus the velocity is: $U = \frac{q}{z_0} = \frac{3.0}{2.32} = 1.29$ m/s

To correctly use the *O'Connor & Dobbins* formula, we transform the units of $z_0$ and $U$:

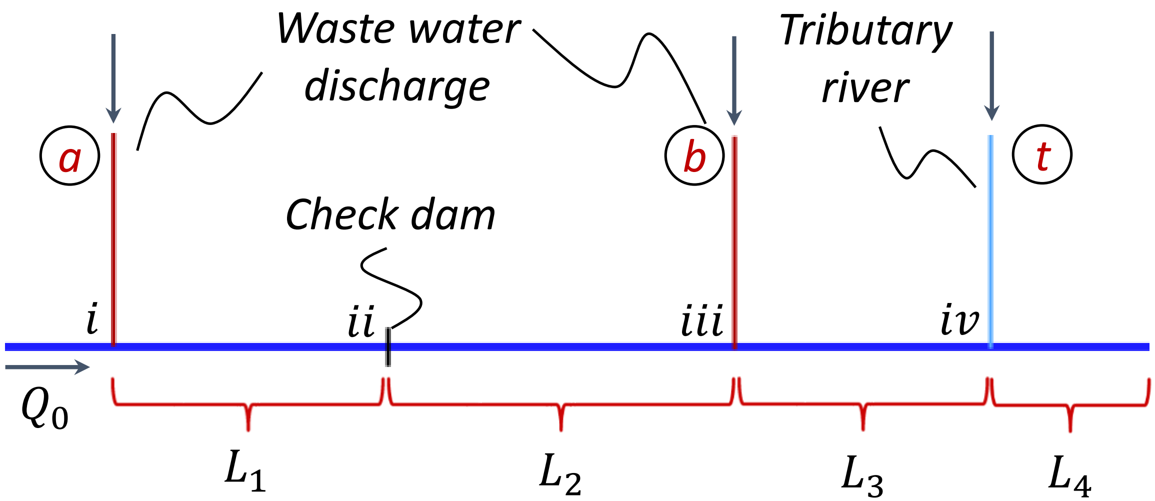
| n | $z_0$ (m) | $R_H$ (m) |
|---|---|---|
| 0 | 2.00 | 1.43 |
| 1 | 2.28 | 1.57 |
| 2 | 2.32 | 1.58 |
| 3 | 2.32 | 1.59 |

$z_0 = 2.32$ m $\xrightarrow{1ft=0.305\,m}$ $z_0 = 7.63$ ft

$U = 1.29$ m/s $\xrightarrow{1ft=0.305\,m}$ $U = 4.23$ ft/s

$\longrightarrow$ $r_a = 12.9 \frac{U^{1/2}}{{z_0}^{3/2}} = 12.9 \frac{4.23^{1/2}}{7.63^{3/2}} = 1.26$ d$^{-1}$

Let's determine $O$ distribution of the case shown in the scheme.




## Water quality data

| | Q (m³/s) | B (mg/l) | O (mg/l) |
|---|---|---|---|
| 0 | 7.0 | 0.0 | 8.0 |
| a | 5.0 | 33.6 | 1.5 |
| b | 3.0 | 44.0 | 0.5 |
| t | 5.0 | 0.0 | 8.0 |

## Transect properties

| Tract | L (km) | U (m/s) | $r_d$ ($d^{-1}$) | $r_a$ ($d^{-1}$) |
|---|---|---|---|---|
| 1 | 40 | 0.30 | 0.4 | 1.2 |
| 2 | 5 | 0.30 | 0.4 | 1.2 |
| 3 | 40 | 0.40 | 0.4 | 1.1 |
| 4 | 15 | 0.45 | 0.4 | 1.0 |

It is worth noting that the maximum $DO$ concentration is the saturated concentration, i.e. $O_s = 8.0$ mg/l.

We can solve the problem assuming:

- Pointed mixing of the solutes
- Negligible diffusion

Thus, we can use S-P model

Tract 1

Let's determine the upstream boundary condition by the mass balance at the node $i$:

$$Q_1 = Q_0 + Q_a = 7 + 5 = 12 \text{ m}^3/\text{s}$$

$$B_1 = \frac{B_0 Q_0 + B_a Q_a}{Q_1} = \frac{0.0 \cdot 7.0 + 33.6 \cdot 5.0}{12.0} = 14.0 \text{ mg/l}$$

$$O_1 = \frac{O_0 Q_0 + O_a Q_a}{Q_1} = \frac{8.0 \cdot 7.0 + 1.5 \cdot 5.0}{12.0} = 5.3 \text{ mg/l} \longrightarrow \tilde{O}_1 = O_s - O_1 = 2.7 \text{ mg/l}$$

