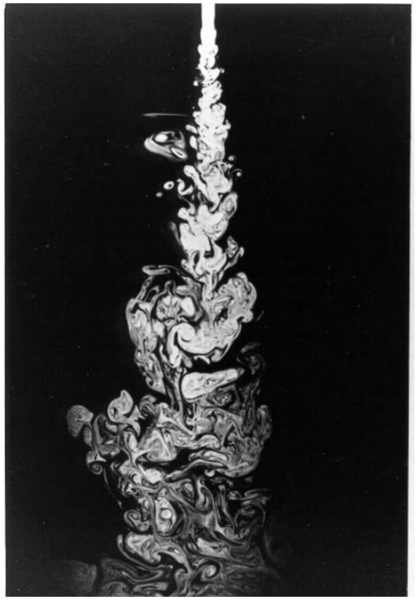
LESSON 1: INTRODUCTION TO THE COURSE

Program of the course:

- <u>Transport dynamics of solute</u>:
  - Advection/Convection
  - Diffusion
  - Dispersion

  Main mechanisms



Turbulence mixing of jet-fluid (Dimotakis, 1993)

Confluence of the Green and Colorandoi Rivers

- Solute/pollutant dynamics in rivers: - conservative solute
  - non-conservative pollutants

Red dye in river

River eutrophication

- <u>Salt wedge and density currents</u>

Density current intrusion in a saline costal ambient. (Photo: I. Wood, Univ. of Canterbury

Main objectives of the course:

- Analyzing pollutants transport mechanisms
- Estimating pollutants/chemical concentrations to determine water quality
- Using advection-dispersion models to analyze future scenarios

Preliminary knowledge.

- Fluid properties: fluid density, dynamic and kinematic viscosity,…
- Continuity equation and Principle of conservation of Energy and Momentum
- Momentum equation of real fluid (Navier-Stokes equation)
- Non-dimensional parameters (Reynolds number, Froude number, etc..)
- Basic principles of open channel flow (uniform flow, supercritic and subcritic flow,…)

Suggested textbook

- White, F. M. (1979). *Fluid mechanics*. Tata McGraw-Hill Education.

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Reference texts:

- S. Lanzoni. Dinamica degli inquinanti nei corpi idrici superficiali. *Lecture notes*.
- Scnoor, J. L. (1996). Environmental Modelling, Fate and Transport of Pollutants in Water, Air, and Soil. *John Wiley & Son Ltd*.
- Rutherford, J. C. (1994). River mixing. *John Wiley & Son Ltd*.
- Fischer, H. B., List, J. E., Koh, C. R., Imberger, J., & Brooks, N. H. (2013). Mixing in inland and coastal waters. *Elsevier*.
- Pedersen, F. B. (2012). *Environmental Hydraulics: Stratified Flows* (Vol. 18). Springer Science & Business Media.
- PPTs of the lectures.

About the exam

Oral exam:	1st question (>6/10)	*concerning the course*
	2nd question (>6/10)	*experimental activities*
	group exercises (>6/10)	

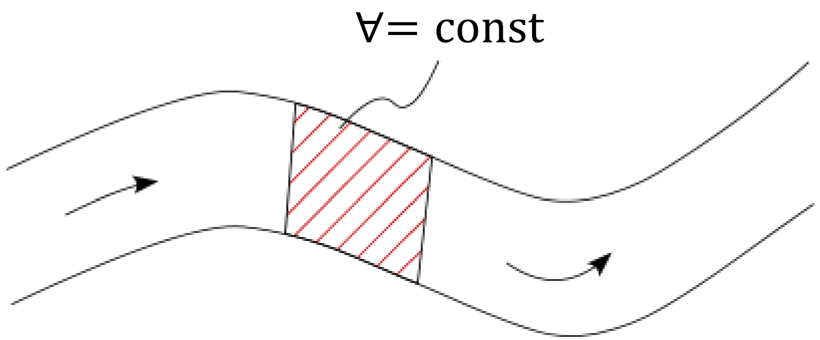
Date:	I exam session:	01/02/2023	9.00 a.m.	room IP	
	II exam session:	14/02/2023	9.00 a.m.	room IP	*These date may*
	III exam session:	20/07/2023	9.00 a.m.	room IP	*change…*
	IV exam session:	05/09/2023	9.00 a.m.	room IP	

