Revenue Management: Capacity Allocation

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Outline

Revenue Management (RM) tactical level:
calculate and update booking limits

Single resource capacity control: optimally allocating capacity of a resource to different classes of demand (in contrast with multiple resources, or Network RM)

Static models: demand arrives in increasing fare order

- 2-Class problem
  - Exact solution
- Multi-Class Problem
  - EMSR Heuristics (= Expected Marginal Seat Revenue)
- Examples

2-Class problem

2 classes: discount *(d)* and full fare *(f)* customers

p_d, p_f : respective fares, with $p_f > p_d > 0$

Hypothesis:

- d pax book before f pax
- Discount demand and full fare demand are independent r.v.
- No cancellations or overbooking or no-shows

➢ What is the optimal booking limit b for d pax?

➢ What is the optimal protection level y for f pax?

Simplification: we optimize only expected revenue. Incremental cost (meal, fuel, fees …) and ancillary contribution (sales on board) are not considered.

Risks

- Booking limit too low → empty seats (*spoilage*)
- Booking limit too high → deny boarding to *f* pax (*dilution*)

C = capacity

D_d = r.v. discount demand, with cdf $F_d(x)$

D_f = r.v. full fare demand, with cdf $F_f(x)$

<u>Marginal analysis</u>: compare two different booking limits, b-1 and b, and the corresponding revenues

Marginal analysis

Δ = revenue variation,
changing the booking limit from b-1 to b

- If $D_d \leq b-1$ then $\Delta = 0$
- If $D_d \geq b$ and $D_f \leq C-b$ then $\Delta = p_d$
- If $D_d \geq b$ and $D_f > C-b$ then $\Delta = p_d - p_f$

$$E[\Delta] = 0 \cdot F_d(b-1) + p_d \cdot (1-F_d(b-1)) \cdot F_f(C-b) + (p_d - p_f) \cdot (1-F_d(b-1)) \cdot (1-F_f(C-b))$$

$$= (1-F_d(b-1)) \cdot [p_d - p_f \cdot (1-F_f(C-b))]$$

Marginal analysis

$E[\Delta] = (1-F_d(b-1))\cdot[p_d-p_f\cdot(1-F_f(C-b))]$

$$E[\Delta] \geq 0 \Leftrightarrow p_d/p_f \geq 1-F_f(C-b)$$

- The sign of $E[\Delta]$ does not depend on F_d !
- $1-F_f(C-b)$ increases as b increases
- C-b = Protection level y for *f* pax
- If $p_d/p_f < 1-F_f(C) = P[D_f>C]$ then $b=0$
  (do not allocate any seats for d-pax)

Littlewood's rule

The optimal discount booking limit b* is such that:

$$1-F_f(C-b^*) = p_d/p_f$$

Equivalently, the optimal full fare protection level y* is such that:

$$1-F_f(y^*) = p_d/p_f$$

Assuming strict monotonicity of F_f:

$$y^* = \text{MIN}\ [C;\ F_f^{-1}(1-p_d/p_f)]$$

b* and y* are independent of the forecast discount demand!!

Example 1

If an airline has set an optimal discount booking limit equal to 80 seats on a 150-seats aircraft, what is the optimal discount booking limit on a 100-seats aircraft for the same flight?

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- If $C = 150$, then $b^* = 80$ and
  $y^* = C - b^* = 150 - 80 = 70$
- If $C = 100$, then $y^* = 70$ still holds, and
  $b^* = C - y^* = 100 - 70 = 30$

Example 2

Assume that D_f is $N(\mu_f = 50; \sigma_f = 100)$, and $C = 100$.

1. What is the maximum fare ratio p_d/p_f such that the optimal booking limit is zero?

2. What is the minimum fare ratio p_d/p_f such that the optimal booking limit is equal to C?

3. What is the optimal booking limit b* if
    - $p_d/p_f = 0.4$?
    - $p_d/p_f = 0.5$?
    - $p_d/p_f = 0.6$?

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Assume that D_f is $N(\mu_f = 50; \sigma_f = 100)$, and $C = 100$.

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Remember:

If $p_d/p_f < 1-F_f(C) = P[D_f>C]$ then $b=0$
(do not allocate any seats for d-pax)

$1-F_f(C) = P[D_f>C] = 0.309$

If $p_d/p_f \leq 0.309$ then $b=0$

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If $b = C = 100$, then
$1\text{-}F_f(C\text{-}b) = 1\text{-}F_f(0) = 0.691$

If $p_d/p_f \geq 0.691$ then $b^* = C$

Example 2: values of b and $1\text{-}F_f(C\text{-}b)$

$p_d/p_f = 0.4$

b	$1\text{-} F_f(C\text{-}b)$	b	$1\text{-} F_f(C\text{-}b)$	b	$1\text{-} F_f(C\text{-}b)$
24	0.397	49	0.496	75	0.599
25	0.401	50	0.500	76	0.603
26	0.405	51	0.504	80	0.618
30	0.421	55	0.520	85	0.637
35	0.440	60	0.540	90	0.655
40	0.460	65	0.560	95	0.674
45	0.480	70	0.579	100	0.691

