1

Es 1

a) $f(x) = \operatorname{arctg}\left(\frac{e^x}{|e^x-1|}\right)$

$D: e^x - 1 \neq 0 \Rightarrow e^x \neq 1 \Rightarrow x \neq 0 \qquad D = (-\infty, 0) \cup (0, +\infty)$

$\lim_{x \to 0} \operatorname{arctg}\left(\frac{e^x \to 1}{|e^x-1| \to 0^+}\right) \to +\infty = \frac{\pi}{2}$ posso estendere per continuità f in $x=0$ ponendo $f(0) = \pi/2$.

$f(x) \geq 0 \iff \frac{e^x}{|e^x-1|} \geq 0 \Rightarrow$ Vero $\forall x \in \mathbb{R} \Rightarrow f(x) > 0 \ \forall x \in \mathbb{R}$.

$f(-x) \neq f(x)$ f non ha simmetrie

$\lim_{x \to +\infty} \operatorname{arctg} \frac{e^x}{e^x - 1} = \lim_{x \to +\infty} \operatorname{arctg} \frac{\cancel{e^x}\ 1}{\cancel{e^x}\left(1 - \frac{1}{e^x}\right)} = \operatorname{arctg} 1 = \frac{\pi}{4}$

NB $|e^x - 1| = \begin{cases} e^x - 1 & x > 0 \\ -(e^x - 1) = -e^x + 1 & x < 0 \end{cases}$

$y = \pi/4$ as. orizz a $+\infty$

$\lim_{x \to -\infty} \operatorname{arctg}\left(\frac{e^x \to 0}{|e^x - 1| \to 1}\right) = \operatorname{arctg} 0 = 0$

$y = 0$ as. orizz a $-\infty$.

derivate

1

$x>0$ $\quad |e^x-1| = e^x-1 \quad f(x) = \operatorname{arctg}\left(\frac{e^x}{e^x-1}\right)$ -

$$f'(x) = \frac{1}{1+\left(\frac{e^x}{e^x-1}\right)^2} \cdot \frac{e^x\cdot(e^x-1) - e^x\cdot e^x}{(e^x-1)^2} = \frac{1}{\frac{(e^x-1)^2+e^{2x}}{\cancel{(e^x-1)^2}}} \cdot \frac{-e^x}{\cancel{(e^x-1)^2}} =$$

$$= \frac{-e^x}{(e^x-1)^2+e^{2x}} < 0 \quad \forall x>0$$

$x<0$ $\quad |e^x-1| = -(e^x-1) = -e^x+1 \quad f(x) = \operatorname{arctg}\left(\frac{e^x}{1-e^x}\right)$

$$f'(x) = \frac{1}{1+\left(\frac{e^x}{1-e^x}\right)^2} \cdot \frac{e^x(1-e^x) - e^x\cdot(-e^x)}{(1-e^x)^2} = \frac{1}{\frac{(1-e^x)^2+e^{2x}}{\cancel{(1-e^x)^2}}} \cdot \frac{e^x}{\cancel{(1-e^x)^2}} =$$

$$= \frac{e^x}{(e^x-1)^2+e^{2x}} > 0 \quad \forall x<0$$

$\boxed{x=0 \text{ pto angoloso}}$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} \frac{-e^x}{(e^x-1)^2+e^{2x}} = \frac{-1}{0+1} = -1$$

$$\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} \frac{e^x}{(1-e^x)^2+e^{2x}} = \frac{1}{0+1} = 1$$

f è derivabile $\forall x \neq 0$, $x=0$ è pto angoloso

$$f' \quad + \quad 0 \quad -$$
$$f \quad \nearrow \qquad \searrow$$

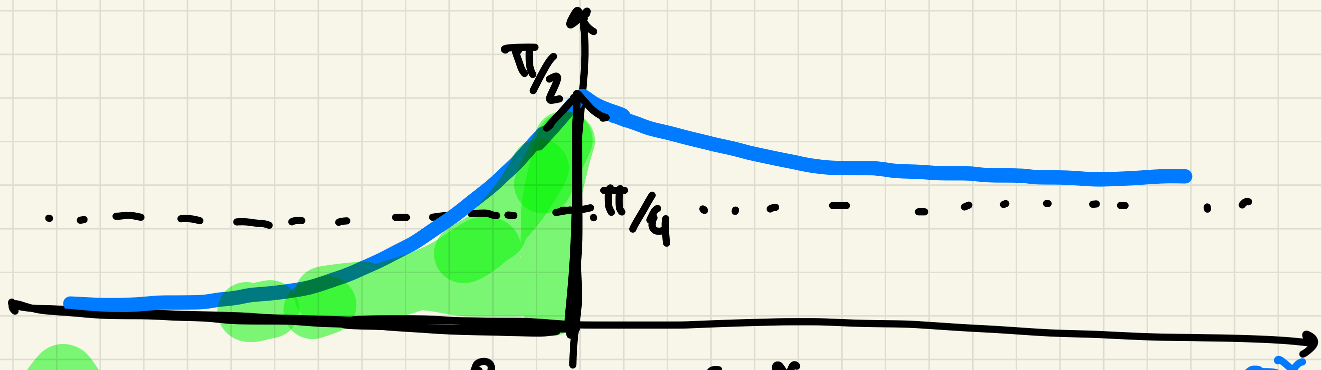
$x=0$ pto di massimo locale (e assoluto)

$\lim_{x\to+\infty} f(x) = \frac{\pi}{4}$ $\quad \lim_{x\to-\infty} f(x) = 0$

$f(0) = \frac{\pi}{2}$

$\max f = \frac{\pi}{2}$

$\inf f = 0$ $\quad$ f non ha MINIMO.



b) $\int_{-\infty}^{0} f(x)\,dx = \int_{-\infty}^{0} \operatorname{arctg}\left(\frac{e^x}{-e^x+1}\right) dx =$

$y = e^x$ $\quad x = \lg y$ $\quad dx = \frac{1}{y}\,dy$

$x=0 \to y=1$

$x \to -\infty \quad y \to 0$

$= \int_0^1 \operatorname{arctg}\left(\frac{y}{1-y}\right) \frac{1}{y}\,dy$

notiamo che

$$g(y) = \operatorname{arctg}\left(\frac{y}{1-y}\right) \cdot \frac{1}{y}$$ è continua in $[0,1)$

e inoltre $\lim_{y\to 0^+} (y) = \lim_{y\to 0^+} \operatorname{arctg}\left(\frac{y}{1-y}\right) \cdot \frac{1}{y} =$

$$= \lim_{y\to 0^+} \left(\frac{y}{1-y} + o(y)\right) \cdot \frac{1}{y} = \lim_{y\to 0} \cancel{y}\left(\frac{1}{1-y} + o(1)\right) \cdot \frac{1}{\cancel{y}} = 1$$

$$\operatorname{arctg}\left(\frac{y}{1-y}\right) = \frac{y}{1-y} + o(y) \quad y \to 0$$

$$\lim_{x\to 1^-} \operatorname{arctg}\underbrace{\left(\frac{y}{1-y}\right)}_{\to 0^+} \cdot \underbrace{\frac{1}{y}}_{\to 1} = \frac{\pi}{2}$$

posso estendere $g(y)$ a una funzione CONTINUA in $[0,1]$ ponendo $g(0) = 1$ $g(1) = \frac{\pi}{2}$

$$\Rightarrow \int_0^1 g(y)\,dy = \int_0^1 \operatorname{arctg}\left(\frac{y}{1-y}\right)\frac{1}{y}\,dy$$ è FINITO.