Fluid (intensive) property ψ does not depend by the fluid mass and it can be:

- Scalar: T, ρ, …
- Vector: ***u***, …

Kinematic approach

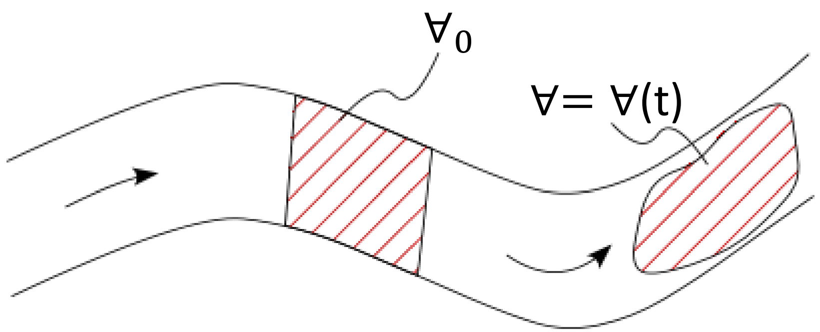
- <u>Eulerian</u>



Euler studies the field properties in a "fixed" domain (***x***)

u=***u***(***x***,*t*)

- <u>Lagrangian</u>



Lagrange studies the properties of the same fluid particles (*ξ*) which are moving along the domain

u=***u***(*ξ*,*t*)

Extensive property of fluid is: $$\Psi = \int_{\forall_S} \psi \cdot \mathrm{d}\forall$$

$\forall_S$ is the System Volume or Material Volume $\longrightarrow$ Conservation of Ψ in $\forall_S$ (it is Lagrangian!)

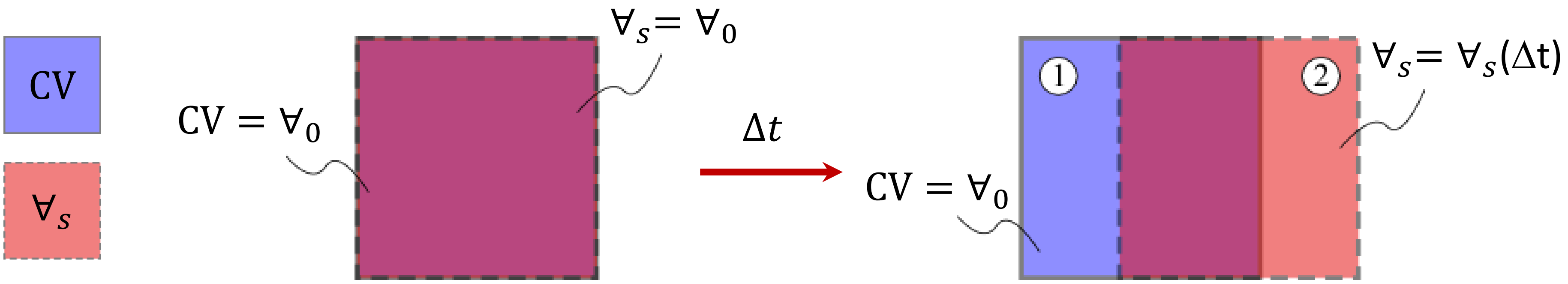
Usually, we work on Control Volume (CV) (It is Eulerian!)

How can we study Ψ in CV?