$b^* = 24$

Example 2: values of b and $1-F_f(C-b)$
$p_d/p_f = 0.5$

b	$1- F_f(C-b)$	b	$1- F_f(C-b)$	b	$1- F_f(C-b)$
24	0.397	49	0.496	75	0.599
25	0.401	50	0.500	76	0.603
26	0.405	51	0.504	80	0.618
30	0.421	55	0.520	85	0.637
35	0.440	60	0.540	90	0.655
40	0.460	65	0.560	95	0.674
45	0.480	70	0.579	100	0.691

b* = 50

Example 2: values of b and $1-F_f(C-b)$
$p_d/p_f = 0.6$

b	$1-F_f(C-b)$	b	$1-F_f(C-b)$	b	$1-F_f(C-b)$
24	0.397	49	0.496	75	0.599
25	0.401	50	0.500	76	0.603
26	0.405	51	0.504	80	0.618
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$b^* = 75$

Special case: Littlewood's Rule with Independent Normal Demands

$F_f(x) = \Phi[(x-\mu_f)/\sigma_f]$ $\Rightarrow$ $\Phi[(C-b^*-\mu_f)/\sigma_f] = 1 - p_d/p_f$

$$b^* = [C - \sigma_f \Phi^{-1}(1-p_d/p_f) - \mu_f]^+$$

$$y^* = \min[\mu_f + \sigma_f \Phi^{-1}(1-p_d/p_f) ; C]$$

b* and y* are <u>linear</u> functions of σ_f (if in (0,C))

- If $p_d/p_f = ½$, then $\Phi^{-1}(1-p_d/p_f) = 0$ $\Rightarrow$ $b^* = C-\mu_f$ and $y^* = \mu_f$ (assuming $\mu_f \in [0, C]$)
- If $p_d/p_f < ½$, then $y^* > \mu_f$ and y* increases with σ_f
- If $p_d/p_f > ½$, then $y^* < \mu_f$ and y* decreases with σ_f

Example 3

A flight has C = 100

2 fare classes

p_f = 300€

F_f is $N(70, \sigma_f)$

If p_d = 150€ then

b* = …

y* = …

Example 3

A flight has C = 100

2 fare classes

p_f = 300€

F_f is $N(70, \sigma_f)$

If p_d = 150€ then

$b^* = C - y^* = 100 – 70 = 30$

$y^* = 70$

Multiple Fare Classes: Assumptions

- n classes
- fares: $p_1 > p_2 > \ldots > p_n$ (class 1 is the highest fare class, class n is the lowest fare class; we say that class 1 is the *highest class*)
- Demand in each class is independent r.v.
- Demands book in increasing fare order
- There are no *cancellations or no-shows* or overbooking

Problem:

Find the booking limit b_j for each class j (and corresponding protection level) in order to maximize expected revenue

Booking process

Period	n	n-1	…	3	2	1	Time
Fare	p_n	p_{n-1}	…	p_3	p_2	p_1	
Bookings	x_n	x_{n-1}	…	x_3	x_2	x_1	
Low fare bookings					High fare bookings		

D_j: demand in class j
$f_j(x)$: probability distribution of D_j
$F_j(x)$: cdf of D_j

Exact vs Heuristic Solutions

- Exact Solution is possible (Dynamic Programming) but computationally intensive
- Exact Solution is NOT used in practice
- Heuristic solutions:
  - EMSR-a
  - EMSR-b

(EMSR: Expected Marginal Seat Revenue)

EMSR-a

To compute y_j^*, the total protection level for class j:

- consider a single class $i \in \{j, j-1, \dots, 1\}$
- compare classes $j+1$ and i using Littlewood's rule: reserve capacity $y_{j+1,i}$ for class i as

$$y_{j+1,i} = F_i^{-1}\left(1 - \frac{p_{j+1}}{p_i}\right)$$

- Repeating for each $i \in \{j, j-1, \dots, 1\}$ and adding:

$$y_j = \sum_{i=1}^{j} y_{j+1,i} = \sum_{i=1}^{j} F_i^{-1}\left(1 - \frac{p_{j+1}}{p_i}\right)$$

- $y_j^* = MIN\left[C; y_j\right]$

EMSR-a

In case of demand normally distributed D_i~$N(\mu_i, \sigma_i)$ obtain the protection level y_j^* as:

$$y_j^* = MIN[C;\ y_j]$$

where y_j is given by:

$$y_j = \sum_{i=1}^{j} \left\{ \mu_i + \sigma_i \, \Phi^{-1} \left(\frac{p_i - p_{j+1}}{p_i} \right) \right\}$$

EMSR-b

- Assume demands are normally distributed $D_i \sim N(\mu_i, \sigma_i)$
- Compute in each period "artificial" average demand μ, price p and standard deviation σ as:
- $\mu = \sum_{i=1}^{j} \mu_i \qquad p = \sum_{i=1}^{j} p_i \mu_i / \mu \qquad \sigma = \sqrt{\sum_{i=1}^{j} \sigma_i^2}$
- $y_j^* = MIN\left[\mu + \sigma\, \Phi^{-1}\left(1 - \frac{p_{j+1}}{p}\right); C\right]$

Example (See file EMSR)

		Demand statistics		Protection levels		
Class	Fare	Mean	Std. Dev.	EMSR-a	EMSR-b	Optimal
1	1050€	17.3	5.8	9.7	9.7	9.7
2	950€	45.1	15.0	50.5	53.3	54.0
3	699€	39.6	13.2	91.6	96.8	98.2
4	520€	34.0	11.3			

Numerical experiments show that EMSR heuristics perform within 1% of the optimal revenue.

Extensions

- Capacity allocation with dependent demands
- Buy up (Sell up): closing a discount fare class leads to increased demand in higher classes
- Cannibalization: opening a discount fare class leads to decreased demand in higher classes
- Dynamic models: relax the assumption that the demand for classes arrives in a strict low-to-high fare order.

See

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