Es 2

2

$$f(x) = \frac{x}{\sqrt{(x^2-2)^3}} = \frac{x}{(x^2-2)^{3/2}}$$

a) D: $(x^2-2)^3 > 0 \Rightarrow x^2-2>0 \Rightarrow x>\sqrt{2},\ x<-\sqrt{2}$

$D = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$

$$f(-x) = \frac{-x}{\sqrt{(x^2-2)^3}} = -f(x)$$ f è dispari

$f(x) \geq 0 \iff x \geq 0$ (dato che $\sqrt{(x^2-2)^3} > 0$)

$$\lim_{x\to+\infty} \frac{x}{\sqrt{(x^2-2)^3}} = \lim_{x\to+\infty} \frac{x}{\sqrt{(x^2)^3}\cdot\sqrt{\left(1-\frac{2}{x^2}\right)^3}} = \lim_{x\to+\infty} \frac{\cancel{x}}{x^{3/2}\sqrt{\left(1-\frac{2}{x^2}\right)^3}} = 0$$

$$\lim_{x\to-\infty} \frac{x}{\sqrt{(x^2-2)^3}} = \lim_{x\to-\infty} \frac{x}{\sqrt{(x^2)^3}\sqrt{\left(1-\frac{2}{x^2}\right)^3}} = \lim_{x\to-\infty} \frac{x}{-x^3\sqrt{\left(1-\frac{2}{x^2}\right)^3}} = 0$$

$y=0$ as. orizz a $+\infty$ e a $-\infty$.

$$\lim_{x\to\sqrt{2}^+} \frac{x}{\underbrace{\sqrt{(x^2-2)^3}}_{\to 0^+}} = +\infty \qquad \lim_{x\to(-\sqrt{2})^-} f(x) = -\infty$$

$x=\sqrt{2}$ as. verticale destro

$x=-\sqrt{2}$ as. verticale sinistro.

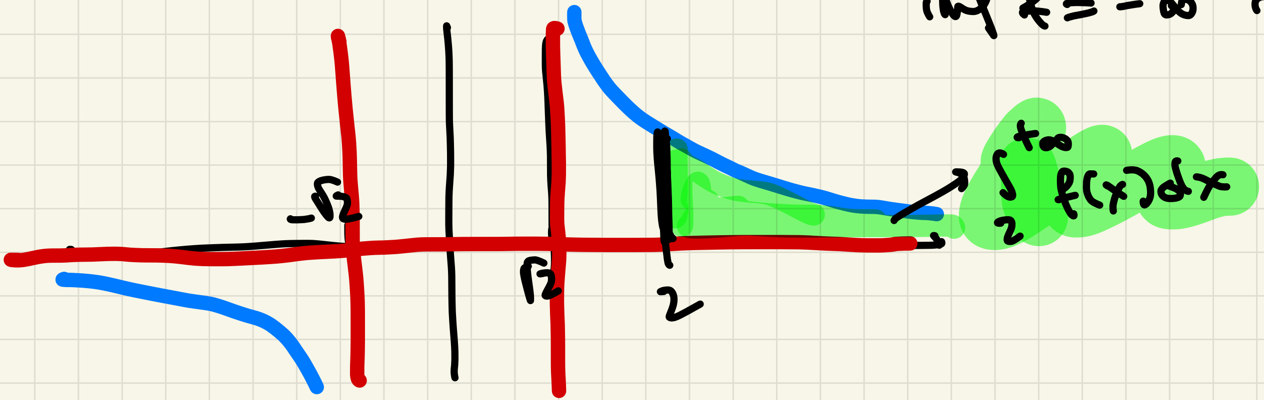
$$f'(x) = \frac{1\cdot(x^2-2)^{3/2} - x\cdot\frac{3}{\cancel{2}}(x^2-2)^{1/2}\cdot\cancel{2}x}{(x^2-2)^3} = \frac{(x^2-2)^{1/2}\left[(x^2-2)-3x^2\right]}{(x^2-2)^3} =$$

2

$$= \frac{-2x^2-2}{(x^2-2)^{5/2}} = \frac{-2(x^2+1)}{(x^2-2)^{5/2}} < 0 \quad \forall x \in D$$

f è strett. decrescente

$\inf f = -\infty \quad \sup f = +\infty.$



b) $\int \frac{x}{\sqrt{(x^2-2)^3}}\,dx \Rightarrow y = (x^2-2)$, $dy = 2x\,dx$ $= \frac{1}{2}\int \frac{1}{y^{3/2}}\,dy = \frac{1}{2}\left[\frac{1}{1-\frac{3}{2}}\,y^{1-\frac{3}{2}} + c\right]$

$$= \frac{1}{2}\left(-2y^{-1/2} + c\right) = -y^{-1/2} + c = -\frac{1}{\sqrt{y}} + c = -\frac{1}{\sqrt{x^2-2}} + c$$

2

$$\int_2^{+\infty} \frac{x}{(x^2-2)^{3/2}}\,dx = \lim_{M\to+\infty}\left[-\frac{1}{\sqrt{x^2-2}}\right]_2^M =$$

$$= \lim_{M\to-\infty} \underbrace{-\frac{1}{\sqrt{M^2-2}}}_{\to 0} - \left(-\frac{1}{\sqrt{4-2}}\right) = \frac{1}{\sqrt{2}}.$$

c) $$\int_2^{+\infty} \frac{x}{(x^2-2)^{\alpha/2}}\,dx$$

$$\frac{x}{(x^2-2)^{\alpha/2}} = \frac{x}{\left[x^2\left(1-\frac{2}{x^2}\right)\right]^{\frac{\alpha}{2}}} = \frac{x}{x^\alpha\left(1-\frac{2}{x^2}\right)^{\frac{\alpha}{2}}} \sim \frac{1}{x^{\alpha-1}}$$

per confronto asint l'integrale converge $\iff \alpha-1>1$

$\iff \underline{\alpha>2}$

Esercizio 3

(3)

$f(x) = \frac{\lg x}{x^3}$

a) D: $x>0$ D. $(0,+\infty)$ f non ha simmetrie

$\lim_{x\to 0^+} \frac{\lg x}{x^3} = \lim_{x\to 0^+} \underbrace{\lg x}_{-\infty} \cdot \underbrace{\frac{1}{x^3}}_{+\infty} = -\infty$ $x=0$ as. verticale destro.

$\lim_{x\to+\infty} \frac{\lg x}{x^3} = 0$ per confronto infiniti $y=0$ as. orizz a $+\infty$.

$f(x)\geq 0 \iff$ visto che $x>0$ nel dominio $x^3>0 \ \forall x\in D$

$\lg x \geq 0 \iff x\geq 1$

$$f'(x) = \frac{\frac{1}{x}\cdot x^3 - \lg x \cdot 3x^2}{x^6} = \frac{x^2(1-3\lg x)}{x^6} = \frac{1-3\lg x}{x^4}$$

f è derivabile $\forall x\in D$.

$f'(x)\geq 0 \iff 1-3\lg x\geq 0 \iff \lg x \leq \frac{1}{3} = \lg e^{1/3} \iff x\leq e^{\frac{1}{3}}$.

3

f' $\quad 0 \quad + \quad e^{1/3} \quad -$

$f \quad \nearrow \quad \searrow$

$\sup f = \max f = f(e^{1/3}) = \dfrac{\frac{1}{3}}{e} = \dfrac{1}{3e}$

$\inf f = -\infty.$

$x = e^{1/3}$ è pto di massimo locale e anche assoluto (visto che $f \to 0 \quad x \to +\infty$, $f \to -\infty \quad x \to 0^+$)

$$f''(x) = \frac{x^4\left(-3\frac{1}{x}\right) - 4x^3(1-3\lg x)}{x^8} = \frac{x^3[-3-4+12\lg x]}{x^8} =$$

$$= \frac{12\lg x - 7}{x^5}$$

f è derivabile 2 volte $\forall\, x \in D$

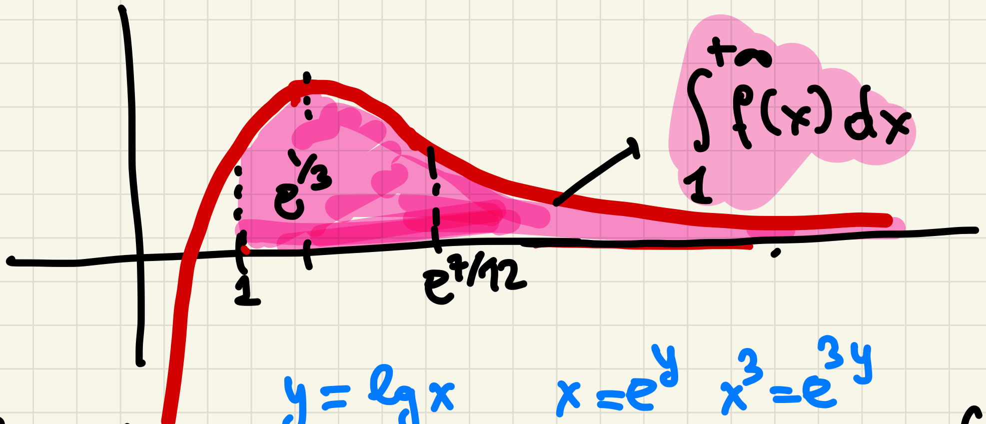
$f''(x) \geq 0 \iff 12\lg x - 7 \geq 0 \iff \lg x \geq \frac{7}{12} = \lg e^{7/12} \Rightarrow x \geq e^{7/12}$