N.B.: Physics experiences are applied to System Volume and not to Control Volume

$$\sum \boldsymbol{F} = m \cdot \boldsymbol{a} \qquad\qquad \sum \boldsymbol{F} \neq m \cdot \boldsymbol{a}$$

System Volume | *Control Volume*



- *t=0* $\Psi_{\forall_s}(0) = \Psi_{\forall_0}(0)$
- *t=Δt* $\Psi_{\forall_s}(\Delta t) = \Psi_{\forall_0}(\Delta t) - \Psi_1(\Delta t) + \Psi_2(\Delta t)$

By subtracting the two expressions and for $\Delta t \to 0$, we have:

$$\frac{\Psi_{\forall_s}(\Delta t) - \Psi_{\forall_s}(0)}{\Delta t} = \frac{\Psi_{\forall_0}(\Delta t) - \Psi_{\forall_0}(0)}{\Delta t} - \psi_1 \frac{\Delta\forall}{\Delta t} + \psi_2 \frac{\Delta\forall}{\Delta t}$$

$$\longrightarrow \quad \underbrace{\frac{\mathrm{d}\Psi}{\mathrm{d}t}}_{\textit{Lagrangian}} = \underbrace{\frac{\partial \Psi}{\partial t} - \dot{\Psi}_1 + \dot{\Psi}_2}_{\textit{Eulerian}}$$

TRANSPORT THEOREM

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = ?$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall(t)} \psi \, \frac{\mathrm{d}\forall_0}{\mathrm{d}\forall_0}\, \mathrm{d}\forall = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall_0} \psi \, \mathrm{J}\, \mathrm{d}\forall_0$$

$$\frac{\mathrm{d}\forall}{\mathrm{d}\forall_0} = \det\begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix} = J$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = \int_{\forall_0} \frac{\mathrm{d}}{\mathrm{d}t}(\psi\, \mathrm{J})\, \mathrm{d}\forall_0 = \int_{\forall_0} \left(\frac{\mathrm{d}\psi}{\mathrm{d}t}\, \mathrm{J} + \frac{\mathrm{dJ}}{\mathrm{d}t}\, \psi\right) \mathrm{d}\forall_0$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = \int_{\forall_0} \left(\frac{\mathrm{d}\psi}{\mathrm{d}t} + \psi\, \nabla\cdot\boldsymbol{u}\right) J\, \mathrm{d}\forall_0 = \int_{\forall(t)} \left(\frac{\mathrm{d}\psi}{\mathrm{d}t} + \psi\, \nabla\cdot\boldsymbol{u}\right) \mathrm{d}\forall$$

$$\frac{\mathrm{dJ}}{\mathrm{d}t} = J\, \nabla\cdot\boldsymbol{u}$$

Let’s rearrange the integrating expression

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} + \psi\, \nabla\cdot\boldsymbol{u} = \frac{\partial\psi}{\partial t} + \boldsymbol{u}\cdot\nabla\psi + \psi\, \nabla\cdot\boldsymbol{u} = \frac{\partial\psi}{\partial t} + \nabla\cdot(\psi\otimes\boldsymbol{u})$$

$$\boldsymbol{a}\otimes\boldsymbol{b} = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall} \psi \,\mathrm{d}\forall = \int_{\forall} \frac{\partial \psi}{\partial t}\,\mathrm{d}\forall + \int_{\forall} \nabla\cdot(\psi\otimes\boldsymbol{u})\,\mathrm{d}\forall$$

Gauss th.: $\int_{\forall} \nabla\cdot\boldsymbol{\varphi}\,\mathrm{d}\forall = \int_{S} \boldsymbol{\varphi}\cdot\boldsymbol{n}\,\mathrm{d}S$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall} \psi \,\mathrm{d}\forall = \int_{\forall} \frac{\partial \psi}{\partial t}\,\mathrm{d}\forall + \int_{S} (\psi\otimes\boldsymbol{u})\cdot\boldsymbol{n}\,\mathrm{d}S$$