Now we can determine the critic condition in $L_1$:

$$x_{c1} = \frac{U_1}{r_{a1} - r_d} \ln\left[\frac{r_{a1}}{r_d}\left(1 - \frac{r_{a1} - r_d}{r_d}\frac{\tilde{O}_1}{B_1}\right)\right]$$

$$= \frac{0.3 \cdot (3600 \cdot 24)}{1.2 - 0.4} \ln\left[\frac{1.2}{0.4}\left(1 - \frac{1.2 - 0.4}{0.4}\frac{2.7}{14.0}\right)\right]\frac{1}{1000} = 19.8 \text{ km}$$

$$\tilde{O}_{c1} = \frac{r_d}{r_{a1}} B_1 e^{-r_d \frac{x_{c1}}{U_1}} = \frac{0.4}{1.2} 14.0 e^{-\frac{0.4}{(3.6 \cdot 24)}\frac{19.8}{0.3}} = 3.4 \text{ mg/l}$$

$$\longrightarrow O_{c1} = O_s - \tilde{O}_{c1} = 8.0 - 3.4 = 4.6 > 4.0 \text{ mg/l}$$

$O$ decreses till $x_{c1} = 19.8$ km, but we do not observe anoxic condition ($O_{c1} = 4.6 > 4.0$ mg/l). For $x > x_{c1}$ $O$ increases, at the end of the tract we find:

$$\tilde{O}_{1e} = \tilde{O}_1 e^{-r_{a1}\frac{x}{U_1}} + \frac{B_1 r_d}{r_{a1} - r_d}\left[e^{-\frac{r_d L_1}{U_1}} - e^{-\frac{r_{a1} L_1}{U_1}}\right] = 3.1 \text{ mg/l} \longrightarrow O_{1e} = O_s - \tilde{O}_{1e} = 4.9 \text{ mg/l}$$

$$B_{1e} = B_1 e^{-\frac{r_d L_1}{U_1}} = 7.6 \quad \text{mg/l}$$

These concentrations are the initial condition of the following tract.

Tract 2

In the upstream section we have a check dam. Downstream of the dam we observe an intense mixing of the water that can easily entrape air.

Thus we can assume that along the short length $L_2$ the flow recovers the saturated condition of oxygen. At the end of the tract 2 we have:

$$O_{2e} = O_s = 8.0 \quad \text{mg/l}$$

$$B_{2e} = B_2 e^{-\frac{r_d L_2}{U_2}} = 7.6 e^{-\frac{0.4 \quad 5.0}{(3.6\cdot 24)0.3}} = 7.0 \text{ mg/l} \qquad \text{with } B_2 = B_{1e}$$

Tract 3

Before to study the $O$ distribution along the segment, we determine the upstream condition through the mass balance in the node $iii$:

$$Q_3 = Q_2 + Q_b = 12 + 3 = 15 \text{ m}^3/\text{s} \qquad \text{with } Q_2 = Q_1$$

$$B_3 = \frac{B_{2e}Q_2 + B_b Q_b}{Q_3} = \frac{7.0 \cdot 12.0 + 44.0 \cdot 3.0}{15.0} = 14.4 \quad \text{mg/l}$$

$$O_3 = \frac{O_{2e}Q_2 + O_b Q_b}{Q_3} = \frac{8.0 \cdot 12.0 + 0.5 \cdot 3.0}{15.0} = 6.5 \text{ mg/l} \;\longrightarrow\; \tilde{O}_3 = O_s - O_3 = 1.5 \quad \text{mg/l}$$

The critic condition is:

$$x_{c3} = \frac{0.4 \cdot (3.6 \cdot 24)}{1.1 - 0.4} \ln\left[\frac{1.1}{0.4}\left(1 - \frac{1.1 - 0.4}{0.4}\frac{1.5}{14.4}\right)\right] = 40.0 \quad \text{km}$$

$$\tilde{O}_{c3} = \frac{0.4}{1.1} 14.4 e^{-\frac{0.4}{(3.6 \cdot 24)}\frac{40.0}{0.4}} = 3.3 \quad \text{mg/l} \longrightarrow O_{c1} = O_s - \tilde{O}_{c3} = 4.7 > 4.0 \quad \text{mg/l}$$

The critic condition is achieved at the end of the tract. $B$ in the last section is:

$$B_{3e} = B_3 e^{-\frac{r_d L_3}{U_3}} = 14.4 e^{-\frac{0.4}{(3.6 \cdot 24)}\frac{40.0}{0.4}} = 9.1 \quad \text{mg/l}$$

Tract 4

Let's determine the initial condition (node $iv$) after the inflow of clean water:

$$Q_4 = Q_3 + Q_t = 15 + 5 = 20 \text{ m}^3/\text{s}$$

$$B_4 = \frac{B_{3e}Q_3 + B_tQ_t}{Q_4} = \frac{9.1 \cdot 15.0 + 0.0 \cdot 5.0}{20.0} = 6.8 \ \text{mg/l}$$

$$O_4 = \frac{O_{3e}Q_3 + O_tQ_t}{Q_4} = \frac{4.7 \cdot 15.0 + 8.0 \cdot 5.0}{20.0} = 5.5 \ \text{mg/l} \longrightarrow \tilde{O}_4 = O_s - O_4 = 2.5 \ \text{mg/l}$$