f'' $\quad 0 \quad - \quad e^{7/12} \quad +$

$f \quad \frown \quad \smile$

$x = e^{7/12}$ pto di flesso

3



b) $\int \frac{\lg x}{x^3}\,dx =$ $\quad y = \lg x \quad x = e^y \quad x^3 = e^{3y} \quad dx = e^y\,dy$

$$= \int \frac{y}{e^{3y}} \cdot e^y \cdot dy =$$

$= \int y e^{-2y}\,dy =$ per parti: $f(y) = y \to f'(y) = 1$, $h(y) = e^{-2y} \to H(y) = -\frac{1}{2}e^{-2y}$

$$= y\left(-\frac{1}{2}e^{-2y}\right) - \int 1 \cdot \left(-\frac{1}{2}e^{-2y}\right)dy = -\frac{1}{2}ye^{-2y} + \frac{1}{2}\int e^{-2y}\,dy =$$

$$= -\frac{1}{2}ye^{-2y} + \frac{1}{2}\left(-\frac{1}{2}e^{-2y}\right) + c = -\frac{1}{2}\lg x\, e^{-2\lg x} - \frac{1}{4}e^{-2\lg x} =$$

$$= -\frac{1}{2}\lg x \cdot \frac{1}{x^2} - \frac{1}{4}\frac{1}{x^2}$$

$$e^{-2\lg x} = e^{\lg x^{-2}} = x^{-2} = \frac{1}{x^2}$$

3

$$\int_1^{+\infty} \frac{\lg x}{x^3}\,dx = \lim_{M\to+\infty} \left[-\frac{1}{2}\lg x \frac{1}{x^2} - \frac{1}{4}\frac{1}{x^2} \right]_1^M =$$

$$= \lim_{M\to+\infty} \underbrace{-\frac{1}{2}\frac{\lg M}{M^2}}_{\downarrow 0} - \underbrace{\frac{1}{4}\frac{1}{M^2}}_{\downarrow 0} + \frac{1}{2}\underbrace{\lg 1}_{=0}\frac{1}{1^2} + \frac{1}{4}\frac{1}{1^2} = \frac{1}{4}$$

c) $\displaystyle\int_1^{+\infty} \frac{\lg x}{x^\alpha}\,dx$

$\lg x = y \quad x = e^y \quad dx = e^y dy$

$x^\alpha = e^{\alpha y}$

$$\int \frac{\lg x}{x^\alpha}dx = \text{stesso cambio di prima} = \int y\, e^{(1-\alpha)y} dy = \text{per parti } f = y,\ h = e^{(1-\alpha)y} =$$

$$= y\cdot\frac{e^{(1-\alpha)y}}{1-\alpha} - \int 1\cdot\frac{1}{(1-\alpha)} e^{(1-\alpha)y} = \frac{y\,e^{(1-\alpha)y}}{1-\alpha} - \frac{1}{(1-\alpha)^2}e^{(1-\alpha)y} + c =$$

$$= \frac{\lg x\, e^{(1-\alpha)\lg x}}{1-\alpha} - \frac{1}{(1-\alpha)^2}e^{(1-\alpha)\lg x} + c = \frac{\lg x\cdot x^{1-\alpha}}{1-\alpha} - \frac{1}{(1-\alpha)^2}x^{1-\alpha}$$

$$\int_1^{+\infty}\frac{\lg x}{x^\alpha}dx = \lim_{M\to+\infty} \frac{\lg M\cdot M^{1-\alpha}}{1-\alpha} - \frac{M^{1-\alpha}}{(1-\alpha)^2} + \frac{1}{(1-\alpha)^2} < +\infty \iff 1-\alpha<0 \iff \alpha>1$$

4

Es 4

a) $$\lim_{x\to 0^+} \frac{3(1-\cos x)\sin^2 x}{x^\alpha \, tg\, x} = \lim_{x\to 0^+} \frac{3\cdot x^2\left(\frac{1}{2}+o(1)\right)\cdot x^2(1+o(1))}{x^\alpha \; x\cdot(1+o(1))} =$$

Taylor $1-\cos x = 1-1+\frac{x^2}{2}+o(x^2) = x^2\left(\frac{1}{2}+o(1)\right)$

$(\sin x)^2 = (x+o(x))^2 = x^2(1+o(1))^2$

$tg\, x = x+o(x) = x(1+o(1))$

$$= \lim_{x\to 0^+} x^{4-\alpha-1}\, \frac{3\cdot\left(\frac{1}{2}+o(1)\right)(1+o(1))^2}{(1+o(1))} = \begin{cases} \alpha=3 & = \frac{3}{2} \\ \alpha>3 \quad (3-\alpha<0) & = +\infty \\ \alpha<3 \quad (3-\alpha>0) & = 0 \end{cases}$$

$4-\alpha-1 = 3-\alpha$

b) $\int_0^1 f(x)\,dx$

DA QUANTO VISTO SOPRA

$$f(x) = x^{3-\alpha}\, \frac{3\cdot\left(\frac{1}{2}+o(1)\right)(1+o(1))^2}{1+o(1)} \sim \frac{1}{x^{\alpha-3}}$$

per confronto asintotico l'integrale tra 0 e 1 converge $\Longleftrightarrow$ $\alpha-3<1 \Longleftrightarrow \boxed{\alpha<4}$

c) $\alpha = 0$ $\quad \int_0^1 \frac{3(1-\cos x)\cdot(\sin x)^2}{\operatorname{tg} x}\,dx =$ $\qquad \operatorname{tg} x = \frac{\sin x}{\cos x}$ $\quad$ 4

$$= \int_0^1 \frac{3(1-\cos x)(\sin x)^{\cancel{2}}}{\frac{\cancel{\sin} x}{\cos x}}\,dx = \int_0^1 3(1-\cos x)\cos x \sin x\,dx$$

$y = \cos x$

$dy = -\sin x\,dx$

$x = 0 \to y = \cos 0 = 1$

$x = 1 \to y = \cos 1$

$$= \int_1^{\cos 1} 3(1-y)\cdot y(-1)\,dy = -\int_1^{\cos 1} 3(1-y)y\,dy = \int_{\cos 1}^1 3(1-y)y\,dy$$

$$= \int_{\cos 1}^1 (3y - 3y^2)\,dy = \left[\frac{3y^2}{2} - \frac{\cancel{3}y^3}{\cancel{3}}\right]_{\cos 1}^1 = \frac{3}{2} - 1 - \frac{3}{2}(\cos 1)^2 + (\cos 1)^3$$

Es 5

5

$$a_n = \frac{n^\alpha \left(\sin \frac{1}{n^2} - \operatorname{tg} \frac{1}{n^2}\right)}{4n - 2\operatorname{arctg} n}$$

a) Taylor $\sin \frac{1}{n^2} = \frac{1}{n^2} - \frac{1}{6}\left(\frac{1}{n^2}\right)^3 + o\left(\frac{1}{n^2}\right)^3$

$$\operatorname{tg} \frac{1}{n^2} = \frac{1}{n^2} + \frac{1}{3}\left(\frac{1}{n^2}\right)^3 + o\left(\frac{1}{n^2}\right)^3$$

$$\sin \frac{1}{n^2} - \operatorname{tg}\frac{1}{n^2} = \cancel{\frac{1}{n^2}} - \frac{1}{6}\frac{1}{n^6} + o\left(\frac{1}{n^6}\right) - \cancel{\frac{1}{n^2}} - \frac{1}{3}\frac{1}{n^6} + o\left(\frac{1}{n^6}\right) =$$

$$= -\frac{1}{2}\frac{1}{n^6} + o\left(\frac{1}{n^6}\right) = \frac{1}{n^6}\left(-\frac{1}{2} + o(1)\right)$$

$$4n - 2\operatorname{arctg} n = n\left[4 - 2\underbrace{\frac{\operatorname{arctg} n}{n}}_{\to 0}\right]$$