$(a\otimes\boldsymbol{b})\cdot\boldsymbol{c} = (\boldsymbol{b}\cdot\boldsymbol{c})a$

$$\longrightarrow \underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall} \psi \,\mathrm{d}\forall}_{\textit{Total variation of } \psi} = \underbrace{\int_{\forall} \frac{\partial \psi}{\partial t}\,\mathrm{d}\forall}_{\textit{Local variation of } \psi} + \underbrace{\int_{S} \psi\,\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S}_{\textit{Flux across } S \textit{ of } \forall}$$

Some definitions:

- Volumetric Concentration: $c_s = \lim_{\delta\forall \to 0} \frac{\delta M_s}{\delta\forall}$ [kg/m^3]

  M_s is the mass of solute in the fluid volume $\forall$

  N.B. With more solutes: $c_s = \sum_i c_{s,i}$

- Mass Concentration: $\hat{c_s} = \lim_{\delta\forall \to 0} \frac{\delta M_s}{\delta M}$ [%]

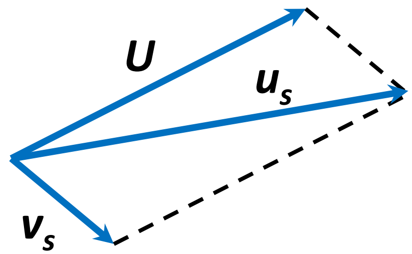
  M is the fluid mass in $\forall$

- Fluid Density: $\rho = \lim_{\delta\forall \to 0} \frac{\delta M}{\delta\forall}$ [kg/m^3]

N.B.: $\hat{c_s} = \frac{c_s}{\rho}$

- Mixture Velocity (water + solute):

  $\boldsymbol{U}$: mean velocity of the mixture
  $\boldsymbol{v_s}$: diffusive velocity of the solute
  $\boldsymbol{u_s}$: absolute velocity of the solute

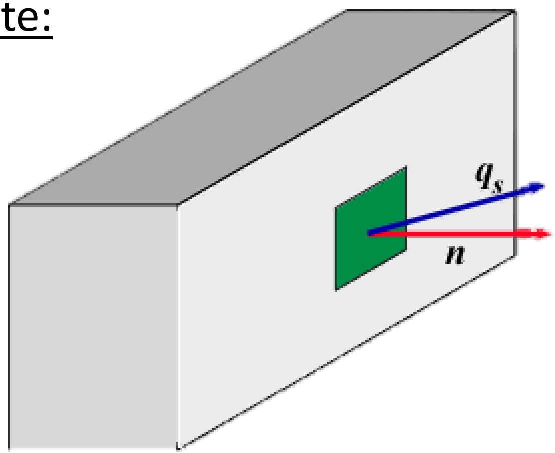


- Mass Flux: $\boldsymbol{q_s} = c_s\,\boldsymbol{u_s}$ [kg/m²s]

  N.B. With more solutes: $\boldsymbol{q_s} = \sum_i \boldsymbol{q_{s,i}}$

- Relative Mass Flux: $\boldsymbol{q_s^r} = c_s\,(\boldsymbol{u_s} - \boldsymbol{U}) = c_s\,\boldsymbol{v_s}$ [kg/m²s]

- Mass Rate:

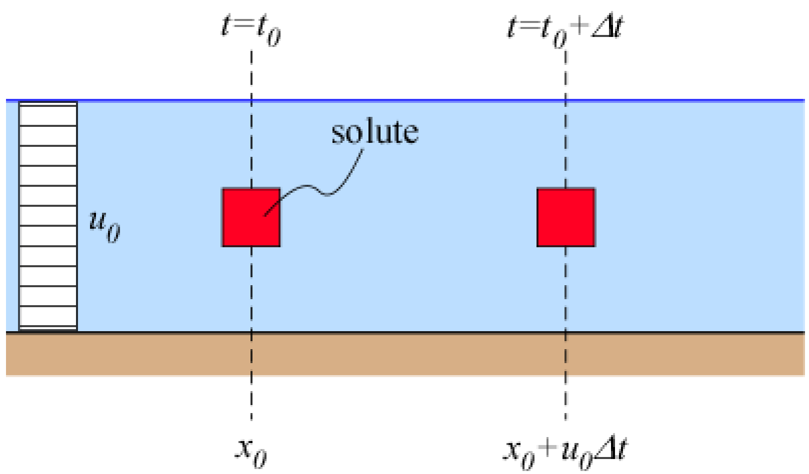


$$Q_s = \int_A \boldsymbol{q_s} \cdot \boldsymbol{n}\,\mathrm{d}A$$ [kg/s]

1. Advection/convection

Advection: transport of a solute by the fluid

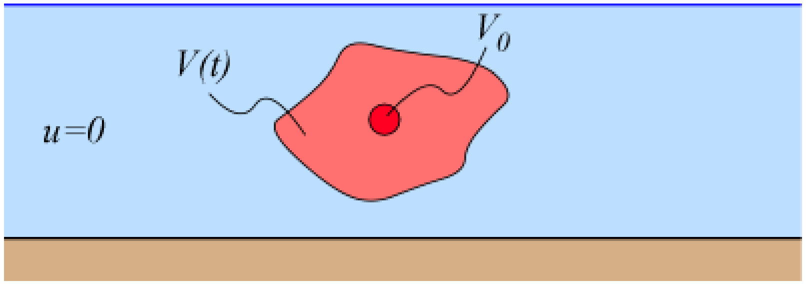
Convection: transport of an intrinsic property of fluid (e.g., momentum and temperature)



Hp: - 1D flow
- Uniform flow: $\boldsymbol{u}(x,t) = u_0$
- Inviscid fluid: $\mu = 0$

→ Solute translate with constant velocity

2. Diffusion

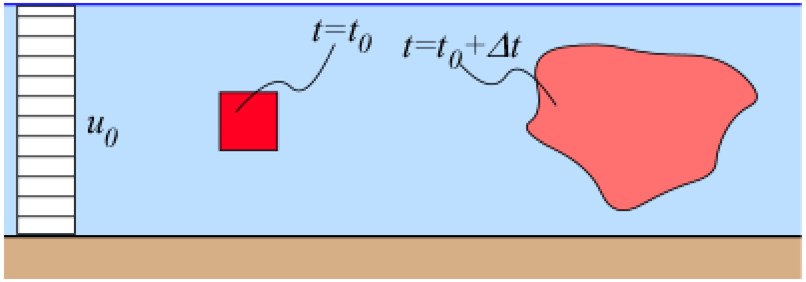


Molecular Diffusion (or Brawnian Motion)

→ Random spreding out of molecules due to termodynamics condition

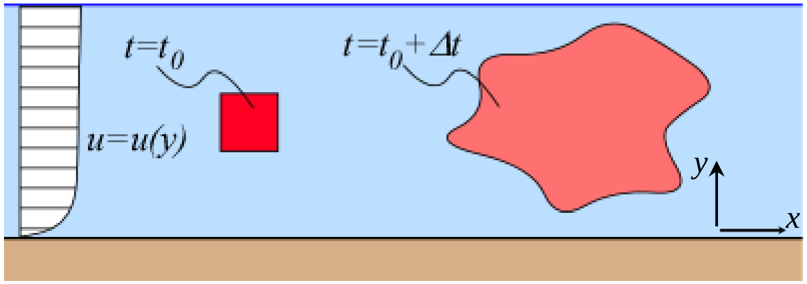
Turbulent Diffusion

→ Solute spreads cause of velocity fluctuation due to turbulence



N.B.: Turbulent Diffusion is 2 orders of magnitude greater than Molecular Diffusion

3. Dispersion



Hp: - real fluid: $\mu \neq 0$

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y}$$

→ Velocity gradient and turbolent diffusion promote the dispersion of the solute

N.B.: Dispersion is much greater than turbulent Diffusion