[29/01/25; 6A e 7]

(1)

// x_{ik}: # scatole $i \in \{A,B,C,D,E\}$ caricate nel jumbo $k \in \{1,2\}$

// $y_i = 1$ se scatola $i \in \{A, B\}$ è caricata su 1 o 2, 0 altrimenti

$\max\ 20(x_{A1}+x_{A2}) + 10(x_{B1}+x_{B2}) + 30(x_{C1}+x_{C2}) + 25(x_{D1}+x_{D2}) + 50(x_{E1}+x_{E2})$

s.t. $x_{C1} \geq 50 \quad x_{D1} \leq 24 \quad x_{D2} \leq 49$

$20x_{A1} + 20x_{B1} + 20x_{C1} + 25x_{D1} + 55x_{E1} \leq 50 \times 10^3$ // $1\,m^3 = 1000\,dm^3$

$20x_{A2} + 20x_{B2} + 20x_{C2} + 25x_{D2} + 55x_{E2} \leq 50 \times 10^3$

$110x_{A1} + 100x_{A2} + 40x_{B1} + 50x_{B2} + 180x_{C1} + 170x_{C2} + 35x_{D1} + 40x_{D2} + 80x_{E1} + 70x_{E2} \leq 180\,000$

$x_{A1}+x_{A2} \leq M y_A \quad x_{B1}+x_{B2} \leq M y_B \quad y_A + y_B \leq 1$ # $M = 100\,000$ (suff. grande COST.)

$1 \cdot x_{A1} + 0{,}5x_{B1} + 1{,}5x_{C1} + 1x_{D1} + 1{,}1x_{E1} \geq 1{,}0\,(x_{A1}+x_{B1}+x_{C1}+x_{D1}+x_{E1})$

$1 \cdot x_{A1} + 0{,}5x_{B1} + 1{,}5x_{C1} + 1x_{D1} + 1{,}1x_{E1} \leq 1{,}2\,(x_{A1}+x_{B1}+x_{C1}+x_{D1}+x_{E1})$

$1 \cdot x_{A2} + 0{,}5x_{B2} + 1{,}5x_{C2} + 1 \cdot x_{D2} + 1{,}1x_{E2} \leq 1{,}2\,(x_{A2}+x_{B2}+x_{C2}+x_{D2}+x_{E2})$

$1 \cdot x_{A2} + 0{,}5x_{B2} + 1{,}5x_{C2} + 1 \cdot x_{D2} + 1{,}1x_{E2} \geq 1{,}4\,(x_{A2}+x_{B2}+x_{C2}+x_{D2}+x_{E2})$

$x_{ij} \in \mathbb{Z}_+ \quad i \in \{A,B,C,D,E\},\ j \in \{1,2\} \qquad y_A, y_B \in \{0,1\}$

[7 CFU: // z: valore assoluto differenza

$z \geq x_{A1}+x_{B1}+x_{C1}+x_{D1}+x_{E1} - (x_{A2}+x_{B2}+x_{C2}+x_{D2}+x_{E2})$ $\quad z \in \mathbb{R}$

$z \leq x_{A2}+x_{B2}+x_{C2}+x_{D2}+x_{E2} - (x_{A1}+x_{B1}+x_{C1}+x_{D1}+x_{E1})$ $\quad z \leq 50$]

(2) $\min\ -6x_1 - 4x_2 - \hat{x}_3$ $\quad \{\hat{x}_3 = -x_3$

s.t. $-x_1 - x_2 - \hat{x}_3 + x_4 = 2$

a) $2x_1 + 2x_2 + x_5 = 3$

$2x_1 - 2x_2 + \hat{x}_3 + x_6 = 2$

$x_1, x_2, \hat{x}_3, x_4, x_5, x_6 \geq 0$

x_1	x_2	$\hat{x}_3$	x_4	x_5	x_6	z	b	
-6	-4	-1	0	0	0	-1	0	
-1	-1	-1	1	0	0	0	2	
2	2	0	0	1	0	0	3	
2	-2	1	0	0	1	0	2	
0	-10	2	0	0	3	-1	6	$+2R_3$
0	-2	-1/2	1	0	1/2	0	3	$+R_3'$
0	4	-1	0	1	-1	0	1	$-R_3$
1	-1	1/2	0	0	1/2	0	1	$R_3/2$
0	0	-1/2	0	5/2	1/2	-1	17/2	$+10R_2'$
0	0	-1	1	1/2	0	0	7/2	$+R_2/2$
0	1	-1/4	0	1/4	-1/4	0	1/4	$R_2/4$
1	0	1/4	0	1/4	1/4	0	5/4	$+R_2'$
2	0	0	0	3	1	-1	11	$+2R_3$
4	0	0	1	3/2	1	0	17/2	$+R_3'$
1	1	0	0	1/2	0	0	3/2	$+R_3$
4	0	1	0	1	1	0	5	$4R_3$

(b)

$z^*_{MAX} = -z^*_{MIN} = -(-11) = +11 = w^*$ (DUALITÀ FORTE)

$x_1^* = 0,\ x_2^* = 3/2,\ x_3^* = -5,\ x_4^* = 17/2,\ x_5^* = x_6^* = 0$

(4) •) Enunciato CCPD ...