$\operatorname{arctg} n$ è LIMITATO per $n \to +\infty$ (NON USO POLINOMI a $+\infty$)

$$a_n = \frac{n^\alpha \cdot \frac{1}{n^6}\left(-\frac{1}{2} + o(1)\right)}{n\left[4 - 2\frac{\operatorname{arctg} n}{n}\right]} = \frac{1}{n^{7-\alpha}} \frac{\left(-\frac{1}{2} + o(1)\right)}{\left(4 - 2\frac{\operatorname{arctg} n}{n}\right)} \to \begin{cases} -\frac{1}{8} & \alpha = 7 \\ 0 & \alpha < 7 \ (7-\alpha > 0) \\ -\infty & \alpha > 7 \ (7-\alpha < 0) \end{cases}$$

b) Da quanto visto prima

$$|a_n| = \frac{1}{n^{7-\alpha}} \frac{\left|-\frac{1}{2}+o(1)\right|}{\left|4+\frac{2\arctan n}{n}\right|} \sim \frac{1}{n^{7-\alpha}}$$

per confronto asintotico la serie $\sum_{n=1}^{\infty} |a_n|$ converge

se $7-\alpha>1 \Rightarrow \alpha<6$

e diverge se $7-\alpha\leq 1$ $\alpha \geq 6$.

Es 6

6

$$f(x) = \frac{3 \sin x}{4(1-\cos x)^\alpha}$$

a) Taylor

$$\sin x = x + o(x) = x(1+o(1))$$

$$1-\cos x = 1 - 1 + \frac{x^2}{2} + o(x^2) = x^2\left(\frac{1}{2} + o(1)\right)$$

$$(1-\cos x)^\alpha = x^{2\alpha}\left(\frac{1}{2}+o(1)\right)^\alpha$$

$$\lim_{x\to 0^+} \frac{3\, x\,(1+o(1))}{4x^{2\alpha}\cdot\left(\frac{1}{2}+o(1)\right)^\alpha} = \lim_{x\to 0^+} \frac{1}{x^{2\alpha-1}} \frac{3(1+o(1))}{4\left(\frac{1}{2}+o(1)\right)^\alpha} =$$

$$= \begin{cases} 2\alpha-1=0 \quad \alpha=\frac{1}{2} & = \frac{3}{4}\frac{1}{\left(\frac{1}{2}\right)^{1/2}} = \frac{3}{4}\sqrt{2} \\ 2\alpha-1>0 \quad \alpha>\frac{1}{2} & = +\infty \\ 2\alpha-1<0 \quad \alpha<\frac{1}{2} & = 0 \end{cases}$$

b) da quanto visto prima $f(x) = \frac{1}{x^{2\alpha-1}}\frac{3}{4}\frac{(1+o(1))}{\left(\frac{1}{2}+o(1)\right)^\alpha} \sim \frac{1}{x^{2\alpha-1}}$

l'integrale tra 0 e 1 converge per confronto as. $\iff 2\alpha-1<1 \iff \alpha<1$

6

c) $\alpha = \frac{1}{2}$ $\quad \int_0^1 \frac{3 \sin x}{4(1-\cos x)^{1/2}} dx = \begin{matrix} y = 1-\cos x \\ dy = \sin x \, dx \end{matrix} =$

$x = 0 \rightarrow y = 1 - \cos 0 = 1 - 1 = 0$

$x = 1 \rightarrow y = 1 - \cos 1$

$$= \int_0^{1-\cos 1} \frac{3}{4} \frac{1}{y^{1/2}} dy = \frac{3}{4} \left[\frac{1}{1-\frac{1}{2}} y^{1-\frac{1}{2}} \right]_0^{1-\cos 1} = \frac{3}{4} \cdot \left[\frac{1}{2} (1-\cos 1)^{\frac{1}{2}} - 0 \right]$$

$$= \frac{3}{8} \sqrt{1-\cos 1} \, .$$

Es 7

7

$$\sum_{n=1}^{\infty} \left(\frac{x+2}{3}\right)^n \sin\left(\frac{1}{n}\right)$$

$0 < \frac{1}{n} \leq 1 < \frac{\pi}{2}$

$\sin \frac{1}{n} > 0$!

Converg. ASSOLUTA

$$|a_n| = \left|\frac{x+2}{3}\right|^n \sin\left(\frac{1}{n}\right)$$

criterio radice n-esima

$$\lim_n \sqrt[n]{\left|\frac{x+2}{3}\right|^n \sin\frac{1}{n}} = \lim_n \left|\frac{x+2}{3}\right| \sqrt[n]{\sin\frac{1}{n}} =$$

$$= \lim_n \left|\frac{x+2}{3}\right| \underbrace{e^{\frac{1}{n}\lg\left(\sin\frac{1}{n}\right)}}_{\downarrow \atop 1} = \left|\frac{x+2}{3}\right|$$

$$\frac{1}{n}\lg\left(\sin\frac{1}{n}\right) \sim \frac{1}{n}\lg\frac{1}{n} = -\frac{\lg n}{n} \to 0$$

$$\left|\frac{x+2}{3}\right| < 1 \Rightarrow |x+2| < 3 \Rightarrow \begin{cases} x+2 < 3 \\ x+2 > -3 \end{cases} \begin{cases} x < 1 \\ x > -5 \end{cases} \Rightarrow -5 < x < 1$$

$-5 < x < 1 \Rightarrow$ la serie converge ASSOLUTAMENTE e anche SEMPLICEMENTE

7

- $x>1$, $x<-5$ la serie diverge assolutam.

infatti $|a_n| = \left|\frac{x+2}{3}\right|^n \sin\frac{1}{n} \to +\infty$

$\left|\frac{x+2}{3}\right| > 1$

dato che $\lim_n |a_n| = +\infty \Rightarrow \lim a_n$ NON PUÒ ESSERE 0

$\Rightarrow$ la serie NON CONVERGE SEMPLICEMENTE

- $x = 1$ $\quad a_n = \left(\frac{1+2}{3}\right)^n \sin\frac{1}{n} = \sin\frac{1}{n} \geq 0 \quad |a_n| = a_n$

$\sin\frac{1}{n} \sim \frac{1}{n} \Rightarrow$ la serie DIVERGE sia assolutamente che semplicemente

- $x = -5$ $\quad a_n = \left(\frac{-5+2}{3}\right)^n \sin\left(\frac{1}{n}\right) = (-1)^n \sin\frac{1}{n}$.

$|a_n| = \sin\frac{1}{n} \Rightarrow$ la serie diverge assolutamente

ma $\left.\begin{array}{l}\sin\frac{1}{n} \to 0 \quad n\to+\infty \\ \sin\frac{1}{n+1} \leq \sin\frac{1}{n}\end{array}\right]$ per il criterio di Leibniz converge semplicemente