The critic condition is:

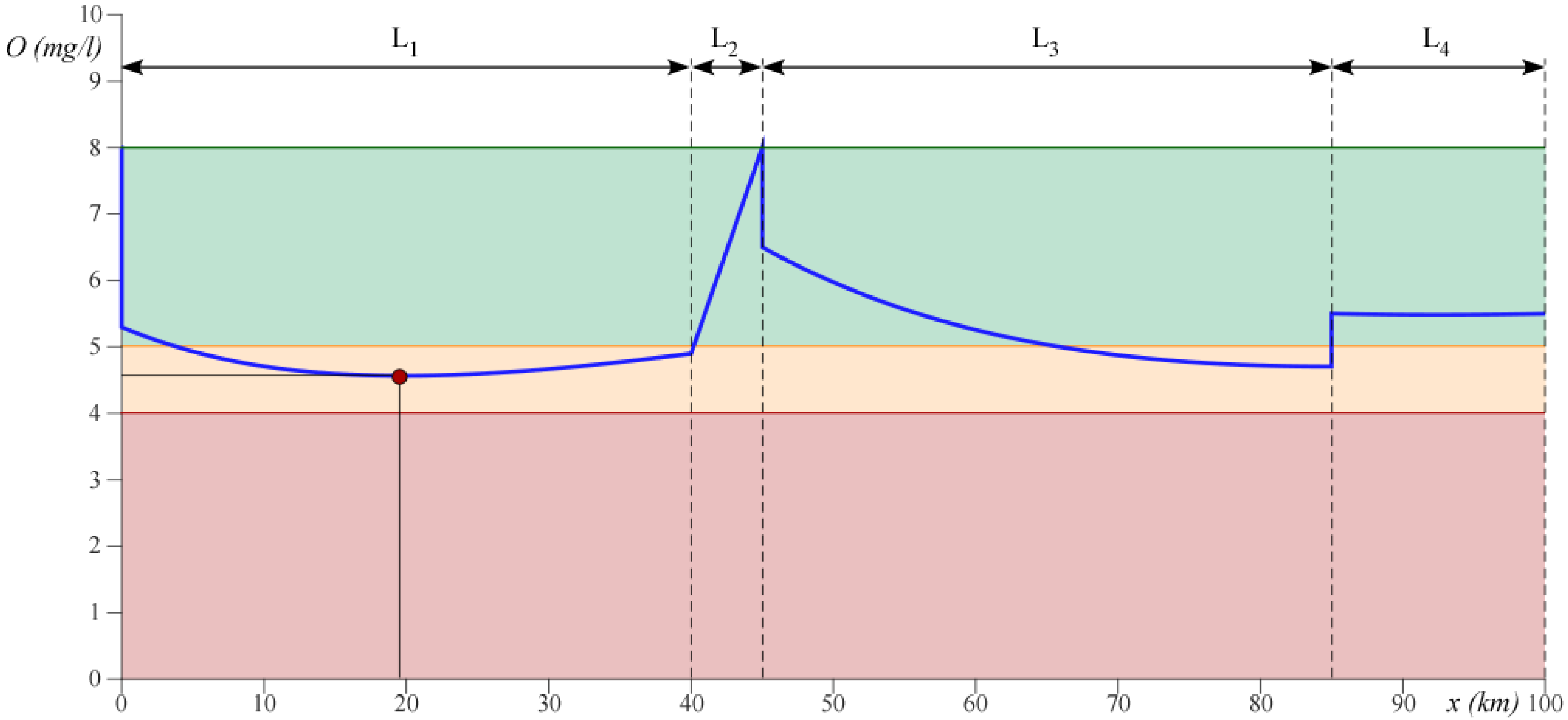
$$x_{c4} = \frac{0.45 \cdot (3.6 \cdot 24)}{1.0 - 0.4} \ln\left[\frac{1.0}{0.4}\left(1 - \frac{1.0 - 0.4}{0.4}\frac{2.5}{6.8}\right)\right] = 7.4 \quad \text{km}$$

$$\tilde{O}_{c4} = \frac{0.4}{1.0} 6.8 e^{-\frac{0.4}{(3.6 \cdot 24)}\frac{7.4}{0.45}} = 2.5 \quad \text{mg/l} \longrightarrow O_{c4} = O_s - \tilde{O}_{c4} = 5.5 > 4.0 \quad \text{mg/l}$$

The reduction of $O$ is negligible in this tract and $O \cong 5.5$ mg/l till the distance $L_4$.

# EXERCISE 3: APPLICATION OF S-P MODEL






| Transect | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $O_{min}$ (mg/l) | 4.6 | 4.9 | 4.7 | 5.5 |
| $\bar{O}$ (mg/l) | 4.7 | 6.5 | 5.2 | 5.5 |

$\bar{O} = 4.7 < 5.0$ mg/l in the segment 1. It is necessary to reduce the waste water discharge in the node $i$.