•) $(0,-2,0)$ è AMM. PRIMALE!

•) duale $\max\ -2u_1 + u_3 + 2u_4$

s.t. $2u_1 + u_2 + 3u_3 - u_4 \leq 3$

$u_1 + u_3 - u_4 \geq -6$

$u_1 + 3u_2 + 2u_3 + 2u_4 = -2$

u_1 lib, $u_2 \geq 0$, $u_3 \leq 0$, $u_4 \geq 0$

•) $0 \cdot (2u_1 + u_2 + 3u_3 - u_4 - 3) = 0 \rightarrow //$

$-2(u_1 + u_3 - u_4 + 6) = 0 \rightarrow \boxed{u_1 + u_3 - u_4 = -6}$

0 libero $\rightarrow$ NO CCPD (A.D.)

$(2x_1 + x_2 + x_3 + 2)u_1 = 0$ per A. Primale

$(x_1 + 3x_3 - 0)u_2 = 0 \rightarrow //$

$(3x_1 + x_2 + 2x_3 - 1)u_3 = 0 \rightarrow \boxed{u_3 = 0}$

$(-x_1 - x_2 + 2x_3 - 2)u_4 = 0 \rightarrow //$

SISTEMA EQ. (*)

$\begin{cases} u_1 + 3u_2 + 2u_3 + 2u_4 = -2 & (A.D.) \\ u_1 + u_3 - u_4 = -6 & (CCPD) \\ u_3 = 0 & (CCPD) \end{cases}$

$\Rightarrow \begin{cases} u_1 = -14/3 - u_2 \\ u_4 = 4/3 - u_2 \\ u_3 = 0 \end{cases}$ TRA LE ∞^1 SOLUZIONI, SE SCELGO $u_2 = 0$ OTTENGO:

$u_1 = -14/3 \quad u_2 = 0 \quad u_3 = 0 \quad u_4 = 4/3$

QUESTA SOLUZIONE

1) SODDISFA (*), QUINDI È IN CCPD CON $(0,-2,0)$

2) È AMMISSIBILE DUALE, infatti

u_1 lib ✓, $u_2 = 0 \geq 0$, $u_3 = 0 \leq 0$, $u_4 = 4/3 \geq 0$,

$2u_1 + u_2 + 3u_3 - u_4 = -28/3 - 4/3 < 3$ ✓

$u_1 + u_3 - u_4 \geq -6$ per costruzione

$u_1 + 3u_2 + 2u_3 + 2u_4 = -2$ per "

3) $\Rightarrow (0,-2,0)$ è OTTIMA!

(4) è max

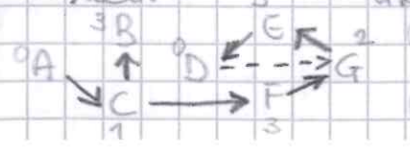
(2) L_2 è tra sol di P_2

(3) non è min

(3) A B C D E F G Acc

	A	B	C	D	E	F	G	Acc
0	0∧	∞∧	∞∧	∞∧	∞∧	∞∧	∞∧	A
1	0∧	4A	1A	∞∧	∞∧	∞∧	∞∧	BC
2	0∧	3C	1A	~~5B~~ 4C	∞∧	3C	∞∧	BDF
3	0∧	3C	1A	4C	∞∧	3C	~~6G~~ 2F	G
4	0∧	3C	1A	4C	3G	3C	2F	E
5	0∧	3C	1A	0E	3G	3C	2F	D
6	0∧	3C	1A	0E	3G	3C	2F	∅

— ALBERO - - - GRAFO



(h=4) D

D ← C ← A

(h=5) = h=6

(5) a) x_6, x_2, x_3 in base ($\exists$ pivot migliorant.)

b) $\frac{100}{37} > 0/19 \Rightarrow$ NON AMMISS.

c) entra x_1, esce x_2

d) x_4 per x_6, x_4 per x_3, x_5 per x_2

e) $x_1, x_2 \rightarrow -30-10 \cdot 0 = -30$

$x_4, x_1 \rightarrow -30-10 \cdot 2 = -50$

(6) a) $2{,}5 \leq U_2 \leq 3{,}8$[(1)] $[2{,}0]$ $L_2 \notin (1{,}1;1{,}5$[(3)]

b) $[2{,}5\,;\,2{,}7]$ $\quad L_2 \leq U_2^{(2)}$ $L_2 \leq 2{,}5^{(2)}$

c) P_5 e P_6 chiusi $\quad [L_2 = 1{,}6$

d) P_3

$U_2 \notin (2{,}5;3{,}9$

[(6) 7CFU] a) $1{,}1 \leq L_2 \leq 1{,}9$; $U_2 \geq 2{,}5$

scelgo $L_2 = 1{,}5 \quad U_2 = 1{,}7$

b) $[1{,}5\,;\,1{,}9]$ c) P_3, P_4, P_5, P_6, CHIUDO P_3 e P_4 e P_5