Es 8 $\sum_{n=1}^{\infty} \frac{n^{n\alpha}}{n!}$ (8)

criterio del rapporto

$$a_{n+1} = \frac{(n+1)^{(n+1)\alpha}}{(n+1)!} = \frac{(n+1)^{n\alpha}\,(n+1)^{\alpha}}{n!\,(n+1)}$$

$$\lim_n \frac{a_{n+1}}{a_n} = \lim_n a_{n+1} \cdot \frac{1}{a_n} = \lim_n \frac{(n+1)^{n\alpha}\,(n+1)^{\alpha}}{\cancel{n!}\,(n+1)} \cdot \frac{\cancel{n!}}{n^{n\alpha}} =$$

$$= \lim_n \left(\frac{n+1}{n}\right)^{n\alpha} (n+1)^{\alpha-1} =$$

$$= \lim_n \underbrace{\left(1+\frac{1}{n}\right)^{n\alpha}}_{\downarrow\, e^{\alpha}} \underbrace{(n+1)^{\alpha-1}}_{\downarrow\, \begin{cases} 0 & \alpha-1<0 \\ 1 & \alpha-1=0 \\ +\infty & \alpha-1>0 \end{cases}} = \begin{cases} (\alpha-1=0 \quad \alpha=1) = e^{1}\cdot 1 = e > 1 \\ (\alpha-1>0 \quad \alpha>1) = e^{\alpha}\cdot +\infty = +\infty > 1 \\ (\alpha-1<0 \quad \alpha<1) = e^{\alpha}\cdot 0 = 0 < 1 \end{cases}$$

la serie converge se $\alpha < 1$ e diverge se $\alpha \geq 1$

9

Es 9

$$f(x) = \frac{1}{x\,(2\lg^2 x - \lg x - 1)}$$

a) D: $x > 0$ $\quad 2\lg^2 x - \lg x - 1 \neq 0$ $\quad y = \lg x \quad 2y^2 - y - 1 \neq 0$

$$y = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \left< \begin{matrix} 1 \\ -\frac{1}{2} \end{matrix} \right.$$

$\lg x \neq 1 \quad \lg x \neq -\frac{1}{2}$

$\downarrow \qquad \downarrow$

$x \neq e \quad x \neq e^{-1/2}$

D: $(0, e^{-1/2}) \cup (e^{-1/2}, e) \cup (e, +\infty)$

f non è simmetrica

$f(x) \geq 0 \iff 2\lg^2 x - \lg x - 1 > 0 \iff x > e,\ x < e^{-1/2}$.

($x > 0$ nel dominio)

$$\lim_{x \to +\infty} \frac{1}{x(2\lg^2 x - \lg x - 1)} = 0$$ $y = 0$ as. orizz a $+\infty$

9

$$\lim_{x\to 0^+} \frac{1}{\underbrace{x\lg^2 x}_{\to 0^+}\,\underbrace{\left[2-\frac{1}{\lg x}-\frac{1}{\lg^2 x}\right]}_{\to 2}} = +\infty$$

$x=0$ as. verticale destro.

$$\lim_{x\to 0^+} x\lg^2 x = 0 \,!$$

$x=e$ as. verticale

$$\lim_{x\to e^-} f(x) = \underbrace{-\infty} \qquad \lim_{x\to e^+} f(x) = \underbrace{+\infty}$$

$x=e^{-1/2}$ as. verticale

$$\lim_{x\to (e^{-1/2})^-} f(x) = \underbrace{+\infty} \qquad \lim_{x\to (e^{-1/2})^+} f(x) = \underbrace{-\infty}$$

ricordarsi lo studio del segno!

$f(x)>0 \quad x>e$

$f(x)<0 \quad e^{-1/2}<x<e$

$f(x)>0 \quad x<e^{-1/2}$

$$f'(x) = \frac{-\left[1\cdot(2\lg^2 x-\lg x-1)+x\cdot\left[2\cdot 2\lg x\cdot\frac{1}{x}-\frac{1}{x}\right]\right]}{x^2\left[2\lg^2 x-\lg x-1\right]^2} =$$

$$= \frac{-\left[2\lg^2 x - \lg x - 1 + \cancel{x} \cdot \frac{1}{\cancel{x}}(4\lg x - 1)\right]}{x^2 (2\lg^2 x - \lg x - 1)^2} =$$

9

$$= \frac{-(2\lg^2 x + 3\lg x - 2)}{\boxed{x^2 (2\lg^2 x - \lg x - 1)^2}} \quad > 0$$

f è derivabile $\forall x \in D$

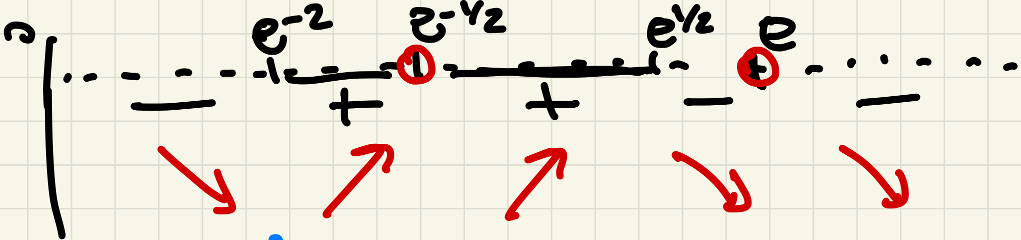
$$f'(x) \geq 0 \iff 2\lg^2 x + 3\lg x - 2 \leq 0 \qquad y = \lg x$$

$$2y^2 + 3y - 2 \leq 0$$

$$y = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} \left\langle \begin{array}{l} -2 \\ \frac{1}{2} \end{array} \right.$$

$$-2 \leq y \leq \frac{1}{2}$$

$$-2 \leq \lg x \leq \frac{1}{2} \iff e^{-2} \leq x \leq e^{\frac{1}{2}}$$



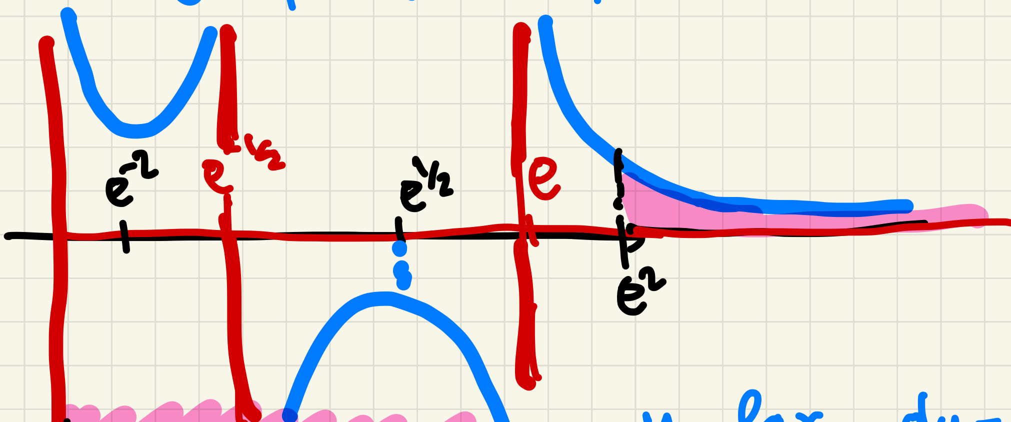
9

$x = e^{-1/2}$, $x = e$ NON STANNO in D !!

$x = e^{-2}$ pto di min locale
$x = e^{1/2}$ pto di max locale

NON ASSOLUTI

$\inf f = -\infty$
$\sup f = +\infty$



b) $\displaystyle\int_{e^2}^{+\infty} \frac{1}{x(2\lg^2 x - \lg x - 1)}\,dx = \int_2^{+\infty} \frac{1}{2y^2 - y - 1}\,dy$

$y = \lg x \qquad dy = \frac{1}{x}dx$
$x = e^2 \quad y = \lg e^2 = 2$
$x \to +\infty \quad y \to +\infty$

$2y^2 - y - 1 = 2(y-1)\left(y+\frac{1}{2}\right)$

fratti semplici $\dfrac{1}{2y^2-y-1} = \dfrac{1}{2}\left(\dfrac{A}{y-1} + \dfrac{B}{y+\frac{1}{2}}\right) = \dfrac{(A+B)y + \frac{1}{2}A - B}{2(y-1)(y+\frac{1}{2})}$

9

$\begin{cases} A+B=0 \\ \frac{1}{2}A - B = 1 \end{cases} \quad \begin{cases} -B = A \\ \frac{3}{2}A = 1 \end{cases} \quad B = -\frac{2}{3} \quad A = \frac{2}{3}$

$$\int_2^{+\infty} \frac{1}{2y^2-y-1}\,dy = \frac{1}{2}\left[\int_2^{+\infty} \frac{\frac{2}{3}}{y-1}\,dy + \int_2^{+\infty} -\frac{2}{3}\,\frac{1}{y+\frac{1}{2}}\,dy\right] =$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left[\int_2^{+\infty} \frac{1}{y-1}\,dy - \int_2^{+\infty} \frac{1}{y+\frac{1}{2}}\,dy\right] =$$

$$= \frac{1}{3}\lim_{M\to+\infty}\left[\lg\frac{|y-1|}{|y+\frac{1}{2}|}\right]_2^M = \frac{1}{3}\lim_{M\to+\infty} \lg\underbrace{\left(\frac{M-1}{M+\frac{1}{2}}\right)}_{1} - \lg\left|\frac{2-1}{2+\frac{1}{2}}\right| =$$

$\lg 1 = 0$

$$= -\frac{1}{3}\lg\frac{1}{\left(\frac{5}{2}\right)} = \frac{1}{3}\lg\frac{5}{2}$$

10

Es 10 $\quad f(x) = x e^{-x^2}$

a) $D = \mathbb{R}$ $\quad f(-x) = -x e^{-(-x)^2} = -x e^{-x^2} = -f(x)$ $\quad$ f DISPARI

$$\lim_{x\to+\infty} x \underbrace{e^{-x^2}}_{\downarrow 0} = \lim_{x\to+\infty} \frac{x}{e^{x^2}} = 0$$ per confronto infiniti

$$\lim_{x\to-\infty} x e^{-x^2} = \lim_{x\to-\infty} \frac{x}{e^{x^2}} = 0$$

$y=0$ as. orizzontale a $+\infty$ e a $-\infty$.

$f(x) \geq 0 \iff x \geq 0$ (visto che $e^{-x^2} > 0$)

$$f'(x) = 1\, e^{-x^2} + x \cdot (-2x) e^{-x^2} = (1-2x^2)\, e^{-x^2}$$

f è derivabile $\forall x \in \mathbb{R}$.

$$f'(x) \geq 0 \iff 1-2x^2 \geq 0 \iff 2x^2-1 \leq 0 \iff -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$-$ $\quad -1/\sqrt{2} \quad$ $+$ $\quad 1/\sqrt{2} \quad$ $-$

$\searrow \qquad \nearrow \qquad \searrow$

$x = -\frac{1}{\sqrt{2}}$ pto di min locale e assoluto

$x = \frac{1}{\sqrt{2}}$ pto di max locale e assoluto.

10

$$\max f = f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} e^{-1/2}$$

$$\min f = f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} e^{-1/2}.$$

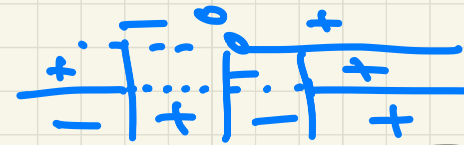
$$f''(x) = -2\cdot 2x\, e^{-x^2} + (1-2x^2)(-2x)e^{-x^2} = e^{-x^2}\left[-4x-2x+4x^3\right] =$$

$$= 2x(2x^2-3)\,e^{-x^2}.$$

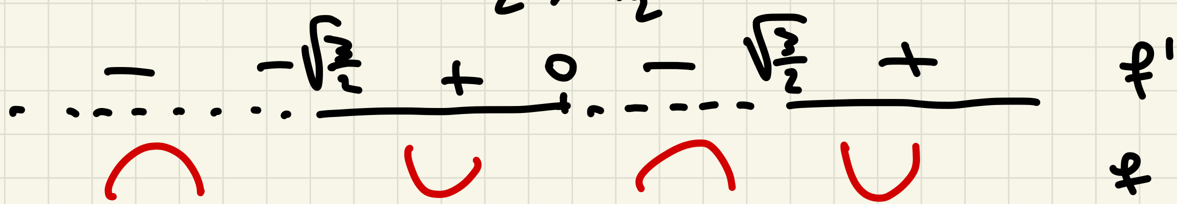
$$f''(x) \geq 0 \iff x\cdot(2x^2-3) \geq 0$$

$x \geq 0$

$2x^2-3 \geq 0 \qquad x \geq \sqrt{\frac{3}{2}} \quad x \leq -\sqrt{\frac{3}{2}}$

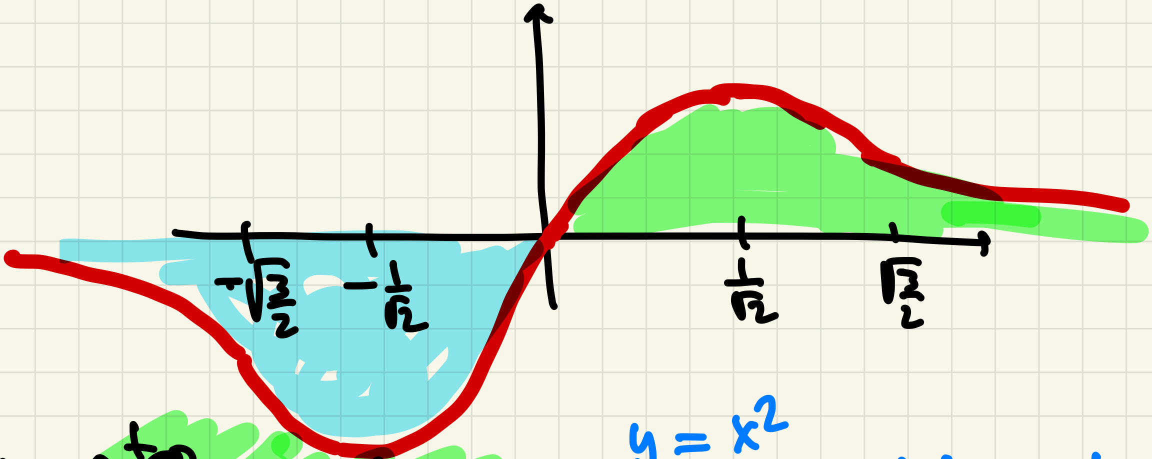


$$f''(x) \geq 0 \iff x \geq \sqrt{\frac{3}{2}}\ ,\ -\sqrt{\frac{3}{2}} \leq x \leq 0$$



$x = -\sqrt{\frac{3}{2}}\ ,\ 0\ ,\ \sqrt{\frac{3}{2}}$ pti di flesso.

10



b) $\displaystyle\int_0^{+\infty} x e^{-x^2} dx =$

$y = x^2$

$dy = 2x\,dx \qquad \frac{1}{2}dy = x\,dx$

$x = 0 \quad y = 0^2 \qquad x \to +\infty \quad y \to +\infty$

$\displaystyle = \int_0^{+\infty} \frac{1}{2} e^{-y} dy =$

$\displaystyle = \frac{1}{2} \lim_{M\to+\infty} \left[-e^{-y}\right]_0^M = \frac{1}{2} \lim_{M\to+\infty} \underbrace{-e^{-M}}_{\to 0} + 1 = \frac{1}{2}$

$\displaystyle\int_{-\infty}^{+\infty} x e^{-x^2} dx = 0$ perché la funzione è DISPARI!

c) $\lim_{x\to 0} \dfrac{x e^{-x^2} - 6\sin x + 5x}{\sinh x + \sin x - 2x} = \lim_{x\to 0} \dfrac{\cancel{x^5}\left(\frac{9}{20} + o(1)\right)}{\cancel{x^5}\left(\frac{1}{60} + o(1)\right)} = \frac{9}{20}\cdot 60 = 27$

(b)

Taylor

$$e^{-x^2} = 1 - x^2 + \frac{1}{2}(-x^2)^2 + o(-x^2)^2 = 1 - x^2 + \frac{1}{2}x^4 + o(x^4)$$

$$\sin x = x - \frac{x^3}{6} + \frac{1}{5!}x^5 + o(x^5)$$

$$\sinh x = x + \frac{x^3}{6} + \frac{1}{5!}x^5 + o(x^5)$$

N: $x\left(1 - x^2 + \frac{1}{2}x^4 + o(x^4)\right) - 6\left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) + 5x =$

$$= \cancel{x} - \cancel{x^3} + \frac{1}{2}x^5 + o(x^5) - \cancel{6x} + \cancel{x^3} - \frac{x^5}{20} + o(x^5) + \cancel{5x} =$$

$$= \frac{9}{20}x^5 + o(x^5) = x^5\left(\frac{9}{20} + o(1)\right)$$

D: $\cancel{x} - \cancel{\frac{x^3}{6}} + \frac{x^5}{120} + o(x^5) + \cancel{x} + \cancel{\frac{x^3}{6}} + \frac{x^5}{120} + o(x^5) - \cancel{2x} = \frac{x^5}{60} + o(x^5) = x^5\left(\frac{1}{60} + o(1)\right)$

Es 11

11

$e^x - \alpha x - 1 = 0 \Rightarrow$ le soluzioni di questa equaz sono $x \in \mathbb{R}$ tali che $f(x) = 0$ con $f(x) = e^x - \alpha x - 1$

$x = 0$ è sempre soluzione

NB $\boxed{f(0) = e^0 - \alpha \cdot 0 - 1 = 0}$

studio $f(x) = e^x - \alpha x - 1$

D: $\mathbb{R}$

$\lim_{x \to +\infty} e^x - \alpha x - 1 = +\infty$ $\forall \alpha$ perché e^x domina

$\lim_{x \to -\infty} \underset{\downarrow 0}{e^x} \underbrace{- \alpha \underset{\downarrow -\infty}{x}} - 1 = \alpha \cdot (+\infty) = \begin{cases} +\infty & \alpha > 0 \\ -\infty & \alpha < 0 \end{cases}$

se $\alpha = 0 \Rightarrow \lim_{x \to -\infty} \underset{\to 0}{e^x} \cancel{- \alpha x} - 1 = -1$

$f'(x) = e^x - \alpha$

$\boxed{e^x - \alpha > 0 \text{ se } \alpha \leq 0 \ \forall x}$

$\boxed{e^x - \alpha > 0 \Rightarrow e^x > \alpha = e^{\lg \alpha} \text{ se } \alpha > 0 \quad x > \lg \alpha}$

11

$\alpha \le 0 \Rightarrow f$ è strett crescente

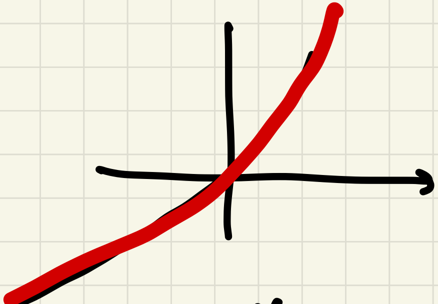
$\alpha > 0 \Rightarrow f$ ha pto di minimo locale e assoluto in $x = \lg \alpha$

$\min f = e^{\lg \alpha} - \alpha \lg \alpha - 1$

$= \alpha - \alpha \lg \alpha - 1 < 0$

$x = \lg \alpha = \begin{cases} 0 & \text{se } \alpha = 1 \\ > 0 & \text{se } \alpha > 1 \\ < 0 & \text{se } \alpha < 1 \end{cases}$

$\alpha < 0$



$f(x) = 0$ ha una sola soluz $x = 0$

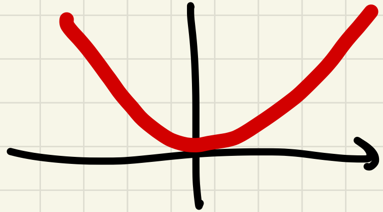
$\alpha = 0$ $\quad f(x) = e^x - 1$

$f(x) = 0$ ha una sola soluzione $x = 0$.

$0 < \alpha < 1$

$f(x) = 0$ ha 2 soluzioni $x = 0$ e $x = c < \lg \alpha < 0$

$\alpha = 1$



$f(x)=0$ ha 1 sola soluz $x=0$.

$\alpha > 1$

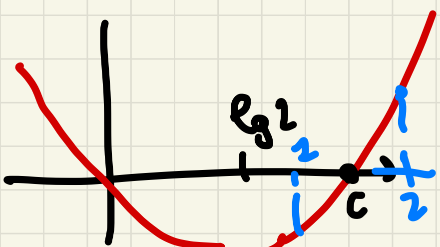
$f(x)=0$ ha 2 soluzioni $x=0$ e $x=c>\lg\alpha>0$.

Se $\alpha = 2$ $f(x)=0$ ha 2 soluzioni $x=0$ e $x=c>\lg 2$

$f(1) = e^1 - 2\cdot 1 - 1 = e-3 < 0$

$f(2) = e^2 - 2\cdot 2 - 1 = e^2 - 5 > 0$

$x=c$ è compresa tra 1 e 2



$f\left(\frac{3}{2}\right) = e^{3/2} - \frac{3}{2}\cdot 2 - 1 = e^{3/2} - 4 > 0$

$x=c$ è compresa tra 1 e $\frac{3}{2}$.

(con il metodo di NEWTON posso essere più precisa →)

11

$a_1 = \frac{3}{2}$ $\quad a_2 = a_1 - \frac{f(a_1)}{f'(a_1)} = \frac{3}{2} - \frac{e^{\frac{3}{2}} - 4}{e^{3/2} - 2} =$

$$= \frac{2e^{3/2} - 2}{2(e^{3/2} - 2)} = \frac{e^{3/2} - 1}{e^{3/2} - 2} = 1,4029.$$

c è compresa tra 1 e 1,403

$a_3 = a_2 - \frac{f(a_2)}{f'(a_2)}$ ... $\quad a_n = a_{n-1} - \frac{f(a_{n-1})}{f'(a_{n-1})}$

$$\boxed{c = \lim_{n \to +\infty} a_n}$$

12

Es 12

$$f(x) = (x+1)\lg\left(\frac{x+1}{x}\right)$$

a) D: $\frac{x+1}{x} > 0$

$x > -1$

$x > 0$

$-1 \quad 0$

$+ \quad - \quad +$

D: $x > 0$
$x < -1$

$D = (-\infty, -1) \cup (0, +\infty)$

f non è simmetrica

$$\lim_{x\to+\infty} (x+1)\lg\left(\frac{x+1}{x}\right) = \lim_{x\to+\infty} (x+1)\lg\left(1+\frac{1}{x}\right) = \quad y = \frac{1}{x} =$$

$$= \lim_{y\to 0^+} \left(\frac{1}{y}+1\right)\lg(1+y) = \lim_{y\to 0^+} (y+1) \cdot \underbrace{\frac{\lg(1+y)}{y}}_{\downarrow 1} = 1$$

limite a $-\infty$ si fa uguale $\lim_{x\to-\infty} f(x) = 1$

$y = 1$ è as. orizzontale a $+\infty$ e a $-\infty$.

$$\lim_{x\to 0^+} \underbrace{(x+1)}_{\downarrow\, 1} \log\underbrace{\left(\frac{x+1}{x}\right)}_{\downarrow\, +\infty} = +\infty$$

$x=0$ as. verticale destro **(12)**

$$\lim_{x\to(-1)^-} \underbrace{(x+1)}_{\downarrow\, 0} \log\underbrace{\left(\frac{x+1}{x}\right)}_{\downarrow\, -\infty} \quad \left(\frac{x+1}{x}\to 0^+\right) = \lim_{x\to(-1)^-} \frac{\log\left(\frac{x+1}{x}\right)}{\frac{1}{x+1}} = \text{Hopital} =$$

$$= \lim_{x\to(-1)^-} \frac{\frac{1}{\frac{x+1}{\cancel{x}}}\cdot\frac{x-(x+1)}{x^{\cancel{2}}}}{-\left(\frac{1}{x+1}\right)^2} = \lim_{x\to(-1)^-} \frac{-\frac{1}{x+1}}{-\frac{1}{(x+1)^2}} = 0$$

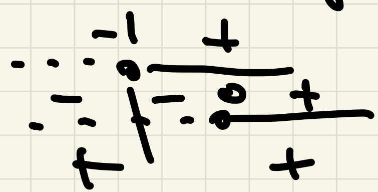
posso estendere f in (-1) ponendo $f(-1)=0$.

$$f(x)\geq 0 \iff (x+1)\log\left(\frac{x+1}{x}\right)\geq 0 \iff$$

$x+1\geq 0 \Rightarrow x\geq -1$

$\log\frac{x+1}{x}\geq 0 \qquad \frac{x+1}{x}\geq 1 \qquad \frac{x+1-x}{x}\geq 0$

$x>0$



$f(x)\geq 0 \qquad x>0$

$\qquad\qquad\quad x<-1$

(quindi $\forall x\in D$.)

$$f'(x) = 1 \cdot \lg\left(\frac{x+1}{x}\right) + \cancel{(x+1)} \cdot \frac{1}{\frac{\cancel{(x+1)}}{\cancel{x}}} \cdot \frac{1 \cdot x - 1(x+1)}{x^{\cancel{2}}} =$$

$$= \lg\left(\frac{x+1}{x}\right) + \frac{x-x-1}{x} = \lg\left(\frac{x+1}{x}\right) - \frac{1}{x}$$

f è derivabile $\forall\, x>0 \quad x<-1$

$$\lim_{x\to -1^-} f'(x) = \lim_{x\to(-1)^-} \lg\underbrace{\left(\frac{x+1}{x}\right)}_{\to 0^+} \underbrace{\left(-\frac{1}{x}\right)}_{\to 1^+} = -\infty$$

$x=-1$ pto a tangente verticale.

$$f''(x) = \frac{1}{\frac{x+1}{x}} \cdot \frac{x-(1+x)}{x^2} - \left(-\frac{1}{x^2}\right) = -\frac{1}{x(x+1)} + \frac{1}{x^2} = \frac{-x+x+1}{x^2(x+1)} =$$

$$= \frac{1}{x^2(x+1)}$$

$f''(x) \geq 0 \iff x+1>0 \quad x>-1$

f'' — -1 //// 0 +

f è convessa per $x>0$ $\Rightarrow$ non ha pti di max locale

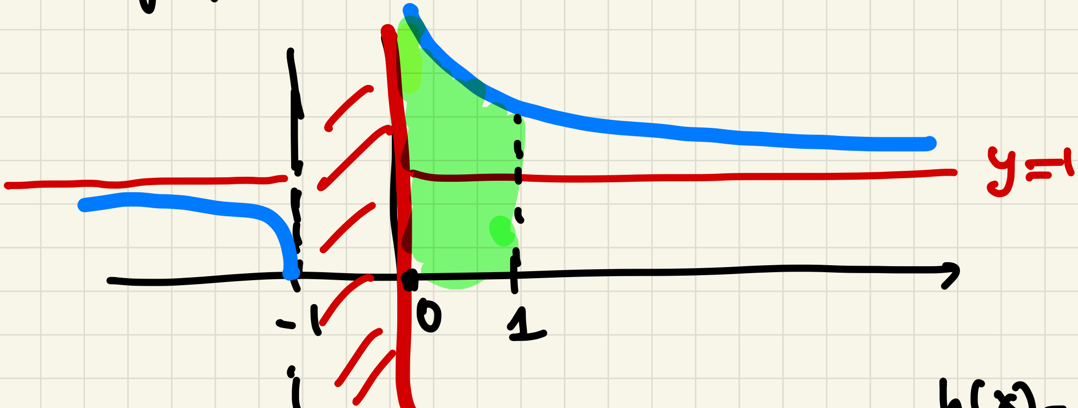
f è concava per $x<-1$ $\Rightarrow$ non ha pti di min locale

$f(x) \geq 0 \quad \forall x \in D$ e $f(-1)=0 \Rightarrow x=-1$ pto di min assoluto

12

$\min f = 0$

$\sup f = +\infty \Longrightarrow f$ non ha massimo



$f'(x) = \frac{1}{\frac{x+1}{\cancel{x}}} \cdot \frac{x-(x+1)}{x^{\cancel{2}}}$

b) $\int (x+1) \lg\left(\frac{x+1}{x}\right) dx =$ per parti

$h(x) = x+1 \qquad H(x) = \frac{x^2}{2}+x$

$f(x) = \lg\left(\frac{x+1}{x}\right) \qquad f'(x) = -\frac{1}{x(x+1)}$

$$= \left(\frac{x^2}{2}+x\right)\lg\left(\frac{x+1}{x}\right) - \int\left(\frac{x^2}{2}+x\right)\cdot\left(-\frac{1}{x(x+1)}\right)dx =$$

$$= x\left(\frac{x}{2}+1\right)\lg\left(\frac{x+1}{x}\right) + \int \frac{x+2}{2(x+1)}\,dx =$$

$\frac{x+2}{x+1} = 1 + \frac{1}{x+1}$

12

$$= x\left(\frac{x}{2}+1\right)\lg\left(\frac{x+1}{x}\right) + \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{1}{x+1}\,dx =$$

$$= x\left(\frac{x}{2}+1\right)\lg\left(\frac{x+1}{x}\right) + \frac{1}{2}x + \frac{1}{2}\lg|x+1| + C.$$

$$\int_0^1 (x+1)\lg\left(\frac{x+1}{x}\right)dx = \lim_{\varepsilon\to 0^+}\left[x\left(\frac{x}{2}+1\right)\lg\left(\frac{x+1}{x}\right) + \frac{1}{2}x + \frac{1}{2}\lg|x+1|\right]_\varepsilon^1 =$$

$$= \lim_{\varepsilon\to 0^+} 1\left(\frac{1}{2}+1\right)\lg\left(\frac{1+1}{1}\right) + \frac{1}{2}\cdot 1 + \frac{1}{2}\lg 2 - \underbrace{\varepsilon\left(\overset{\to 1}{\frac{\varepsilon}{2}+1}\right)\lg\left(\frac{\varepsilon+1}{\varepsilon}\right)}_{\downarrow 0} - \underbrace{\frac{1}{2}\varepsilon}_{\downarrow 0} +$$

$$- \underbrace{\frac{1}{2}\lg(\varepsilon+1)}_{\downarrow 0} =$$

$$\varepsilon\cdot\lg\left(\frac{\varepsilon+1}{\varepsilon}\right) = \underbrace{\varepsilon\lg(\varepsilon+1)}_{\downarrow 0} - \underbrace{\varepsilon\lg\varepsilon}_{\downarrow 0}$$

$$= \frac{3}{2}\lg 2 + \frac{1}{2} + \frac{1}{2}\lg 2 = 2\lg 2 + \frac{1}{2}.$$

Es 13

13

1) $f(0) = e^0 + \frac{1}{0-1} = 1 - 1 = 0$

$f'(x) = e^x - \frac{1}{(x-1)^2}$ $\quad f'(0) = e^0 - \frac{1}{(0-1)^2} = 1 - 1 = 0$

$f''(x) = e^x - \frac{(-2)}{(x-1)^3} = e^x + \frac{2}{(x-1)^3}$ $\quad f''(0) = e^0 + \frac{2}{(-1)^3} = 1-2 = -1$

$f'(0) = 0 \quad f''(0) = -1 < 0$ $\quad x=0$ pto di massimo locale.

b) $\lim_{x \to 0} \frac{e^x + \frac{1}{x-1}}{1 - \cos x^\alpha}$

$f(x) = e^x + \frac{1}{x-1}$

TEOREMA DI TAYLOR

$f(x) = f(0) + f'(0)x + \frac{1}{2} f''(0) x^2 + o(x^2)$

$e^x + \frac{1}{x-1} = 0 + 0 \cdot x + \frac{1}{2}(-1)x^2 + o(x^2) =$

$= x^2\left(-\frac{1}{2} + o(1)\right)$

$$1-\cos x^{\alpha} = 1-\left(1-\frac{x^{2\alpha}}{2}+o(x^{2\alpha})\right) = x^{2\alpha}\left(\frac{1}{2}+o(1)\right)$$

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$$\lim_{x\to 0} \frac{x^2\left(-\frac{1}{2}+o(1)\right)}{x^{2\alpha}\left(\frac{1}{2}+o(1)\right)} = \lim_{x\to 0} x^{2-2\alpha}\,\frac{\left(-\frac{1}{2}+o(1)\right)}{\left(\frac{1}{2}+o(1)\right)} = \lim_{x\to 0} x^{2(1-\alpha)}\,\frac{\left(-\frac{1}{2}+o(1)\right)}{\left(\frac{1}{2}+o(1)\right)}$$

$$= \begin{cases} \alpha=1 & = -1 \\ \alpha>1 \ (2-2\alpha<0) & = -\infty \\ \alpha<1 \ (2-2\alpha>0) & = 0 \end{cases}$$